Fitting ACE Structural Equation Models to Case-Control Family Data

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Abstract

Investigators interested in whether a disease aggregates in families often collect case-control family data, which consist of disease status and covariate information for families selected via case or control probands. Here, we focus on the use of case-control family data to investigate the relative contributions to the disease of additive genetic effects (A), shared family environment (C), and unique environment (E). To this end, we describe a ACE model for binary family data and then introduce an approach to fitting the model to case-control family data. The structural equation model, which has been described previously, combines a general-family extension of the classic ACE twin model with a (possibly covariate-specific) liability-threshold model for binary outcomes. Our likelihood-based approach to fitting involves conditioning on the proband’s disease status, as well as setting prevalence equal to a pre-specified value that can be estimated from the data themselves if necessary. Simulation experiments suggest that our approach to fitting yields approximately unbiased estimates of the A, C, and E variance components, provided that certain commonly-made assumptions hold. These assumptions include: the usual assumptions for the classic ACE and liability-threshold models; assumptions about shared family environment for relative pairs; and assumptions about the case-control family sampling, including single ascertainment. When our approach is used to fit the ACE model to Austrian case-control family data on depression, the resulting estimate of heritability is very similar to those from previous analyses of twin data.
1 INTRODUCTION

To study familial aggregation of a disease, investigators often sample families through case and control probands selected based on their disease status. This design is particularly efficient for rarer diseases. We refer to the resulting data, which consist of disease status and covariate information for the probands and their relatives, as case-control family data. The data are typically used to investigate the presence and magnitude of familial aggregation, a necessary first step for establishing the presence of genetic or shared family environmental effects [Thomas, 2004]. However, the pattern of familial aggregation in the data can sometimes also be used to investigate the relative contributions of additive genetic, shared family environmental, and unique environmental effects. This latter goal is our focus here.

Numerous existing methods can be applied to the case-control family data to first establish the presence and magnitude of familial aggregation. These methods fall into two categories: regression methods and multivariate methods. Regression methods involve fitting regression models that treat either the probands’ disease statuses or the relatives’ disease statuses as the outcomes. A popular example of the latter approach, which is to be preferred [Laird and Cuenco, 2003], is using logistic regression to model the probability of each relative’s disease status as a function of proband disease status [Khoury et al., 1993; Hudson et al., 2001; Laird and Cuenco, 2003], with generalized estimating equations used to handle the dependence between relatives belonging to the same family [Liang and Pulver, 1996]. Multivariate methods, which in certain special cases can be effectively equivalent to regression methods, involve fitting multivariate models for the probands’ and relatives’ disease statuses, with some accommodation made for ascertainment based on proband
After establishing that familial aggregation is present, investigators may be interested in using the pattern of familial aggregation in the data to disentangle the relative contributions of latent genetic and environmental effects to the disease. Some of the aforementioned models for familial aggregation can be adapted for this purpose by modifying them to incorporate genealogical information in the form of relationship-pair-specific family aggregation parameters [Zhao et al., 1998]. Here, however, we focus on models specifically developed to investigate the contributions of genetic and environmental effects. These models can be formulated, equivalently, as path analysis models or as variance components models, and, further, the variance components models can be formulated and fitted within either a structural equation modeling framework or a mixed-effects modeling framework [McArdle and Prescott, 2005]. The best known of these models is the classic ACE model for twin data, which is used to investigate genetic and environmental effects on complex (i.e., non-Mendelian) outcomes. In its variance components formulation, the ACE model is used to partition the variance in a continuous outcome of interest into variance due to Additive genetic effects, to Common (or shared) family environment, and to unique (or individual) Environment. For a binary outcome of interest like disease status, the classic ACE model is combined with a liability-threshold model [Falconer 1965] such that the variance in an unobserved continuous ‘liability’ variable hypothesized to underlie the binary variable is partitioned into variance due to A, C, and E.

The classic ACE model for twin data can be extended to make it appropriate for more general family data. Numerous papers have proposed ACE or ACE-type models for (single outcome) binary family data, as well as approaches to fitting these models. Several papers
written in the 1970s and 1980s [Morton and MacLean, 1974; Lalouel et al., 1983; Curnow and Smith, 1975] described ACE-type models for binary family data within the structural equation modeling framework; these models were developed to investigate the effects of a major gene locus, in addition to the effects of A, C, and E. More recent papers have described ACE or ACE-type models for binary family data within the mixed effects modeling framework, fitted using a likelihood-based approach [Rabe-Hesketh et al., 2008; Noh et al., 2006; McArdle and Prescott, 2005; Pawitan et al., 2004; Guo and Wang, 2002; Pfeiffer et al., 2001] or a Bayesian approach [Burton et al., 1999].

There are numerous papers that address the issue of ascertainment in pedigree analysis in general [Thompson, 1986, Chapter 8], as well as several papers, including some of the ones cited above, that address the use of non-randomly-ascertained binary family data with ACE-type models in particular. For example, Morton and MacLean [1974] (and its extension [Lalouel and Morton, 1981]) and Pfeiffer et al. [2001] describe how to adjust the likelihood for their models for certain types of non-random ascertainment. In addition, Bowden et al. [2007], Burton [2003], Glidden and Liang [2002] (including comments by Epstein [2002] and Burton [2002] and rejoinder by Glidden [2002]), Epstein [2002], and Burton et al. [2001] describe and evaluate various methods of adjusting for ascertainment when fitting genetic variance components models. Most of these papers address the scenario where all families (or sibships) with $k$ or more affected individuals are ascertained, and most conclude that valid estimates of the variance components for the general population can be obtained from the resulting data if appropriate correction for ascertainment is made and if the underlying genetic variance components model is correct. However, to our knowledge, no paper specifically addresses the use of ACE-type models with singly-ascertained case-
control family data (i.e., case-control family data ascertained from a population sufficiently large relative to the number of probands sampled and relative to the sizes of the families comprising the population to assume that no family is selected via more than one proband).

To fill this void, we describe a likelihood-based approach to fitting an ACE model to singly-ascertained case-control family data. The model we fit, which has been proposed previously, combines a general-family extension of the classic ACE model for twin data with a (possibly covariate-specific) liability-threshold model for binary outcomes. Our ascertainment-adjusted approach to fitting takes its cue from Neale and Maes [2004], who note that, although using proband-ascertained data to fit variance components models can increase power to detect effects for rarer diseases, doing so requires “good information on the base rate in the population studied.” Thus, our approach to fitting involves not only adjusting the likelihood contribution of each family by conditioning on the disease status of the proband, but also fixing prevalence to a (good!) pre-specified value during fitting. In particular, for situations where the prevalence of the disease is not already well-known for the population of interest, we fix prevalence to estimates obtained from the case-control data themselves [Javaras et al., 2009].

In Section 2, we describe the case-control family data, including some necessary notation. In Section 3, we describe the ACE model for binary family data. In Section 4, we introduce our approach to fitting the model and also discuss likelihood ratios tests for comparing the fit of the full ACE model to reduced variants. In Section 5, we describe the results of simulation experiments designed to investigate the performance of our fitting approach. In Section 6, we employ our approach to fit an ACE model to the actual data from a case-control family study of major depressive disorder (MDD) in Austria. Finally,
in Section 7, we discuss the advantages and limitations of our approach, focusing on the assumptions that must hold for it to yield valid estimates.

2 CASE-CONTROL FAMILY DATA

The sample consists of case-control sampled probands and their relatives, who we refer to collectively as subjects. There are $n_a$ ‘affected’ probands (e.g., MDD present) and $n_u$ ‘unaffected’ probands (e.g., MDD absent). For the sake of convenience, the families with affected probands will be indexed by $i = 1, \ldots, n_a$ and the families with unaffected probands by $i = n_a + 1, \ldots, n_a + n_u$. Family $i$ has $n_i$ sampled subjects; thus, the total number of sampled subjects is $n = \sum_{i=1}^{n_a+n_u} n_i$. A sampled subject from family $i$ is indexed by $ij$, where $j = 1, \ldots, n_i$ and $j = 1$ if the subject is the proband. Note that our notation implies that each sampled family has only one proband and that each sampled individual belongs to only one sampled family, as would be the case in the sufficiently large population described in the Introduction.

The data for the sample contain information on the subjects’ disease statuses, and possibly also on covariates such as sex and age. We let $Y_{ij}$ denote disease status (i.e., the binary outcome) for subject $ij$: for example, $Y_{ij} = 1$ if subject $ij$ has ever had MDD and $Y_{ij} = 0$ otherwise. Further, the length $n$ vector $Y$ contains the disease statuses for all subjects. Similarly, the length $q$ vector $X_{ij}$ and the $n$ by $q$ matrix $X$ will refer to the $q$ covariates of interest for subject $ij$ and for all subjects, respectively.

Finally, the data also contain information on the familial relationships between the subjects. For the sake of simplicity, we focus here on samples that include only first-degree
relatives of probands. We use $T_{ij,ij'}$ to denote the relationship between subject $ij$ and subject $ij'$. Thus, $T_{i1,ij'}$ denotes the relationship of proband $i1$ to relative $ij'$ and, since we restrict ourselves here to first-degree relatives, can take values ‘parent-child,’ ‘child-parent,’ or ‘sibling-sibling’ (or ‘self’ for $j' = 1$). For $j \neq 1$ and $j' \neq 1$, $T_{ij,ij'}$ denotes the relationship between relative $ij$ and relative $ij'$ and, for first-degree relative sampling, can take the values ‘spouse-spouse,’ ‘parent-child’ (or vice versa), ‘sibling-sibling’, ‘grandparent-grandchild’ (or vice versa), and ‘aunt/uncle-niece/nephew’ (or vice versa). We will assume that knowing $T_{i1,ij}$ and $T_{i1,ij'}$ is sufficient for determining $T_{ij,ij'}$. In other words, two relatives are related only through their proband, which disallows relationships such as double cousins.

3 ACE MODEL FOR BINARY FAMILY DATA

Here, we describe an ACE model for binary family data similar (or identical) to models described previously [e.g., Rabe-Hesketh et al., 2008; Pawitan et al., 2004; Burton et al., 1999]. As noted above, the model combines a general-family extension of the classic ACE model with a (possibly covariate-specific) liability-threshold model for binary outcomes. Further, it is formulated and fitted within the structural equation modeling framework.

In the liability-threshold model for the binary outcomes, it is assumed that

$$Y_{ij} = \begin{cases} 
0 & \text{if } Y_{ij}^* < t_{X_{ij}} \\
1 & \text{if } Y_{ij}^* \geq t_{X_{ij}} 
\end{cases},$$

(1)

where $Y_{ij}^*$ is the unobserved, continuous liability underlying the observed binary outcome $Y_{ij}$, and where $t_{X_{ij}}$ is a covariate-specific threshold. The covariate-specific threshold can be modeled as a linear function of covariates thought to influence disease liability [Rice et
where $\beta$ is a length $q$ parameter vector quantifying how changes in $X_{ij}$ affect $t_{X_{ij}}$. Note that the thresholds are functions of disease prevalence, as we will discuss in Section 4.

The liability is then represented as the following sum, as in the classic ACE model:

$$Y_{ij}^* = aA_{ij} + cC_{ij} + eE_{ij} \text{ for } i = 1, \ldots, n_a + n_u \text{ and } j = 1, \ldots, n_i,$$

where $A_{ij}$, $C_{ij}$, and $E_{ij}$ are latent additive genetic, shared family environmental, and unique environmental error components, respectively, for subject $ij$. We set the means and variances of $A_{ij}$, $C_{ij}$ and $E_{ij}$, which are arbitrary, to 0 and 1, respectively. In addition, we set the variance of $Y_{ij}^*$ to 1. Note that the model in (3) does not include a random error term because it would be confounded with $E_{ij}$, which implies that any measurement error in $Y_{ij}$ will be reflected in the unique environmental component of the model. In addition, note that the model is additive, which implies the assumption that $A_{ij}$, $C_{ij}$, and $E_{ij}$ do not interact with each other in their effect on the liability to the disease. In addition, we assume that $\text{Cov}(A_{ij}, C_{ij}) = 0$, $\text{Cov}(A_{ij}, E_{ij}) = 0$, and $\text{Cov}(C_{ij}, E_{ij}) = 0$. These assumptions imply, for example, that genes do not shape environment, either directly or indirectly, and they allow the variance in liability to be partitioned into separate additive genetic, shared environmental, and unique environmental variance components, denoted as $a^2$, $c^2$, and $e^2$, respectively. Since $\text{var}(Y_{ij}^*) = 1$, these variance components can be interpreted as percentages of the variance in the underlying liability.

Members of the same family may have similar or even identical values for $A_{ij}$ or $C_{ij}$, a fact that is reflected in the within-family correlations of the $A_{ij}$s and $C_{ij}$s, which are
discussed below. If it is assumed that family members’ outcomes do not directly affect each other, then the indirect effects of similar genes and shared environment on family members’ liabilities are the sole source of the observed association between their outcomes, such that:

\[
\text{Cor}(Y_{ij}^*, Y_{ij'}^*) = a^2 \text{Cor}(A_{ij}, A_{ij'}) + c^2 \text{Cor}(C_{ij}, C_{ij'}). \tag{4}
\]

Equation (4) means that, once the within-family correlations have been specified, the observed associations between family members’ outcomes can be used to estimate \(a^2\) and \(c^2\) (and \(e^2\) from \(1 - a^2 - c^2\)).

Specifying the within-family correlations for the additive genetic component requires several assumptions commonly made for general family data as well as for twin data. More specifically, we assume that the genetics of the disease are not influenced by dominance, epistasis, or assortative mating, which implies that \(\text{Cor}(A_{ij}, A_{ij'}) = \frac{1}{2d(ij, ij')}\), where \(d(ij, ij')\) equals the degree of the relationship between \(ij\) and \(ij'\) (i.e., 1 for first-degree relatives, 2 for second-degree relatives, etc.). For spouse-spouse pairs, we assume that \(d(ij, ij')\) is very large (i.e., spouses are not close relatives); this assumption, combined with that of no assortative mating, implies that \(\text{Cor}(A_{ij}, A_{ij'}) = 0\) for spouse-spouse pairs.

Specifying the within-family correlations for the shared environmental component is more complicated for general family data than for twin data. With twin data, it is typical to assume simply that dizygotic twins (reared together) share a family environment to the same extent as monozygotic twins (reared together). However, with more general family data, it is not plausible to assume that all types of family member pairs share a family environment to the same extent, and it can be difficult to determine the extent to which
any pair of family members share a family environment. This complication can be handled in two ways. First, we could simply assume that shared environment has no effect on the outcome (i.e., $c^2 = 0$), which may be a reasonable assumption for some diseases. Second, we could attempt to measure $c^2$ by making certain assumptions about Cor$(C_{ij}, C_{ij'})$. For instance, we could assume that Cor$(C_{ij}, C_{ij'})$ is a known function of the amount of time that $ij$ and $ij'$ lived together [Hopper and Matthew, 1982]. Alternatively, we could set Cor$(C_{ij}, C_{ij'})$ equal to the same (pre-specified or estimated) value for all pairs with the same $T_{ij,ij'}$ value. For example, it could be assumed that all sibling-sibling, parent-child, and spouse-spouse pairs share family environments to differing extents reflected in the correlations $\gamma_{sib}$, $\gamma_{par}$, and $\gamma_{mar}$, respectively, and that other types of relative pairs (who do not typically live together, at least at some point) do not share a family environment and thus have Cor$(C_{ij}, C_{ij'}) = 0$ [Thomas 2004, p. 98]. In this example, it would be necessary to impose one constraint (e.g., $\gamma_{sib} = 1$) to identify the model.

4 MODEL FITTING

We take a likelihood-based approach to fitting. A likelihood for the variance components formulation of the ACE model in (3) is

$$L(a^2, c^2, \gamma, t_x|Y, X) \propto \prod_{i=1}^{n_u+n_u} f(Y_{i2}, \ldots, Y_{im}|Y_{i1}, X_{i1}, \ldots, X_{im}),$$

where $\gamma$ is a vector containing any shared family environmental correlations being estimated, and where $t_x$ is a vector containing the possibly covariate-specific thresholds (the $t_{X_{ij}}$s) or else the parameters describing the relationship between the covariates and the covariate-specific thresholds (e.g., $\beta$ from equation (2)). The likelihood addresses the case-
control ascertainment by conditioning on the proband’s outcome, following, for example, Hopper and Matthews [1982]. It does not condition on the family’s, or the family members’, ascertainment status(es) because, as demonstrated by Tosteson et al. [1991], ascertainment status can be ignored under single ascertainment. In addition, the likelihood does not include a term modeling family size or structure alongside the term modeling family members’ disease statuses, as would be done in the full likelihood approach (see Thompson [1986, Section 8.2]); this omission implies an assumption that family size and structure do not convey information about the parameters of interest ($a^2$ and $c^2$).

The conditional probabilities in (5) can be obtained from the joint probabilities of the outcomes, which can be calculated once we assume a distribution function for the $Y_{ij}^*$s. Here, we assume that the joint distribution of $Y_{i1}^*, \ldots, Y_{in_i}^*$ is a $n_i$-dimensional multivariate normal (e.g., Curnow and Smith, 1975). Thus,

$$f(Y_{i1}, Y_{i2}, \ldots, Y_{in_i} | X_{i1}, \ldots, X_{in_i}) = \int_{D(Y_{i1})} \cdots \int_{D(Y_{in_i})} MVN(\mu, \Sigma) \, dY_{i1}^* \cdots Y_{in_i}^*$$

(6)

where

$$D(Y_{ij}) = \begin{cases} \left[ -\infty, t_{X_{ij}} \right] & \text{if } Y_{ij} = 0 \\ \left[ t_{X_{ij}}, \infty \right] & \text{if } Y_{ij} = 1 \end{cases},$$

(7)

where

$$\mu = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T,$$

(8)

and where

$$\Sigma_{ij,ij'} = \begin{cases} 1 & \text{for } j = j' \\ a^2 \text{Cor}(A_{ij}, A_{ij'}) + c^2 \text{Cor}(C_{ij}, C_{ij'}) & \text{for } j \neq j' \end{cases}.$$

(9)

The mean vector $\mu$ is set to zero in order to identify the model because $\mu$ is completely confounded with the $t_{X_{ij}}$s. Finally, note that, in (7), the same (covariate-specific) thresholds
are used for probands and for relatives, which implies an assumption that case (control) probands are randomly selected from among affected (unaffected) population members.

4.1 PARAMETER ESTIMATION

We choose to fix the threshold(s) before fitting the ACE model to the case-control family data. The reason for this choice is that the alternative approach—jointly estimating the threshold alongside $a^2$, $c^2$, and $\gamma$—produces different threshold estimates depending on the model’s values for the other parameters. More specifically, for reduced variants of the ACE model that specify low or zero correlations between family members (e.g., the E model, where $a^2 = c^2 = 0$), lowering the threshold estimate improves the model’s fit to data that exhibit some familial aggregation. However, since the threshold is a function of disease prevalence and is therefore theoretically the same in all models, comparisons between different variants of the ACE model should be based on how well they fit the data for the same threshold value.

In applications where there is no supplemental information on prevalence, it will need to be estimated in order to determine the threshold(s). It is possible to obtain valid estimates of prevalence from case-control family data if certainly commonly-made assumptions hold (see Javaras et al. [2009]). As an example, suppose that there is a single, categorical covariate thought to affect disease liability, and suppose that subject $ij$ has value $X_{ij} = x$ for that covariate. (This covariate could be the result of coarsening continuous covariates into categorical variables and/or combining multiple categorical covariates by crossing their levels.) To obtain an estimate of the prevalence corresponding to the $x$ stratum, which we
denote \( \pi^x \), we use the following equation:

\[
\hat{\pi}^x = p_A^x \hat{\pi} + p_U^x (1 - \hat{\pi}),
\]  

(10)

where

\[
\hat{\pi} = \frac{p_U}{1 - p_A + p_U},
\]  

(11)

where \( p_A^x \) (\( p_U^x \)) is the proportion of case (control) probands' relatives who have covariate value \( x \) and are affected, and where \( p_A \) (\( p_U \)) is the proportion of case (control) probands' relatives who are affected. Then, the corresponding threshold can be obtained from \( \hat{\pi}^x \) using the following equation, which relies on the assumption (stated above) that the liabilities follow a normal distribution:

\[
t_{X_{ij}} = \Phi^{-1} (1 - \hat{\pi}^x).
\]  

(12)

The estimated threshold in (12) is approximately equal to the value obtained by jointly estimating the threshold alongside the other parameters in the true variant of the ACE model. In contrast, estimating the threshold using a reduced (e.g., \( a^2 = 0 \)) variant of the true ACE model results in a smaller estimate of the threshold (or, equivalently, a larger estimate of the prevalence). For example, when the E model is fitted, the joint threshold estimate is approximately equal to \( \Phi^{-1} (1 - p^x) \), where \( p^x \) is the proportion of (all) relatives who have covariate value \( x \) and are affected, a quantity that is larger than \( \hat{\pi}^x \) and upwardly biased for the true prevalence when the disease aggregates in families.

Once the threshold(s) have been fixed, estimates of \( a^2 \), \( c^2 \), and \( \gamma \) can be obtained by maximizing (5) subject to the constraints that \( a^2 \) and \( c^2 \) are each between 0 and 1 and that \( a^2 + c^2 \) is less than 1. Further, elements of \( \gamma \) are constrained to be between \(-1\) and 1 or,
if, as in the Austrian example, it is reasonable to assume that shared family environment cannot make outcomes negatively correlated, between 0 and 1.

4.2 INFEERENCE

We can form a normal-theory-based confidence interval (CI) for \( a^2 \) (or \( c^2 \)) using the standard asymptotic distribution of the parameter estimates, which is normal with mean equal to the true parameter values and variance equal to the inverted information matrix. Since the normal approximation tends to be more appropriate for quantities with an unrestricted range, we choose to form a CI for a Fisher z-transformation of \( a \) \((z = 0.5 \ln \frac{1+a}{1-a})\) and then re-transform the upper and lower bounds to obtain a CI for \( a^2 \).

In addition, we may want to constrain the full ACE model in order to test various hypotheses, including whether the disease aggregates at all within families (Model E: \( a^2 = c^2 = 0 \)) or whether it is affected by additive genetic effects (Model CE: \( a^2 = 0 \)) or by shared family environment (Model AE: \( c^2 = 0 \)). We can test these hypotheses by calculating the usual likelihood ratio test (LRT) statistic for the constrained and unconstrained models. However, the LRTs for tests such as \( H_0: \) Model AE versus \( H_1: \) Model ACE will not have the standard \( \chi^2 \) asymptotic distributions because the null hypotheses constrain parameters to be on the boundary of the parameter space. \( P \)-values calculated from the standard \( \chi^2 \) distribution will be conservative (i.e., too big), meaning that the standard LRT or related procedures like AIC [Akaike, 1987] will choose overly parsimonious models that result in overestimates of the retained variance components [Sullivan and Eaves, 2002]. For the hypotheses mentioned above, the true distributions of the LRT statistics are mixtures of
χ² distributions with different degrees of freedom. The exact mixture has been derived for a number of different situations [Chernoff, 1954; Self and Liang, 1987; Stram and Lee, 1994 and 1995; Verbeke and Molenberghs, 2003], including the classic ACE model for continuous twin data [Dominicus et al., 2005], but not for the ACE model for case-control family data. Thus, we recommend using a Monte Carlo test [see Ripley, 1987] in which the p-value is determined by comparing the actual LRT statistic to the distribution of LRT values calculated from a large number of datasets simulated under the null hypothesis. Alternatively, a less time-consuming possibility is to use the p-values from the standard χ² distribution, but with some modification, as Dominicus et al. [2005] recommend for continuous twin data. The simulation experiments described in Section 5 suggest that, for case-control family data, using half the standard p-value works well (i.e., results in actual rejection levels approximately equal to the nominal levels) for comparing AE versus ACE, but that using the standard p-value (unhalved) works well for comparing CE versus ACE or E versus either AE or CE.

5 SIMULATION EXPERIMENTS

We conducted simulation experiments to investigate whether the approach to fitting described in Section 4.1 yields valid (i.e., approximately unbiased) estimates when the true variant of the ACE model is fitted. In addition, the simulation experiments investigated whether LRTs and AIC (see Section 4.2) permit identification of the true ACE variant.

The experiments were designed to mimic the Austrian case-control family study of MDD described in Section 6, which is at the small end of case-control family studies. We
created seven fictional populations, each with a different combination of additive genetic and shared family environmental effects. Each population contained approximately 500,000 individuals, a number that corresponds roughly to the number of 18 – 70 years olds living in the Tyrol region of Austria in 2003, the catchment area for the Austrian study [Statistik Austria, 2003]. For each population, we generated data for approximately 125,000 families by following three steps in order: (a) we generated family sizes (from 2 to 9 members) based roughly on the distribution of family sizes in the Austrian data; (b) we generated the relationships between and sexes of family members based on the distribution of family relationships and sex in the Austrian data and the percentage of females (50.5%) between 18 and 70 years old in the Tyrolean population in 2003, and; (c) we used the ACE model for binary family data described in Section 3 to generate lifetime disease statuses for the family members conditional on their sexes and relationships, for various combinations of $a^2$ and $c^2$. In step (c), we allowed prevalence to differ by gender, using values (5.9% for males and 11.5% for females) similar to those obtained by applying Equation (10) to the actual Austrian data. Also, we assumed that shared family environmental correlations equaled 1 for siblings and 0 for all other relatives pairs. Finally, we set $a^2$ equal to one of four values (0, 0.20, 0.40, or 0.70) and $c^2$ equal to one of two values (0 or 0.20), for a combination of seven different populations (the eighth combination, where $a^2 = 0.7$ and $c^2 = 0.2$, was not used because it seemed unrealistic). The values for the variance components were chosen to make our simulation experiments comparable to those of Kuhnert and Do [2003], who compared the performance of various methods for fitting ACE models to binary twin data.

Next, we sampled 1,000 small case-control family datasets from each of the seven fictional populations. Each dataset was formed by selecting 64 case probands and 58 control
probands (the numbers in the Austrian study), and then including all of the probands’ first-degree family members. For each sampled dataset, Equations (10) and (12) were used to estimate the male and female prevalences and thresholds, and then the ACE, AE, CE and E models were fitted to the data using the approach described in Section 4.1.

In the 7,000 case-control family datasets sampled, the number of sampled individuals (relatives plus probands) ranged between approximately 340 and 440. Even for this relatively small sample size, the population was not sufficiently large to ensure single ascertainment, but the extent of multiple ascertainment was extremely small (only about 0.05% of the sampled families were multiply ascertained). In these instances, the first family member to be selected as a proband was retained as the sole proband for his or her family. Although ignoring proband status can result in biases, we felt comfortable doing so here because the number of doubly-ascertained families was so small.

Results for the simulation experiments can be seen in Table 1. We first examine the estimates of the variance components in the second vertical section of the table. Beginning with the italicized numbers, which are estimates produced using the correct variant of the ACE model, we see that our fitting approach yields approximately unbiased estimates of the variance components when the true variant is fitted, even for a case-control family dataset as small as the Austrian dataset. Furthermore, in simulation experiments performed with a larger population size (approximately 2,000,000 members) and a larger sample size (150 case and 150 control probands), estimates (not reported here) of the variance components were even less biased. For example, for $a^2 = 0.2$ and $c^2 = 0.2$, the resulting estimates of $a^2$ and $c^2$ were 0.205 and 0.198, respectively, in the larger simulation experiments, compared to 0.231 and 0.185 in the smaller simulation experiments presented here. Turning to the un-
italicized numbers, which are estimates produced using the incorrect variant of the ACE model, we see that fitting the incorrect model yields biased estimates of the remaining (non-zero) variance components, as would be expected.

Turning to model selection, we first examine the bold numbers in Table 1’s third vertical section, which correspond to LRTs where the true model is the null hypothesis (e.g., for $a^2 = 0.4$ and $c^2 = 0$, the test of $H_0$: AE vs $H_1$: ACE). As noted in Section 4.2, these numbers suggest that using half the standard p-value yields appropriate rejection levels when comparing the AE model versus the ACE model, whereas using the standard p-value (unhalved) yields appropriate rejection levels when comparing the CE model versus the ACE model or the E model versus either the AE or CE models. Second, we examine the bold-italicized numbers in Table 1’s third vertical section, which correspond to LRTs where the true model is the alternative hypothesis (e.g., for $a^2 = 0.4$ and $c^2 = 0$, the test of $H_0$: E vs $H_1$: AE). These numbers suggest that power to detect non-zero variance components is low for small (e.g., 0.2) or even moderate (e.g., 0.4) effects, especially for shared family environmental effects. Third, the italicized numbers in Table 1’s fourth vertical section, which correspond to the percentage of times that AIC identified the true model, suggest that AIC does very well at identifying true AE models with large $a^2$ (e.g., 0.7), and reasonably well at identifying true AE and CE models with small to moderate $a^2$ or $c^2$ and true E models. However, AIC does poorly at identifying true ACE models, unless both $a^2$ and $c^2$ are large. The finding that LRTs and AIC have limited power to detect ACE models with small to moderate $a^2$ and $c^2$ is not surprising, especially for such small datasets. In fact, Kuhnert and Do [2003, p. 441] found that, for binary twin data with 1000 monzygotic and 1000 dizygotic pairs, both a Bayesian fitting method and a maximum likelihood fitting
method “had difficulty in detecting the correct model when the additive genetic effect was low (between 10 and 20%) or of moderate range (between 20 and 40%). Furthermore, neither method could adequately detect a correct model that included a modest common environmental effect (20%) even when the additive genetic effect was large (50%).”

6 AUSTRIAN CASE-CONTROL FAMILY STUDY

We used our method to analyze the actual data from the Austrian study [Hudson et al., 2003]. 64 affected probands with a current DSM-IV [APA, 1994] diagnosis of MDD and 58 unaffected probands without a current or past MDD diagnosis were recruited at Innsbruck University Clinics. Adult first-degree relatives of probands were eligible for the study. Probands and relatives were interviewed using the Structured Clinical Interview for DSM-IV [First et al., 1994].

Our analyses included a total of 122 probands (one per family) and 330 first-degree relatives, all interviewed in person. Disease status was measured by a variable indicating whether the subject had been diagnosed with lifetime MDD (i.e., had a diagnosis of MDD at any point during their life up to the present time). Overall, 13.6% of the relatives have a lifetime diagnosis of MDD, but the estimate of overall prevalence from Equation (11) was 8.8%, with estimates of 6.0% for males and 11.3% for females from Equation (10).

We first performed preliminary analyses to investigate whether MDD aggregates in families and whether the level of aggregation differs by type of relative pair (see Table 2). The overall familial aggregation odds ratio (OR) is significantly greater than 1 for both the 330 proband-relative pairs (OR = 2.7) and the 360 relative-relative pairs (OR
indicating that MDD does aggregate in families. Turning to the relative-type-specific ORs, the aunt/uncle - niece/nephew OR is highly significant, which suggests that genetics account for at least part of the familial aggregation of MDD if we assume that aunt/uncle - niece/nephew pairs do not typically share family environments. Further, this OR is not smaller than the parent - child and sibling - sibling ORs, which suggests that shared family environment plays a very small role, if any, in the familial aggregation of MDD. Finally, note that the spouse-spouse pairs do not contain enough information to estimate $\gamma_{mar}$ when fitting the ACE models.

We fitted ACE, AE, CE, and E variants of the ACE model to the data, with thresholds fixed at values estimated from (12). The thresholds differed by sex only because a logistic regression analysis of the relatives’ disease status revealed that the odds of having lifetime MDD differs significantly by sex, but not by age, in our data. In addition, for model variants that included C, $\gamma_{sib}$ was fixed to one, $\gamma_{par}$ was either fixed to zero or estimated, and Cor($C_{ij}, C_{ij'}$) for all other types of relatives pairs was fixed to zero. Model fitting was performed in R using a function written by the authors. The integrals in (6) were calculated using R’s pmvnorm() function, which implements the algorithms proposed by Genz [1992], and maximization of $\ln(L)$ was performed using R’s optim() function, which implements the technique of Byrd et al. [1995]. For each model, the log-likelihood was unimodal, and the maximum was easily found. Standard errors were calculated from a finite difference approximation to the Hessian matrix at the maximum likelihood values, with the variance components on a Fisher-transformed scale.

Table 3 presents the results from fitting the different variants of the ACE model. Comparing the fit of the different variants reveals that the AE model is clearly the best-fitting
model, as one might expect from examination of the relative-type-specific ORs. (In contrast, the E model is clearly the worst-fitting model, further supporting the finding that MDD aggregates in families.) The AE model fits better than the CE model with a sibling-sibling and a parent-child shared family environment, despite the fact that the latter model has one more parameter than the AE model. The AE model fits slightly worse than either of the ACE models, but the difference in $-2 \log(L)$ is only 0.3, which is far from significant according to either a Monte Carlo LRT, the standard chi-square LRT, or the chi-square LRT with the p-value halved. In the best-fitting AE model, $a^2$ is estimated as approximately 0.52, and the 95% confidence interval for $a^2$ is approximately [0.24, 0.72]. The length of this confidence interval reflects the small size of the Austrian study.

In conclusion, our analysis suggests that the familial aggregation of adult MDD can be explained almost completely by additive genetic effects, which account for approximately one half the variance in the liability to MDD. These findings are consistent with the meta-analysis performed by Sullivan et al. [2000], who found that previous studies of MDD heritability suggest a range of 0.31 – 0.42, and with the recent results of Rabe-Hesketh et al. [2008], who found that the AE model (with heritability estimated as 0.43) best fit a large twin dataset on MDD.

7 DISCUSSION

The simulation experiments and Austrian MDD application reveal that our approach to fitting ACE models makes it possible to use case-control family data to obtain approximately unbiased estimates of the population (i.e., non-ascertainable) ACE variance components,
provided that certain commonly-made assumptions hold. This advance is a boon to investigators seeking to determine whether a disease has a genetic component before they proceed to molecular genetic studies. First, using general family data instead of twin data has several advantages [Pawitan et al., 2004]. For one, case-control family data are easier to collect from scratch than twin data. In addition, if families have more than two members, family data contain more relative pairs, and thus more power to detect variance components, than twin data with the same number of subjects. Further, using family data makes it possible to estimate effects than cannot be estimated from standard twin data (e.g., parent-child shared family environmental effects). Second, using case-control-sampled data instead of population-sampled data offers greater power, especially for rarer diseases.

Of course, our approach is not without limitations. First, we choose to fit the models within a structural equation modeling framework rather than a mixed effects modeling framework. The former does confer several advantages, namely easily interpretable parameters and straightforward adjustment for case-control sampling, but the latter is easier to implement with standard statistical software, makes it easier to incorporate covariates, and involves less computation because only three- or four-dimensional integrals need to be calculated regardless of family sizes (see Rabe-Hesketh et al. [2008, p. 286]). Second, we choose to fix the threshold value(s) during model fitting, which means that the standard errors for the variance component estimates will not reflect the uncertainty surrounding the thresholds. Of course, sensitivity analysis can be performed by refitting the model using different threshold values that define a reasonable range of prevalences. Third, our approach will not perform well for very rare diseases because not enough cases will be available to get reliable estimates of the variance components (or, in some cases, even the
prevalence). Fourth, our approach is limited by the assumptions required for it to yield valid estimates, which we now discuss in the following three paragraphs.

For one, the assumptions of the ACE model itself may not be valid for a particular disease. As an example, the assumption of no gene-environment interactions may not be entirely appropriate for some diseases, such as depression [Moffit et al., 2005]. As another example, estimates of genetic components derived from family members who are unmatched in age may be biased downwards if different genetic factors account for the variation in liability at different ages [Maes et al., 1997]. Similarly, the assumptions surrounding the liability threshold model may not be valid. For example, treating disease status as binary may not be appropriate for late-onset diseases if the data contain many young subjects whose outcomes will be effectively censored. In this case, using an ACE model developed for survival outcomes [e.g., Pitkäniemi et al., 2007] may be more appropriate, although the ACE model for binary family data could still be used if age were incorporated as a threshold-shifting covariate. However, even if disease status can reasonably be treated as a binary outcome, some authors [e.g., Kraemer, 1997; Hopper, 1993] have raised objections to the validity of the liability-threshold model. Further, Glidden and Liang [2002] show that normal-distribution-based estimates of variance components from sibships are biased if family members’ liabilities actually follow a multivariate $t$ distribution with 5 degrees of freedom, with the bias being particularly severe when only those sibships with one or more affected individuals are ascertained. Reassuringly, though, the assumption of normal liabilities would be appropriate for a complex disease resulting from the sum of many small genetic and environmental effects [Lange, 1978].

In addition, the assumptions about shared family environment may not be valid, which
can result in biased estimates of the various variance components. As noted in Section 3, specifying the shared family environmental correlations is more difficult for general family members than for twins. In fact, Gjessing and Lie [2008] point out that, for various pedigree types typically included in family data, patterns of shared family environment “easily mimic genetic transmission,” making it difficult or impossible to separate the two. In our Austrian example, the ACE model had difficulty separating the effects of $a^2$ and $c^2$, as reflected in a large negative correlation between the estimates of $a^2$ and $c^2$. In other examples, it could be impossible to separate $a^2$ and $c^2$, for example if the data consist of only probands and their siblings.

Last, some of the assumptions surrounding the case-control family sampling may not be valid. In addition to single ascertainment, these assumptions include case (control) probands being representative of affected (unaffected) population members, sampled relatives being representative of all relatives, and family size having no correlation with the disease status of its members. Violations of these assumptions can result in biased estimates of the variance components. This is partly because violations can result in biased estimates of prevalence (see Javaras et al. [2009]), which in turn results in biased estimates of the variance components. However, even if the prevalence of the disease is well-known and does not need to be estimated, violations of the assumptions can still result in biased estimates of the ACE variance components. For example, if affected relatives are less likely to be sampled, then estimates of $a^2$ or $c^2$ may be too small.

These limitations aside, our approach to fitting ACE models to case-control family data performs very well when used with an actual dataset: the results from the Austrian example are in line with previous studies investigating genetic and environmental effects on MDD.
REFERENCES


Hudson JI, Mangweth B, Pope Jr HG, De Col C, Hausmann A, Gutweniger S, Laird NM,


### Table 1: Results From Simulation Experiments$^a$

<table>
<thead>
<tr>
<th>True Values</th>
<th>Estimates$^b$</th>
<th>Likelihood Ratio Tests$^c$</th>
<th>AIC$^b$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>ACE</td>
<td>AE</td>
<td>CE</td>
</tr>
<tr>
<td>$a^2$</td>
<td>$c^2$</td>
<td>$\tilde{a}^2$</td>
<td>$\tilde{c}^2$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.071</td>
<td>0.031</td>
</tr>
<tr>
<td>0</td>
<td>0.20</td>
<td>0.108</td>
<td>0.153</td>
</tr>
<tr>
<td>0.20</td>
<td>0</td>
<td>0.186</td>
<td>0.044</td>
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<tr>
<td>0.20</td>
<td>0.20</td>
<td>0.231</td>
<td>0.185</td>
</tr>
<tr>
<td>0.40</td>
<td>0</td>
<td>0.353</td>
<td>0.056</td>
</tr>
<tr>
<td>0.40</td>
<td>0.20</td>
<td>0.408</td>
<td>0.192</td>
</tr>
<tr>
<td>0.70</td>
<td>0</td>
<td>0.633</td>
<td>0.061</td>
</tr>
</tbody>
</table>

---

$a$ All results based on 1000 simulated datasets.

$b$ Estimates and AIC percentages corresponding to the true model appear in un-bolded italics.

$c$ LRT rejection rates corresponding to the true model appear in bold (null hypothesis) or bolded italics (alternative hypothesis).

$d$ Value fixed rather than estimated.
Table 2: Unadjusted Familial Aggregation Odds Ratios (OR) for the Austrian MDD Data

<table>
<thead>
<tr>
<th>Number of Pairs</th>
<th>Familial Aggregation OR(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All Pairs</strong></td>
<td></td>
</tr>
<tr>
<td>Proband-Relative Pairs</td>
<td>330</td>
</tr>
<tr>
<td>Relative-Relative Pairs</td>
<td>360</td>
</tr>
<tr>
<td><strong>Sibling-Sibling Pairs</strong></td>
<td></td>
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<tr>
<td>Proband-Relative Pairs</td>
<td>144</td>
</tr>
<tr>
<td>Relative-Relative Pairs</td>
<td>166</td>
</tr>
<tr>
<td><strong>Parent-Child Pairs</strong></td>
<td></td>
</tr>
<tr>
<td>Proband-Relative Pairs</td>
<td>186</td>
</tr>
<tr>
<td>Relative-Relative Pairs</td>
<td>92</td>
</tr>
<tr>
<td><strong>Spouse - Spouse Pairs</strong></td>
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</tr>
<tr>
<td>Proband-Relative Pairs</td>
<td>0</td>
</tr>
<tr>
<td>Relative-Relative Pairs</td>
<td>30</td>
</tr>
<tr>
<td><strong>Grandparent - Grandchild Pairs</strong></td>
<td></td>
</tr>
<tr>
<td>Proband-Relative Pairs</td>
<td>0</td>
</tr>
<tr>
<td>Relative-Relative Pairs</td>
<td>3</td>
</tr>
<tr>
<td><strong>Aunt/Uncle - Niece/Nephew Pairs</strong></td>
<td></td>
</tr>
<tr>
<td>Proband-Relative Pairs</td>
<td>0</td>
</tr>
<tr>
<td>Relative-Relative Pairs</td>
<td>69</td>
</tr>
</tbody>
</table>

\(^a\) OR = Odds(MDD | Relative has MDD)/Odds(MDD | Relative does not have MDD)

\(^b\) Based on 332 ordered sibling-sibling pairs.

\(^c\) Cannot estimate OR due to small sample size or small number affected.
<table>
<thead>
<tr>
<th>Model</th>
<th>Estimates</th>
<th>Variance Components</th>
<th>$-2 \ln(L)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: ACE</td>
<td>$\gamma_{sib} = 1^b$</td>
<td>$\hat{a}^2 = 0.443$</td>
<td>240.8</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_{par} = 0$</td>
<td>$\hat{c}^2 = 0.067$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\hat{e}^2 = 0.490$</td>
<td></td>
</tr>
<tr>
<td>2: ACE</td>
<td>$\gamma_{sib} = 1^b$</td>
<td>$\hat{a}^2 = 0.443$</td>
<td>240.8</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{par} = 0^b$</td>
<td>$\hat{c}^2 = 0.067$</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>$\hat{e}^2 = 0.490$</td>
<td></td>
</tr>
<tr>
<td>3: AE</td>
<td>$\gamma_{sib} = NA$</td>
<td>$\hat{a}^2 = 0.517$</td>
<td>241.1</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{par} = NA$</td>
<td>$\hat{c}^2 = 0^b$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\hat{e}^2 = 0.483$</td>
<td></td>
</tr>
<tr>
<td>4: CE</td>
<td>$\gamma_{sib} = 1^b$</td>
<td>$a^2 = 0^b$</td>
<td>241.9</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_{par} = 0.672$</td>
<td>$\hat{c}^2 = 0.293$</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>$\hat{e}^2 = 0.707$</td>
<td></td>
</tr>
<tr>
<td>5: CE</td>
<td>$\gamma_{sib} = 1^b$</td>
<td>$a^2 = 0^b$</td>
<td>246.3</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{par} = 0^b$</td>
<td>$\hat{c}^2 = 0.297$</td>
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<td></td>
<td></td>
<td>$\hat{e}^2 = 0.703$</td>
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<td>6: E</td>
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<td>$a^2 = 0^b$</td>
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<td>$\gamma_{par} = NA$</td>
<td>$c^2 = 0^b$</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$e^2 = 1^b$</td>
<td></td>
</tr>
</tbody>
</table>

$a$ Shared family environmental correlation assumed to equal zero for all spouse-spouse, grandparent-grandchild, and aunt/uncle - niece/nephew pairs.

$b$ Value fixed rather than estimated.