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Lachenbruch's Method for Determining the Sample Size Required for Testing Interactions: How It Compares to nQuery Advisor and O'Brien's SAS UnifyPow.

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Abstract

Lachenbruch (1988) proposed a simple method based on the use of orthogonal contrasts to determine the sample size or power for testing main effects and interactions, and uses the normal distribution instead of the non-central F distribution. This method can be used for factorial designs of various size. The example illustrated in this paper considers a 2 x 2 factorial design. This paper will determine both sample size and power of a particular study design with anticipated (assumed) means for each cell of the 2 x 2 factorial design. Lachenbruch's method will be compared to nQuery Advisor 6.0 (2005) and UnifyPow, a macro for the SAS System (O'Brien, 1998), O'Brien, RG and Muller, KE (1993).

Introduction

Lachenbruch (1988) proposed a simple method based on the use of orthogonal contrasts to determine the sample size or power for testing main effects and interactions, and uses the normal distribution instead of the non-central F distribution. This method can be used for factorial designs of various size. The example illustrated in this paper considers a 2 x 2 factorial design. This paper will determine both sample size and power of a particular study design with anticipated (assumed) means for each cell of the 2 x 2 factorial design. Lachenbruch's method will be compared to nQuery Advisor 6.0 (2005) and UnifyPow, a macro for the SAS System (O'Brien, 1998), O'Brien, RG and Muller, KE (1993).

Method

Consider the following 2 x 2 factorial design:

	a	**A**
b	μ_{ba}	μ_{bA}
B	μ_{Ba}	μ_{BA}

2 factors with two levels:

Factor 1 with levels a and A [columns].
Factor 2 with levels b and B [rows].

2 main effects and 1 interaction:

Factor 1 ---------------- main effect
Factor 2 ---------------- main effect
Factor 1 x Factor 2 ---- interaction

The contrast for the interaction effect is (**1, -1, -1, 1**) where the means are ordered as $(\mu_{ba}, \mu_{bA}, \mu_{Ba}, \mu_{BA})$.

If one assumes normally distributed observations (or means that are asymptotically normal), the distribution of this contrast will be normal with **mean M** and **variance V** where

$$\mathbf{M} = \mu_{ba} - \mu_{bA} - \mu_{Ba} + \mu_{BA}$$

and

$$\mathbf{V} = \sigma^2(4/n),$$

where one assumes the same number of observations, n, per cell.

Under the null hypothesis of no interaction, **M = 0**.

For any value of M (not 0), one can find a sample size from the usual normal formula:

$$n = (z_{1-\beta} + z_{1-\alpha/2})^2 4\sigma^2/M^2 .$$

In general, the alternative hypothesis has the form $\mathbf{M} \neq \mathbf{0}$.

Thus, a two-sided test of the contrast is appropriate.

Refer to Appendix A for a listing of Z values for N(0,1).

Example

	a	A
b	$\mu_{ba} = 0.0$	$\mu_{bA} = 0.5$
B	$\mu_{Ba} = 1.0$	$\mu_{BA} = 3.0$

Here, the interest is in testing the null hypothesis of no interaction, $\mathbf{M = 0}$.
The alternative hypothesis is $\mathbf{M \neq 0}$.

Significance level is α **= 0.05, two-sided**. Thus, $z_{1-\alpha/2} = 1.960$.

Power to detect the alternative hypothesis: **1-** β **= 0.80**. Thus, $z_{1-\beta} = 0.842$.

Note the means ($\mu_{ba}, \mu_{bA}, \mu_{Ba}, \mu_{BA}$) are anticipated values of the true cell means.

$\sigma^2 = 1.0$, which is based on the literature or pilot data, etc.

Using $M = \mu_{ba} - \mu_{bA} - \mu_{Ba} + \mu_{BA}$, one finds $M = 0.0 - 0.5 - 1.0 + 3.0 = 1.5$.

Using $n = (z_{1-\beta} + z_{1-\alpha/2})^2 4\sigma^2/M^2$, one finds $n = (0.842 + 1.960)^2 4(1)/(1.5)^2 = 13.9 = 14$.

Thus, 14 observations per cell are required, giving 14 x 4 = 56 total observations required in order to test the null hypothesis of no interaction, $\mathbf{M = 0}$.

To determine the sample size required for testing the main effects, the following contrasts are used:

Columns --- Factor 1 (levels a, A): **(1, -1, 1, -1)**.

Rows --------Factor 2 (levels b, B)**: (1, 1, -1, -1)**.

$M_{Factor\ 1} = 0.0 - 0.5 + 1 - 3 = -2.5$

and

$M_{Factor\ 2} = 0.0 + 0.5 - 1 - 3 = -3.5$.

The required number of observations to test each main effect are:

$n_{Factor\ 1} = (0.842 + 1.960)^2 4(1)/(-2.5)^2 = 5.02 = 6$; **thus 6 x 4 = 24 total observations.**

and

$n_{Factor\ 2} = (0.842 + 1.960)^2 4(1)/(-3.5)^2 = 2.56 = 3$; **thus 3 x 4 = 12 total observations**.

Thus, the interaction test is the one, which controls the sample size, not either of the main effect tests.
Note: This is not always the case, just for this particular example.

How Lachenbruch's Method Compares to nQuery and O'Brien's SAS Module UnifyPow.sas.

Power Fixed, Determine the Required Sample Size.

Method	n per cell	Total N	Power
Lachenbruch			
Factor 1 [col]	6	24	80
Factor 2 [row]	3	12	80
Interaction	14	56	80
nQuery			
Factor 1 [col]	6	24	80
Factor 2 [row]	4	16	80
Interaction	15	60	80
UnifyPow			
Factor 1 [col]	6	24	80
Factor 2 [row]	4	16	80
Interaction	15	60	80

Sample Size n Per Cell Fixed, Determine the Power.

Method	n per cell	Total N	Power
Lachenbruch			
Factor 1 [col]	6	24	80
Factor 2 [row]	3	12	80
Interaction	14	56	80
nQuery			
Factor 1 [col]	6	24	82
Factor 2 [row]	3	12	75
Interaction	14	56	78
UnifyPow			
Factor 1 [col]	6	24	83
Factor 2 [row]	3	12	76
Interaction	14	56	79

Note: nQuery and O'Brien's SAS UnifyPow use the non-central F distribution; Lachenbruch's method uses the normal distribution.

Conclusion

Lachenbruch's method is simple to use. The computations are easily done by hand; no sophisticated software is required. The results are not as accurate as those based on the non-central F distribution but are close enough to get approximate sample sizes.

References

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O'Brien, RG (1998), A Tour of UnifyPow: A SAS Module/Macro for Sample-Size Analysis, **Proceedings of the 23rd SAS Users Group International Conference**, Cary NC: SAS Institute, pp. 1346-1355.

O'Brien, RG and Muller, KE (1993), Unified Power Analysis for t Tests through Multivariate Hypotheses, in Edwards, EK. (Ed.), **Applied Analysis of Variance in the Behavioral Sciences**, New York: Marcel Dekker, pp. 297-344.

Appendix A. Z values for N(0,1)

Error Level	1-tail	2-tail
0.500	0.000	0.674
0.400	0.253	0.842
0.300	0.524	1.036
0.200	0.842	1.282
0.100	1.282	1.645
0.050	1.645	1.960
0.025	1.960	2.248
0.010	2.326	2.576
0.005	2.576	2.813