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Proof of the Lower Bound Formula for the Probability of Correct Binomial Subset Selection with the Levin-Robbins-Leu Sequential Elimination and Recruitment Procedure in the Case b=2, c=4.

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Cheng-Shiun Leu and Bruce Levin

Abstract

We prove the validity of a lower bound for the probability of correct subset selection for choosing the best b=2 binomial populations out of c=4. The procedure considered combines sequential elimination of inferior populations and simultaneous sequential recruitment of superior populations. This extends the results of Leu and Levin (2004) to this new procedure.

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PROOF OF THE LOWER BOUND FORMULA FOR THE PROBABILITY OF CORRECT BINOMIAL SUBSET SELECTION WITH THE LEVIN-ROBBINS-LEU SEQUENTIAL ELIMINATION AND RECRUITMENT PROCEDURE IN THE CASE $b = 2$, $c = 4$.

BY

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1. Introduction.

In 2004, Leu and Levin introduced a sequential identification procedure for selecting the best subset of b binomial populations from c candidates with sequential elimination of inferior populations (which we henceforth think of as coins). We also introduced a related procedure that sequentially recruits superior coins to the subset of b best coins, which we called a sequential recruitment procedure. The published version of these procedures will appear as Leu and Levin (2007) which is available online (see reference). The purpose of this report is to supplement Technical Report #91 by Leu and Levin (2004) in which we proved the validity of a lower bound formula for the probability of correct selection for the procedure with elimination of inferior coins (call it $\mathcal{E}$) in the special case $b = 2$, $c = 4$. As shown in Leu and Levin (2007) the same formula applies to the sequential recruitment procedure (call it $\mathcal{R}$). In Leu and Levin (2007a), we introduce a new procedure for the same problem with both sequential elimination of inferior coins *and* recruitment of superior coins (call it $\mathcal{E}/\mathcal{R}$). In this report, we extend the lower bound result to procedure $\mathcal{E}/\mathcal{R}$ still in the special case $b = 2$, $c = 4$. We assume the reader is familiar with the above reports.

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2. Proof of the Theorem.

The theorem we are trying to prove is actually a corollary of the following.

Theorem: For any integer $r \geq 1$ and any configuration $\mathbf{d}$ of $c = 4$ coins of b-depth $\delta \leq r$,

$$\begin{aligned} & P_r^{(C)}[12|N_\delta^{(C)} = N^{(C)}(\mathbf{u}) \text{ for some } \mathbf{u} \in S_{\mathbf{d}}^{(C)}] \\ \geq\ & (w_1w_2)^r/\{(w_1w_2)^r + (w_1w_3)^r + (w_2w_3)^r + (w_1w_4)^r + (w_2w_4)^r + (w_3w_4)^r\} \\ =\ & 1/\{1 + w_{13}^{-r} + w_{23}^{-r} + w_{14}^{-r} + w_{24}^{-r} + w_{13}^{-r}w_{24}^{-r})\} = L_+(2,4) \end{aligned}$$

where $P_r^{(C)}[12|N_\delta^{(C)} = N^{(C)}(\mathbf{u})$ for some $\mathbf{u} \in S_{\mathbf{d}}^{(C)}]$ is the probability of selecting coins 1 and 2 as the best two coins at the end of procedure $\mathcal{E}/\mathcal{R}$ given that, at or before the time of first elimination and/or recruitment, the first time the sample path reaches any configuration with b-depth δ, it does so at one of the configurations in $S_{\mathbf{d}}$. The probability of correct selection for $b = 2$, $c = 4$ follows from the Theorem by taking $\mathbf{d} = (0000)$ of b-depth 0.

For the case $b = 1$, procedure $\mathcal{E}/\mathcal{R}$ is in fact equivalent to procedure $\mathcal{E}$, and Leu and Levin (1999) prove that for the inequality $P_r^{(C)}[1|N_\delta^{(C)} = N^{(C)}(\mathbf{u})$ for some $\mathbf{u} \in S_{\mathbf{d}}^{(C)}] \geq L_+(1,c)$ to hold for any $\mathbf{d}$ of b-depth δ, it suffices to show $P_r^{(C)}[1|N_r^{(C)} = N^{(C)}(\mathbf{u})$ for some $\mathbf{u} \in S_{\mathbf{a}}^{(C)}] \geq L_+(1,c)$ for any configuration $\mathbf{a}$ of b-depth r at the time of first elimination. We refer the reader to Leu and Levin (1999) for details. A similar argument holds here, where now $b = 2$ and $\mathbf{a}$ is of the form $(r+s, r, r-t, 0)$ with $s \geq 0$,

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$r \geq t \geq 0$, and $s+t \leq r$. Notice that by considering $\mathbf{a}$ to be of the form $(r+s, r, r-t, 0)$ we seem to rule out the possibility of a "recruitment only" event (the first time either elimination or recruitment occurs). For recruitment only event $\mathbf{a}$ should be of the form $(r+s, t, s, 0)$ with $r > t \geq s \geq 0$. To see why it suffices to consider elimination-only events, we argue as follows. We note that the $\mathcal{E}/\mathcal{R}$ procedure satisfies an invariance property: for any sample path leading to a selection of coins i_1 and i_2 and rejection of coins i_3 and i_4, there is a corresponding sample path that leads to a selection of coins i_3 and i_4 and rejection of coins i_1 and i_2: simply interchange all heads with tails and tails with heads (the parity transformation). Thus the set of four coins are separated into two subsets of size $b = 2$ identically irrespective of which binary outcome we label as heads or tails. Thus the conditional probability of correct selection in any sequence leading for the first time to a recruitment-only event is the same as the conditional probability of correct selection in the transformed problem (with coins of transformed probability $q_c, ..., q_1$). But after parity transformation, a recruitment-only event of the form $\mathbf{a} = (r+s, t, s, 0)$ becomes an elimination-only event of the form $(r+s, r, r.t, 0)$. Since we will prove that the conditional probability of correct selection with procedure $\mathcal{E}/\mathcal{R}$ given passage through an elimination-only event for the first time satisfies the lower bound, it follows that the same is true given a recruitment-only event.

If recruitment and elimination happen at the same time at or before the procedure

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terminates, $\mathbf{a}$ is still of the form $(r+s, r, r-t, 0)$ with $0 \leq t \leq r$ and $s+t=r$. Therefore, in all cases without loss of generality, we may take $\mathbf{a}$ of the form $(r+s, r, r-t, 0)$ at the time of first elimination and/or recruitment in the following discussion.

At the time of first elimination and/or recruitment, there are four cases to consider for a future correct selection:

Case 1: elimination and recruitment happen at the same time and the procedure terminates (i.e., we eliminate two bad coins and, at the same time, recruit two good coins).

Case 2: elimination and recruitment happen at the same time but the procedure does not terminate (i.e., we eliminate one bad coin and, at the same time, recruit one good coin).

Case 3: elimination only (i.e., we eliminate one bad coin and do not recruit any coin).

Case 4: recruitment only (i.e., we recruit one good coin and do not eliminate any coin).

For Case 1, if the time of first elimination/recruitment occurs at the time that the procedure terminates, $\mathbf{a}$ is of the form $(r+s, r, r-t, 0)$ with $s=0$ and $t=r$. The Theorem has been proven to be true in this case — see Leu and Levin (2004, pp. 7-8).

For Case 2, $\mathbf{a}$ is then of the form $(r+s, r, s, 0)$ with $s>0$. Let $G_{\mathbf{a}} = \{(r+s, r, s, 0), (r, r+s, s, 0), (r+s, r, 0, s), (r, r+s, 0, s)\}$. For any transposition (ij) and any subset of configurations E, define $(ij)E = \{(ij)\mathbf{e} | \mathbf{e} \in E\}$ where $(ij)\mathbf{e}$ is the configuration with the ith and j^{th} component of $\mathbf{e}$ transposed. The reader may check that

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$G_{\mathbf{a}} = \{\mathbf{a}, (12)\mathbf{a}, (34)\mathbf{a}, (12)(34)\mathbf{a}\}$ and that $S_{\mathbf{a}}$ can be expressed as the union of $G_{\mathbf{a}}$, $(13)G_{\mathbf{a}}$, $(23)G_{\mathbf{a}}$, $(14)G_{\mathbf{a}}$, $(24)G_{\mathbf{a}}$, and either $(13)(24)G_{\mathbf{a}}$ or $(14)(23)G_{\mathbf{a}}$. In fact, it will be most convenient to use one or the other as follows: $(13)(24)\{\mathbf{a}, (12)(34)\mathbf{a}\} \cup (14)(23)\{(12)\mathbf{a}, (34)\mathbf{a}\}$.

Now by definition

$$
\begin{aligned}
& P[12|N_r^{(C)} = N^{(C)}(\mathbf{u}) \text{ for some } \mathbf{u} \in S_{\mathbf{a}}^{(C)}] \\
= \; & \frac{1}{\sum\limits_{\mathbf{u}\in S_{\mathbf{a}}} P[N_r^{(C)} = N^{(C)}(\mathbf{u})]} \times \\
& \sum_{\mathbf{u}\in S_{\mathbf{a}}} P[N_r^{(C)} = N^{(C)}(\mathbf{u})] \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}(\mathbf{u})]. \qquad (1)
\end{aligned}
$$

When elimination occurs, if coin 1 or coin 2 has been eliminated, i.e. the sample path passes through any configuration $\mathbf{u}$ with u_1 or u_2 equal to 0, we have $P_r^{(C)}[12|N_r^{(C)} = N^{(C)}(\mathbf{u})] = 0$. Similarly, when recruitment occurs, if coin 3 or coin 4 have been recruited, i.e. the sample path passes through any configuration $\mathbf{u}$ with u_3 or u_4 equal to $r+s$, we have $P_r^{(C)}[12|N_r^{(C)} = N^{(C)}(\mathbf{u})] = 0$. Thus, (1) can be rewritten as

$$
\begin{aligned}
& \Big(\frac{1}{\sum\limits_{\mathbf{u}\in S_{\mathbf{a}}} P[N_r^{(C)} = N^{(C)}(\mathbf{u})]}\Big) \times \\
& \Big(\sum_{i=2,3} P[N_r^{(C)} = N^{(C)}((i3)\mathbf{a})] \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}((i3)\mathbf{a})] \\
+ & \sum_{i=1,3} P[N_r^{(C)} = N^{(C)}((i3)(12)\mathbf{a})] \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}((i3)(12)\mathbf{a})] \\
+ & \sum_{i=2,4} P[N_r^{(C)} = N^{(C)}((i4)(34)\mathbf{a})] \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}((i4)(34)\mathbf{a})] \\
+ & \sum_{i=1,4} P[N_r^{(C)} = N^{(C)}((i4)(12)(34)\mathbf{a})] \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}((i4)(12)(34)\mathbf{a})]\Big)
\end{aligned}
$$

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and this expression is equivalent to (2) below by definition.

$$
\begin{aligned}
& \Big(\frac{1}{\sum\limits_{\mathbf{u}\in S_{\mathbf{a}}} P[N_r^{(C)} = N^{(C)}(\mathbf{u})]}\Big) \times \\
& [(\sum_{i=2,3} P[N_r^{(C)} = N^{(C)}((i3)\mathbf{a})]) \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}((i3)\mathbf{a}) \text{ for some } i = 2,3] \\
+ \; & (\sum_{i=1,3} P[N_r^{(C)} = N^{(C)}((i3)(12)\mathbf{a})]) \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}((i3)(12)\mathbf{a}) \text{ for some } i = 1,3] \\
+ \; & (\sum_{i=2,4} P[N_r^{(C)} = N^{(C)}((i4)(34)\mathbf{a})]) \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}((i4)(34)\mathbf{a}) \text{ for some } i = 2,4] \\
+ \; & (\sum_{i=1,4} P[N_r^{(C)} = N^{(C)}((i4)(12)(34)\mathbf{a})]) \cdot P_r^{(C)}[12|N_r^{(C)} = N^{(C)}((i4)(12)(34)\mathbf{a}) \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for some } i = 1,4])] \qquad (2)
\end{aligned}
$$

We use this expression because after eliminating one coin and, at the same time, recruiting another coin, the "reduced" game with the remaining two coins continues in one of the four symmetric subsets of configurations deriving from $\{(i3)\mathbf{a} : i = 2,3\}$, $\{(i3)(12)\mathbf{a} : i = 1,3\}$, $\{(i4)(34)\mathbf{a} : i = 2,4\}$, and $\{(i4)(12)(34)\mathbf{a} : i = 1,4\}$. Note that in the reduced game, the procedure is equivalent to Wald's SPRT; therefore we have that (2) is equal to

$$
\begin{aligned}
& \Big(\frac{1}{\sum\limits_{\mathbf{u}\in S_{\mathbf{a}}} P[N_r^{(C)} = N^{(C)}(\mathbf{u})]}\Big) \cdot [(\sum_{i=2,3} P[N_r^{(C)} = N^{(C)}((i3)\mathbf{a})]) \cdot \frac{1}{1+w_{23}^{-r}} \\
+ \; & (\sum_{i=1,3} P[N_r^{(C)} = N^{(C)}((i3)(12)\mathbf{a})]) \cdot \frac{1}{1+w_{13}^{-r}} \\
+ \; & (\sum_{i=2,4} P[N_r^{(C)} = N^{(C)}((i4)(34)\mathbf{a})]) \cdot \frac{1}{1+w_{24}^{-r}} \\
+ \; & (\sum_{i=1,4} P[N_r^{(C)} = N^{(C)}((i4)(12)(34)\mathbf{a})]) \cdot \frac{1}{1+w_{14}^{-r}}]. \qquad (3)
\end{aligned}
$$

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Using Lemma 1 of Leu and Levin (2004) p.5), (3) equals

$$
\begin{aligned}
& \Big(\frac{1}{\sum\limits_{\mathbf{u}\in S_{\mathbf{a}}} P[N_r^{(C)} = N^{(C)}(\mathbf{u})]}\Big) \cdot [P[N_r^{(C)} = N^{(C)}(\mathbf{a})] \cdot \frac{1 + w_{23}^{-(r-s)}}{1 + w_{23}^{-r}} \\
+ \quad & P[N_r^{(C)} = N^{(C)}(\mathbf{a})] \cdot w_{12}^{-s} \cdot \frac{1 + w_{13}^{-(r-s)}}{1 + w_{13}^{-r}} \\
+ \quad & P[N_r^{(C)} = N^{(C)}(\mathbf{a})] \cdot w_{34}^{-s} \cdot \frac{1 + w_{24}^{-(r-s)}}{1 + w_{24}^{-r}} \\
+ \quad & P[N_r^{(C)} = N^{(C)}(\mathbf{a})] \cdot w_{12}^{-s} w_{34}^{-s} \cdot \frac{1 + w_{14}^{-(r-s)}}{1 + w_{14}^{-r}}]. \qquad (4)
\end{aligned}
$$

To complete the proof, it suffices to show that (4) is greater than or equal to

$$
\begin{aligned}
& \Big(\frac{1}{\sum\limits_{\mathbf{u}\in S_{\mathbf{a}}} P[N_r^{(C)} = N^{(C)}(\mathbf{u})]}\Big) \cdot P[N_r^{(C)} = N^{(C)}(\mathbf{a})] \times \\
& [\frac{1 + w_{13}^{-r} + w_{23}^{-(r-s)} + w_{14}^{-(r+s)} + w_{24}^{-r} + w_{13}^{-r} w_{24}^{-r}}{1 + w_{13}^{-r} + w_{23}^{-r} + w_{14}^{-r} + w_{24}^{-r} + w_{13}^{-r} w_{24}^{-r}} \\
+ \quad & w_{12}^{-s} \cdot \frac{1 + w_{13}^{-(r-s)} + w_{23}^{-r} + w_{14}^{-r} + w_{24}^{-(r+s)} + w_{14}^{-r} w_{23}^{-r}}{1 + w_{13}^{-r} + w_{23}^{-r} + w_{14}^{-r} + w_{24}^{-r} + w_{14}^{-r} w_{23}^{-r}} \\
+ \quad & w_{34}^{-s} \cdot \frac{1 + w_{14}^{-r} + w_{24}^{-(r-s)} + w_{13}^{-(r+s)} + w_{23}^{-r} + w_{14}^{-r} w_{23}^{-r}}{1 + w_{13}^{-r} + w_{23}^{-r} + w_{14}^{-r} + w_{24}^{-r} + w_{14}^{-r} w_{23}^{-r}} \\
+ \quad & w_{12}^{-s} w_{34}^{-s} \cdot \frac{1 + w_{14}^{-(r-s)} + w_{24}^{-r} + w_{13}^{-r} + w_{23}^{-(r+s)} + w_{13}^{-r} w_{24}^{-r}}{1 + w_{13}^{-r} + w_{23}^{-r} + w_{14}^{-r} + w_{24}^{-r} + w_{13}^{-r} w_{24}^{-r}}], \qquad (5)
\end{aligned}
$$

since, again by Lemma 1, (5) is exactly equal to the lower bound formula, $L_+(2,4) = 1/(1 + w_{13}^{-r} + w_{23}^{-r} + w_{14}^{-r} + w_{24}^{-r} + w_{13}^{-r} w_{24}^{-r})$. Note in (5) we freely use the identity $w_{13}^{-r} w_{24}^{-r} = w_{14}^{-r} w_{23}^{-r}$.

To demonstrate (4) $\geq$ (5), it is equivalent to show that $(I) + (II) + (III) + (IV) \geq 0$,

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where

$$
\begin{aligned}
(I) &= \frac{1+w_{23}^{-(r-s)}}{1+w_{23}^{-r}} - \frac{1+w_{13}^{-r}+w_{23}^{-(r-s)}+w_{14}^{-(r+s)}+w_{24}^{-r}+w_{13}^{-r}w_{24}^{-r}}{1+w_{13}^{-r}+w_{23}^{-r}+w_{14}^{-r}+w_{24}^{-r}+w_{13}^{-r}w_{24}^{-r}}, \\
(II) &= w_{12}^{-s}\cdot\frac{1+w_{13}^{-(r-s)}}{1+w_{13}^{-r}} \\
&\quad -w_{12}^{-s}\cdot\frac{1+w_{13}^{-(r-s)}+w_{23}^{-r}+w_{14}^{-r}+w_{24}^{-(r+s)}+w_{14}^{-r}w_{23}^{-r}}{1+w_{13}^{-r}+w_{23}^{-r}+w_{14}^{-r}+w_{24}^{-r}+w_{14}^{-r}w_{23}^{-r}}, \\
(III) &= w_{34}^{-s}\cdot\frac{1+w_{24}^{-(r-s)}}{1+w_{24}^{-r}} \\
&\quad -w_{34}^{-s}\cdot\frac{1+w_{14}^{-r}+w_{24}^{-(r-s)}+w_{13}^{-(r+s)}+w_{23}^{-r}+w_{14}^{-r}w_{23}^{-r}}{1+w_{13}^{-r}+w_{23}^{-r}+w_{14}^{-r}+w_{24}^{-r}+w_{14}^{-r}w_{23}^{-r}}, \text{ and} \\
(IV) &= w_{12}^{-s}w_{34}^{-s}\cdot\frac{1+w_{14}^{-(r-s)}}{1+w_{14}^{-r}} \\
&\quad -w_{12}^{-s}w_{34}^{-s}\cdot\frac{1+w_{14}^{-(r-s)}+w_{24}^{-r}+w_{13}^{-r}+w_{23}^{-(r+s)}+w_{13}^{-r}w_{24}^{-r}}{1+w_{13}^{-r}+w_{23}^{-r}+w_{14}^{-r}+w_{24}^{-r}+w_{13}^{-r}w_{24}^{-r}}.
\end{aligned}
$$

Although it suffices to show $(I)+(II)+(III)+(IV) \geq 0$, we can actually demonstrate that (I) through (IV) are all non-negative. We prove $(I) \geq 0$ as follows. Since $(1+w_{23}^{-(r-s)})/(1+w_{23}^{-r}) \geq 1$ for all $s > 0$, it is clear that

$$
\begin{aligned}
& \frac{1+w_{23}^{-(r-s)}}{1+w_{23}^{-r}} \\
\geq\ & \frac{1+w_{23}^{-(r-s)}+w_{13}^{-r}+w_{14}^{-(r+s)}+w_{24}^{-r}+w_{13}^{-r}w_{24}^{-r}}{1+w_{23}^{-r}+w_{13}^{-r}+w_{14}^{-(r+s)}+w_{24}^{-r}+w_{13}^{-r}w_{24}^{-r}} \\
\geq\ & \frac{1+w_{23}^{-(r-s)}+w_{13}^{-r}+w_{14}^{-(r+s)}+w_{24}^{-r}+w_{13}^{-r}w_{24}^{-r}}{1+w_{23}^{-r}+w_{13}^{-r}+w_{14}^{-r}+w_{24}^{-r}+w_{13}^{-r}w_{24}^{-r}}.
\end{aligned}
$$

This thus implies $(I) \geq 0$. The other inequalities $(II) \geq 0$, $(III) \geq 0$, and $(IV) \geq 0$ are handled similarly. This completes the proof of the Theorem for Case 2.

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For Cases 3 and 4, we can use exactly the same proof as in Leu and Levin (2004) for procedure $\mathcal{E}$ in the case $s+t<r$. This is because a certain inequality was proven in Leu and Levin (2004) — that equation (4) at p.11 for the reduced $\mathcal{E}$ game is greater than or equal to the lower bound formula. But the analogous equation (4) for the reduced $\mathcal{E}/\mathcal{R}$ game is exactly the same.

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