Web Appendix A: Assessment of conditions B_{min} and B_{root} for different measures of projected study value

We provide here details of some calculations and results concerning when conditions B_{min} and B_{root} hold, along with details of how these were calculated for Figure 1 and Table 2. Condition B_{min} holds for any n^* if $v_0 \geq 0$ and v_n is concave in n over $[0,\infty)$, i.e., if $\frac{\partial^2 v_n}{\partial n^2} \leq 0$ for all n. In addition, condition B_{min} holds for any n^* such that $\frac{\partial}{\partial n}(n^{-1}v_n) \leq 0$ for all $n \geq n^*$. Similarly, condition B_{root} holds for any n^* such that $\frac{\partial}{\partial n}(n^{-1/2}v_n) \leq 0$ for all $n \geq n^*$.

A.1. Proof of Proposition 8.

To examine the relevant properties of power as a measure of projected study value, we focus on the simple situation of a two-sided, one-sample Z-test. The power for standardized effect size $\Delta>0$ with sample size n is $P_n = \Phi(\Delta n^{1/2} - Z_{1-\alpha/2})$ for Φ the standard normal cumulative distribution function and $Z_x = \Phi^{-1}(x)$. Inverting this shows that $n^{1/2} = (Z_{P_n} + Z_{1-\alpha/2})/\Delta$. For v_n proportional to P_n, condition B_{min} follows from concavity in n, which we evaluate by examining the second derivative:

$$\frac{\partial P_n}{\partial n} = \frac{\Delta}{2} n^{-1/2} \varphi(\Delta n^{1/2} - Z_{1-\alpha/2})$$

$$\frac{\partial^2 P_n}{\partial n^2} = -\frac{1}{2}\frac{\Delta}{2} n^{-3/2} \varphi(\Delta n^{1/2} - Z_{1-\alpha/2}) - \frac{\Delta}{2} n^{-1/2} (\Delta n^{1/2} - Z_{1-\alpha/2}) \varphi(\Delta n^{1/2} - Z_{1-\alpha/2}) \frac{\Delta}{2} n^{-1/2}$$

$$= -\frac{\Delta}{4} n^{-3/2} \varphi(\Delta n^{1/2} - Z_{1-\alpha/2}) \{1 + \Delta n^{1/2} (\Delta n^{1/2} - Z_{1-\alpha/2})\},$$

with φ the standard normal density. The sign of this is determined by the sign of the expression in braces, which can be written in terms of P_n as:

$$1 + (Z_{P_n} + Z_{1-\alpha/2}) Z_{P_n}$$

$$= Z_{p_n}^2 + Z_{1-\alpha/2} Z_{P_n} + 1.$$

Proposition 8 follows from the positivity of this expression whenever $Z_{1-\alpha/2}<2$, or equivalently $\alpha>0.0455$. Note that this does not depend on Δ. For $\alpha=0.01$, v_n is convex only up to n such that $P_n = 0.018$.

A.2. Interval estimation. Consider Bayesian estimation of a normal mean μ, with unknown variance σ^2 and conjugate prior:

$$\mu|\sigma^2 \sim \text{Normal}(\mu_0, \sigma^2/n_0), \quad 1/\sigma^2 = \tau \sim \text{Gamma}(n_0/2, \beta), \qquad \text{(A1)}$$

where n_0 is the sample size equivalent of the prior information and we have assumed that the amount of prior information is the same for both the mean and the variance. Use of expected interval width (Joseph and Belisle, 1997) suggests a study value that is a linear function of width. Such a utility based on the width of a credible interval (Lindley, 1997; Pham-Gia, 1997) is

$$k_1 - k_2 W + k_3 \delta(\mu) ,$$

where k_1+k_3 is the maximum possible utility (from knowing the mean exactly), W is the width of the interval, $\delta(\mu)$ indicates whether the interval includes the true value, k_2 determines the impact of width on the utility, and all of $k_{1\text{-}3}$ are positive. The expected value from a sample of size n is the increase in this utility over the prior utility. Letting w_0 denote the width of the prior credible interval and w_n the expected width of the posterior interval, and assuming that the coverage probability $E[\delta(\mu)]$ is fixed by design for all n (including 0), we have

$$v_n = k_2(w_0 - w_n) , \tag{A2}$$

so that projected study value is proportional to the expected reduction in credible interval width. This form for projected value also holds if utility is any other linear function of width that has narrower intervals better than wider ones.

Under the conjugate normal set up (A1), the expected 1-α credible interval width formula from Joseph and Belisle (1997, equation 8) becomes

$$w_n \propto t_{n+n_0,1-\alpha/2}(n+n_0)^{-1} \frac{\Gamma\left(\frac{n+n_0}{2}\right)}{\Gamma\left(\frac{n+n_0-1}{2}\right)} . \tag{A3}$$

This can be evaluated using an approximation based on Stirling's formula (Graham, Knuth, and Patashnik, 1994),

$$\frac{\Gamma(J+1/2)}{\Gamma(J)} \approx \sqrt{J}\left(1 - \frac{1}{8J} + \frac{1}{128J^2} + \frac{5}{1024J^3} - \frac{21}{32768J^4}\right) . \tag{A4}$$

Figure 1 shows value (A2) calculated using (A3) and (A4) for n_0=2 and n_0=100. From (A3) and (A4) we see that w_n is very nearly proportional to $(n+n_0)^{-1/2}$, so we use this to evaluate when B_{root} holds. From equation (A2) we have

$$\frac{\partial}{\partial n}(n^{-1/2}v_n) \propto \frac{\partial}{\partial n}[n^{-1/2}\{n_0^{-1/2} - (n+n_0)^{-1/2}\}]$$

$$= -\frac{1}{2}n^{-3/2}\{n_0^{-1/2} - (n+n_0)^{-1/2}\} + \frac{1}{2}(n+n_0)^{-3/2}n^{-1/2}$$

$$\propto n^{-3/2}n_0^{-1/2}(n+n_0)^{-3/2}\{-(n+n_0)^{3/2} + n_0^{1/2}(n+n_0) + nn_0^{1/2}\} .$$

The sign of this expression is determined by the term in braces. Letting $n=kn_0$, we evaluate this term as

$$-n_0^{3/2}(k+1)^{3/2}+n_0^{3/2}(k+1)+kn_0^{3/2} = n_0^{3/2}\{2k+1-(k+1)^{3/2}\}.$$

To determine where this changes from positive to negative, we set the term in braces to zero and solve by equating the positive and negative terms and squaring, finding the single positive root at $k=(1+\sqrt{5})/2$, or $n \approx 1.6n_0$. Following similar steps for $\frac{\partial}{\partial n}(n^{-1}v_n)$ produces the expression $[3k+2-2(k+1)^{3/2}]$, which is negative for any $k>0$, implying that B_{min} holds for any n^* no matter how large n_0 is.

Use of expected width in planning studies has also been proposed for frequentist confidence intervals (Goodman and Berlin, 1994). In this case, no width for $n=0$ is defined, but using the reduction from the expected width with $n=2$ as a measure of study value would produce results very similar to Bayesian credible intervals with low prior information. We therefore investigate an alternative definition of study value as proportional to $1/W$, where W now denotes the width of a confidence interval rather than a credible interval. To find $v_n=E(W^{-1})$ for W the width of the usual confidence interval for the mean, $\bar{x} \pm Z_{1-\alpha/2}\, S/\sqrt{n}$, with $\bar{x}$ the sample mean and S^2 the sample variance, we note that

$$W = 2Z_{1-\alpha/2}S/\sqrt{n} = 2Z_{1-\alpha/2}\sigma(n-1)^{-1/2}U^{1/2}n^{-1/2}, \text{ where}$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 \text{ and } U = \frac{n-1}{\sigma^2}S^2 \sim \chi^2_{n-1}.$$

We then have

$$v_n \propto n^{1/2}(n-1)^{1/2}E(U^{-1/2}) \text{ and}$$

$$E(U^{-1/2}) = \int u^{-1/2}\left\{\Gamma\left(\frac{n-1}{2}\right)\right\}^{-1}\left(\frac{1}{2}\right)^{(n-1)/2} u^{(n-1)/2-1}e^{-u/2}du$$

$$=\left\{\Gamma\left(\frac{n-1}{2}\right)\right\}^{-1}\left(\frac{1}{2}\right)^{(n-1)/2}\int u^{(n-2)/2-1}e^{-u/2}du$$

$$=\Gamma\left(\frac{n-2}{2}\right)\left\{\Gamma\left(\frac{n-1}{2}\right)\right\}^{-1}\left(\frac{1}{2}\right)^{1/2},$$

because the integral corresponds to a χ^2_{n-2} density. Using (A4) to evaluate this with $J=(n-2)/2$, and simplifying, we obtain:

$$v_n \propto n^{1/2}\left(\frac{n-1}{n-2}\right)^{1/2}\left(1-\frac{1}{4(n-2)}+\frac{1}{32(n-2)^2}+\frac{5}{128(n-2)^3}-\frac{21}{2048(n-2)^4}\right)^{-1}.$$

This meets B_{root} for all n, because the terms multiplying $n^{1/2}$ in the above expression converge downward to 1 as n increases. This is shown in Figure 1.

Because exact computation of Bayesian credible interval width is difficult for a difference in proportions (Joseph, et al., 1997), we approximate the reduction in width for Table 2 as proportional to $(n_0)^{-1/2} - (n_0+n)^{-1/2}$. We also approximate the reciprocal of confidence interval width as proportional to $n^{1/2}$. With this approximation, n_{root} always has performance 100% of optimal, by definition.

A.3 Bayesian decision theory. Value based on empirical choice between two treatments within a Bayesian framework has been considered by many authors (e.g., Tan and Smith, 1998; Gittins and Pezeshk, 2000a,b; Halpern, et al., 2001; Willan and Pinto, 2005). We consider the case with outcome either normal or Bernoulli, independent conjugate priors, and value a linear function of outcome with higher values more desirable. Letting μ_1 and μ_2 denote the mean outcome under treatments 1 and 2, and I_n the indicator that treatment 1 is chosen by a study of size n per group, the study value is proportional to

$$\mu_1 I_n + \mu_2(1- I_n) - \mu_1 I_0 - \mu_2(1- I_0) = (\mu_1 - \mu_2)(I_n - I_0).$$

For treatment i, i=1 or 2, the posterior mean is a weighted average of the prior mean, μ_{0i}, and the observed mean in the study $\bar{x}_{ni}$, with the weighting determined by the sample size equivalent of the prior information, n_{0i}:

$$\mu_{ni} = \frac{n_{0i}\mu_{0i} + n\bar{x}_{ni}}{n_{0i} + n}.$$

For the Bernoulli case, conjugate priors are Beta(α_i, β_i), with $n_{0i} = \alpha_i + \beta_i$. The optimal post-study action is to choose the treatment with higher posterior mean, so

$$E(I_n \mid \mu_1,\ \mu_2,\ \sigma_1^2,\ \sigma_2^2) = \text{Prob}\{\ \mu_{n1} > \mu_{n2}\ \} = \text{Prob}\{\ \mu_{n2} - \mu_{n1} < 0\}.$$

Given specific values of μ_i, the posterior mean μ_{ni} is either exactly (in the normal case) or approximately (in the Bernoulli case) normally distributed with:
mean $\frac{n_{0i}\mu_{0i} + n\mu_i}{n_{0i} + n}$ and variance $\frac{n}{(n_{0i} + n)^2}\sigma_i^2$, where σ_i^2 is a given value of the variance in the normal case or μ_i (1- μ_i) in the Bernoulli case. We therefore have for given μ_1, μ_2, σ_1^2, σ_2^2,

$$\mu_{n2} - \mu_{n1} \sim \text{Normal}\ (-d_n,\ S_n^2),\ \text{where}$$

$$d_n = \frac{n_{01}\mu_{01} + n\mu_1}{n_{01} + n} - \frac{n_{02}\mu_{02} + n\mu_2}{n_{02} + n},\ \text{and}\quad S_n^2 = \frac{n}{(n_{01} + n)^2}\sigma_1^2 + \frac{n}{(n_{02} + n)^2}\sigma_2^2.$$

This implies

$E(I_n \mid \mu_1,\ \mu_2,\ \sigma_1^2,\ \sigma_2^2) = \Phi(d_n / S_n)$, so that the projected study value is proportional to:

$$E_{\sigma_1^2,\sigma_2^2}(E_{\mu_1,\mu_2}[E\{(\mu_1 - \mu_2)(I_n - I_0) \mid \mu_1,\ \mu_2\} \mid \sigma_1^2,\ \sigma_2^2])$$

$$= \iiiint (\mu_1 - \mu_2)\{\Phi(d_n / S_n) - I_0\} \times g_{\mu_1}(\mu_1 \mid \sigma_1) g_{\mu_2}(\mu_2 \mid \sigma_2) d\mu_1 d\mu_2 g_{\sigma_1}(\sigma_1) g_{\sigma_2}(\sigma_2) d\sigma_1 d\sigma_2 , \qquad \text{(A5)}$$

for the normal case, where the functions g are the conjugate priors (A1) described above. For the Bernoulli case, the two outer integrals do not occur, the g_{μ_i} are not conditional, and $E(I_n \mid \mu_1,\ \mu_2)$ is only approximated by $\Phi(d_n / S_n)$. In the case of equipoise, $\mu_{01} = \mu_{02} = \mu_0$, we were able to confirm B_{min} for the normal case using symbolic mathematics software. The details are in Web Appendix B. We examined conditions B_{min} and B_{root} for the Bernoulli case by numerical calculations of (A5) over a range of values for n. For μ_{01} equal to each of 0.05, 0.1, 0.3, and 0.5, we examined from 27 to 60 different scenarios resulting from 3 or 4 different assumed prior standard deviations for μ_1, 3 to 5 different values of μ_{02}, and the prior standard deviation of μ_2 being 0.5, 1, or 2 times as large as the prior standard deviation for μ_1. For each pair of priors for μ_1 and μ_2, we evaluated (A5) by averaging the integrand u_n over 100,000 pairs of values for (μ_1, μ_2) generated randomly from the priors. The cases meeting B_{root} for all $n^*>4$ included all the equipoise cases and most of scenarios where at least one prior had a standard deviation as large as its mean. The program used for these evaluations is given in Web Appendix C, and Web Table 1 gives the detailed results.

The curve in Figure 1 and results in Table 2 result from applying the computer program by Halpern, et al. (2001) to treatments with conjugate beta priors with mean (standard deviation) 0.4 (0.05) and 0.4 (0.10) for the success rates.

A.4 Shannon information. Lindley (1997) and Bernardo (1997) consider a study's utility defined in terms of the improvement of the posterior for a parameter θ over its prior. Restricting attention to utility functions such that the usual Bayesian posterior given observed data **x** optimizes the utility given **x** (proper functions) and that depend only on the true value of θ (local functions) leads to defining a study's expected value as proportional to:

$$\iint \log[p_\theta(\theta|\mathbf{x})/p_\theta(\theta)]\, p_\theta(\theta|\mathbf{x})\, d\theta\ \ p_{\mathbf{x}}(\mathbf{x})\, d\mathbf{x} ,$$

where **x** is the observed data in a sample of n observations, $p_{\mathbf{x}}(\mathbf{x})$ is the prior distribution of **x**, $p_\theta(\theta)$ is the prior distribution of θ, and $p_\theta(\theta|\mathbf{x})$ is the posterior distribution of θ given **x**. This is the expected gain in Shannon information. Bernardo (1997) provides a large sample approximation to this that is proportional to:

$$\log(1 + n/n_0) \qquad \text{(A6)}$$

and notes that for well behaved $p_\theta(\theta)$, this is a very good approximation even for "rather moderate" n. If θ is the mean of a normal distribution with known variance and conjugate prior,

then the expected value of the study under this definition is exactly proportional to (A6). Using (A6), we have for evaluation of B_{root}:

$$\frac{\partial}{\partial n}(n^{-1/2}v_n) \propto \frac{\partial}{\partial n}\{n^{-1/2}\log(1+n/n_0)\}$$

$$= -(1/2)n^{-3/2}\log(1+n/n_0) + n^{-1/2}n_0^{-1}(1+n/n_0)^{-1}$$

$$= (1/2)n^{-3/2}n_0^{-1}(1+n/n_0)^{-1}\{-n_0(1+n/n_0)\log(1+n/n_0)+2n\}.$$

The sign of this is determined by the sign of the expression in braces. Writing $n=kn_0$, we see that this expression is negative when $\log(1+k) > 2k/(1+k)$. This implies that B_{root} holds for $k>3.9$, or $n^*>3.9n_0$. Following the same steps for evaluation of B_{min} shows that it holds for k such that $\log(1+k) > k/(1+k)$, i.e., for any $k>0$ and therefore for any n^*.

Results shown in Figure 1 and Table 2 use (A6).

A.5 Estimation with squared error loss. Although expected squared error loss is not usually considered when choosing sample size, it is frequently used as a metric for assessing performance of estimators or model selection methods and is therefore an important measure of value. For the conjugate normal case (A1), the expected reduction in squared error from use of the prior mean is

$$v_n = E\{(\mu_0-\mu)^2\} - E\left\{\left(\frac{n_0\mu_0 + n\bar{x}_n}{n_0+n} - \mu\right)^2\right\}, \qquad \text{(A7)}$$

because the posterior mean is the weighted average, shown in the second term, of the prior mean μ_0 and the observed mean $\bar{x}_n$. Note that the first term is just var(μ). Using the standard identities $\mathrm{var}(\mu) = E\{\mathrm{var}(\mu|\sigma^2)\} + \mathrm{var}\{E(\mu|\sigma^2)\} = E(\sigma^2)/n_0$, expected squared error equals variance plus squared bias, and unconditional expectation equals the expectation of conditional expectation, we have:

$$v_n = E(\sigma^2)/n_0 - E\left\{\mathrm{var}\left(\frac{n\bar{x}_n}{n_0+n}\right) + E^2\left(\frac{n_0\mu_0 + n\bar{x}_n}{n_0+n} - \mu\right) \mid \mu\right\}$$

$$= E(\sigma^2)/n_0 - \left(\frac{n}{n_0+n}\right)^2 \frac{E(\sigma^2)}{n} - E\left\{\left(\frac{n_0\mu_0 + n\mu}{n_0+n} - \mu\frac{n_0+n}{n_0+n}\right)^2\right\}$$

$$= E(\sigma^2)\left(\frac{1}{n_0} - \frac{n}{(n_0+n)^2}\right) - E\left\{\left(\frac{n_0(\mu_0-\mu)}{n_0+n}\right)^2\right\}$$

$$= E(\sigma^2)\left(\frac{1}{n_0} - \frac{n}{(n_0+n)^2}\right) - \left(\frac{n_0}{n_0+n}\right)^2 E\{(\mu_0-\mu)^2\}$$

$$= E(\sigma^2)\left(\frac{1}{n_0} - \frac{n}{(n_0+n)^2} - \frac{n_0^2}{n_0(n_0+n)^2}\right)$$

$$\propto \frac{(n_0+n)^2 - n_0 n - n_0^2}{n_0(n_0+n)^2}$$

$$= \frac{n}{n_0(n_0+n)}$$

$$\propto \frac{n}{n_0+n}. \qquad \text{(A8)}$$

For evaluating when condition B_{root} holds, we have

$$\frac{\partial}{\partial n}(n^{-1/2}v_n) \propto \frac{\partial}{\partial n}\{n^{1/2}(n+n_0)^{-1}\}$$

$$= (1/2)n^{-1/2}(n_0+n)^{-1} - n^{1/2}(n_0+n)^{-2}$$

$$= n^{-1/2}(n_0+n)^{-2}\{(1/2)(n_0+n) - n\}$$

$$= (1/2)n^{-1/2}(n_0+n)^{-2}(n_0-n).$$

This is negative when the last term is negative, i.e., whenever $n>n_0$, and B_{root} therefore holds for $n^*>n_0$. Similar evaluation for condition B_{min} shows that

$$\frac{\partial}{\partial n}(n^{-1}v_n) \propto -(n+n_0)^{-2} \quad ,$$

so B_{min} holds for any n^*.

Because (A7) also holds for the case where μ is the probability of a Bernoulli outcome and has a conjugate beta prior, a similar derivation shows that (A8) also holds for study value in this case, implying the same results for when B_{root} and B_{min} hold.

For a frequentist definition of study value, we use the reduction in expected squared error from using $\bar{x}_n$ rather than $\bar{x}_1$. We immediately have $v_n = \sigma^2(1-n^{-1})$ for any distribution with finite variance σ^2. Direct calculation shows that this meets B_{root} for $n^*>2$ and B_{min} for $n^*>1$. Figure 1 shows this measure and uses (A8) for the Bayesian case.

Web Appendix B: Calculations for Propositions 9 and 10

From Proposition 5, the cost efficiency of n_{root} relative to any other n is

$$R = \{ P_{n_{root}} /(2F)\}/\{P_n/(F + nc)\} \ . \tag{B1}$$

Also, n can be expressed as a multiple of n_{root} that depends on $P_{n_{root}}$ and P_n but not on Δ:

$$n = \frac{F}{c}\left(\frac{Z_{P_n} + Z_{0.975}}{Z_{root} + Z_{0.975}}\right)^2 ,$$

where Z_{root} is the $P_{n_{root}}$ point of the standard normal and we have used the identity from Proposition 5, $n_{\text{root}} = F/c$. Substituting this into (B1) leads to

$$R = \frac{P_{n_{root}}}{2P_n}\left\{1 + \left(\frac{Z_{P_n} + Z_{0.975}}{Z_{root} + Z_{0.975}}\right)^2\right\}.$$

Numerical evaluation of this expression verifies Propositions 9 and 10. The bound in Proposition 10 of 81% is attained when $P_{n_{root}}$=0.19, in which case the optimal cost efficiency is at $P_n = 0.52$.

Web Appendix C: Supplemental material on Bayesian decision theory comparing normal means with conjugate priors

This provides analytical confirmation that study value defined by (A5) for deciding between two treatments meets condition B_{min} under equipoise with unequal amounts of prior information and potentially different variances. The Maple 9.00 (Maplesoft, Inc., Waterloo, Canada) commands below were used to verify concavity via the second derivative condition.

```
> assume(n>0);assume(tau>0);assume(tau0>0);assume(tau1>0); assume(tau2>0);
assume(tau01>0); assume(tau02>0);
> normpdf:=(x, tau)->exp(-x*x/2*tau^2)/sqrt(2*Pi)*tau;
#define normal density with precision tau
```

$$normpdf := (x, tau) \rightarrow \frac{e^{\left(-x\,x\,\frac{1}{2}\,\tau^2\right)}\,\tau}{\sqrt{2\,\pi}}$$

```
> int(normpdf(x, tau), x=-infinity..infinity);#check
```

$$1$$

```
> capphi:=x->erf(sqrt(2)*x/2)/2+1/2;
#Define normal cdf
```

$$capphi := x \rightarrow \frac{1}{2}\,\mathrm{erf}\left(\frac{1}{2}\sqrt{2}\,x\right) + \frac{1}{2}$$

```
> evalf(capphi(-1.645));evalf(capphi(1.645));#check
```

$$0.0499849056$$

$$0.9500150944$$

```
> #define integral elements
```

```
dn:=(mu1,mu2,mu0, n)->(tau01^2*mu0+n*tau1^2*mu1)/(tau01^2+n*tau1^2)-
(tau02^2*mu0+n*tau2^2*mu2)/(tau02^2+n*tau2^2);
```

$$dn := (mu1, mu2, mu0, n) \rightarrow \frac{\tau01^2\,\mu0 + n\,\tau1^2\,\mu1}{\tau01^2 + n\,\tau1^2} - \frac{\tau02^2\,\mu0 + n\,\tau2^2\,\mu2}{\tau02^2 + n\,\tau2^2}$$

Note that we have changed notation in order to simplify the calculations:

$\tau1^2 = 1/\sigma_1^2$ $\quad$ $\tau01^2 = n_{01}/\sigma_1^2$ $\quad$ $\tau2^2 = 1/\sigma_2^2$ $\quad$ $\tau02^2 = n_{02}/\sigma_2^2$

```
➢ sn2:=n->n*tau1^2/(tau01^2+n*tau1^2)^2 + n*tau2^2/(tau02^2+n*tau2^2)^2;
```

$$sn2 := n \rightarrow \frac{n\,\tau1^2}{(\tau01^2 + n\,\tau1^2)^2} + \frac{n\,\tau2^2}{(\tau02^2 + n\,\tau2^2)^2}$$

```
➢ #calculate integral of interest, after taking second derivative inside integrations
```

The command below evaluates the inner two integrals of (A5) after taking the second derivative of the integrand u_n with respect to n. Note that we have assumed without loss of generality that μ_0=0, and diff(· ,n$2) takes the second derivative.

```
➢ answ:=int(simplify(int((mu1-mu2)*diff(capphi(dn(mu1,mu2,0,n)/sqrt(sn2(n))),n$2)
*normpdf(mu1, tau01), mu1=-infinity..infinity))*normpdf(mu2, tau02), mu2=-
infinity..infinity);
```

$$answ :=$$

$$-(\sqrt{2}\,\tau02\sim(4\,n\sim^6\,\tau2\sim^8\,\tau1\sim^8 + 2\,\tau02\sim^6\,\tau01\sim^6\,\tau2\sim^2\,\tau1\sim^2 + \tau02\sim^2\,\tau01\sim^{10}\,\tau2\sim^4 + \tau01\sim^2\,\tau02\sim^{10}\,\tau1\sim^4$$
$$+ 4\,n\sim^6\,\tau2\sim^{10}\,\tau1\sim^6 + n\sim^5\,\tau1\sim^8\,\tau2\sim^6\,\tau01\sim^2 - 4\,n\sim^5\,\tau1\sim^8\,\tau2\sim^6\,\tau02\sim^2 + 20\,n\sim^5\,\tau1\sim^6\,\tau2\sim^8\,\tau02\sim^2$$
$$+ n\sim^5\,\tau2\sim^{10}\,\tau01\sim^2\,\tau1\sim^4 - 8\,n\sim^4\,\tau1\sim^8\,\tau2\sim^4\,\tau02\sim^4 + 8\,n\sim^4\,\tau1\sim^6\,\tau2\sim^6\,\tau01\sim^4$$

$$+40\,n\sim^{4}\,\tau1\sim^{6}\,\tau2\sim^{6}\,\tau02\sim^{4}+26\,n\sim^{5}\,\tau1\sim^{6}\,\tau2\sim^{8}\,\tau01\sim^{2}+40\,n\sim^{3}\,\tau1\sim^{6}\,\tau2\sim^{4}\,\tau02\sim^{6}$$

$$+10\,n\sim^{3}\,\tau1\sim^{4}\,\tau2\sim^{6}\,\tau01\sim^{6}+10\,n\sim^{3}\,\tau2\sim^{8}\,\tau01\sim^{6}\,\tau1\sim^{2}+20\,n\sim^{2}\,\tau1\sim^{6}\,\tau2\sim^{2}\,\tau02\sim^{8}$$

$$-3\,n\sim^{4}\,\tau1\sim^{8}\,\tau2\sim^{4}\,\tau01\sim^{2}\,\tau02\sim^{2}+26\,n\sim^{4}\,\tau1\sim^{6}\,\tau2\sim^{6}\,\tau01\sim^{2}\,\tau02\sim^{2}$$

$$+5\,n\sim^{4}\,\tau1\sim^{4}\,\tau2\sim^{8}\,\tau01\sim^{2}\,\tau02\sim^{2}+32\,n\sim^{4}\,\tau1\sim^{4}\,\tau2\sim^{8}\,\tau01\sim^{4}+20\,n\sim^{2}\,\tau1\sim^{4}\,\tau2\sim^{4}\,\tau02\sim^{4}\,\tau01\sim^{4}$$

$$+10\,n\sim^{2}\,\tau1\sim^{4}\,\tau2\sim^{4}\,\tau02\sim^{6}\,\tau01\sim^{2}+10\,n\sim^{2}\,\tau1\sim^{4}\,\tau2\sim^{4}\,\tau02\sim^{2}\,\tau01\sim^{6}$$

$$+10\,n\sim^{2}\,\tau2\sim^{6}\,\tau02\sim^{2}\,\tau01\sim^{6}\,\tau1\sim^{2}+4\,n\sim\tau02\sim^{10}\,\tau1\sim^{6}+10\,n\sim^{3}\,\tau1\sim^{6}\,\tau2\sim^{4}\,\tau02\sim^{4}\,\tau01\sim^{2}$$

$$+10\,n\sim^{3}\,\tau1\sim^{4}\,\tau2\sim^{6}\,\tau02\sim^{4}\,\tau01\sim^{2}+40\,n\sim^{3}\,\tau1\sim^{4}\,\tau2\sim^{6}\,\tau01\sim^{4}\,\tau02\sim^{2}$$

$$+10\,n\sim^{2}\,\tau1\sim^{6}\,\tau2\sim^{2}\,\tau02\sim^{6}\,\tau01\sim^{2}-3\,n\sim\tau01\sim^{10}\,\tau2\sim^{6}+5\,n\sim\tau1\sim^{4}\,\tau2\sim^{2}\,\tau01\sim^{2}\,\tau02\sim^{8}$$

$$+12\,n\sim\tau1\sim^{4}\,\tau2\sim^{2}\,\tau02\sim^{6}\,\tau01\sim^{4}+2\,n\sim\tau1\sim^{2}\,\tau2\sim^{4}\,\tau01\sim^{6}\,\tau02\sim^{4}+8\,n\sim\tau1\sim^{2}\,\tau2\sim^{4}\,\tau02\sim^{2}\,\tau01\sim^{8})$$

$$\tau1\sim^{2}\,\tau01\sim)\Big/\Big(8\,(n\sim\tau1\sim^{4}\,\tau02\sim^{4}+2\,n\sim^{2}\,\tau1\sim^{4}\,\tau02\sim^{2}\,\tau2\sim^{2}+n\sim^{3}\,\tau1\sim^{4}\,\tau2\sim^{4}+\tau01\sim^{2}\,\tau1\sim^{2}\,\tau02\sim^{4}$$

$$+2\,\tau01\sim^{2}\,\tau1\sim^{2}\,\tau02\sim^{2}\,n\sim\tau2\sim^{2}+n\sim^{2}\,\tau2\sim^{4}\,\tau01\sim^{2}\,\tau1\sim^{2}+\tau01\sim^{6}\,\tau2\sim^{2}+2\,\tau01\sim^{4}\,\tau2\sim^{2}\,n\sim\tau1\sim^{2}$$

$$+\tau01\sim^{2}\,\tau2\sim^{2}\,n\sim^{2}\,\tau1\sim^{4})\text{^}(3/2)\,(\tau01\sim^{2}+n\sim\tau1\sim^{2})\,\sqrt{\pi}\,n\sim^{(3/2)}\quad \mathrm{sqrt}($$

$$
\begin{aligned}
&(n\sim\tau2\sim^{4}\tau01\sim^{4}+n\sim^{2}\tau2\sim^{4}\tau01\sim^{2}\tau1\sim^{2}+\tau02\sim^{2}\tau1\sim^{2}n\sim^{2}\tau2\sim^{4}+2\tau02\sim^{4}\tau1\sim^{2}n\sim\tau2\sim^{2}\\
&+\tau01\sim^{2}\tau1\sim^{2}\tau02\sim^{2}n\sim\tau2\sim^{2}+\tau02\sim^{6}\tau1\sim^{2}+\tau02\sim^{2}\tau2\sim^{2}\tau01\sim^{4})(\tau1\sim^{2}n\sim^{2}\tau2\sim^{4}\\
&+2\tau1\sim^{2}\tau02\sim^{2}n\sim\tau2\sim^{2}+\tau2\sim^{2}\tau01\sim^{2}n\sim\tau1\sim^{2}+\tau1\sim^{2}\tau02\sim^{4}+\tau2\sim^{2}\tau01\sim^{4}))\Big)\\
&-(\tau01\sim\tau2\sim^{2}(\tau2\sim^{6}\tau01\sim^{16}+3\tau2\sim^{2}\tau02\sim^{8}\tau01\sim^{8}\tau1\sim^{4}+\tau02\sim^{12}\tau01\sim^{4}\tau1\sim^{6}\\
&+3\tau2\sim^{4}\tau01\sim^{12}\tau02\sim^{4}\tau1\sim^{2}+7n\sim^{6}\tau1\sim^{10}\tau2\sim^{8}\tau01\sim^{4}+12n\sim^{6}\tau1\sim^{10}\tau2\sim^{8}\tau02\sim^{4}\\
&+20n\sim^{6}\tau1\sim^{10}\tau2\sim^{8}\tau01\sim^{2}\tau02\sim^{2}-4n\sim^{6}\tau1\sim^{8}\tau2\sim^{10}\tau01\sim^{2}\tau02\sim^{2}-6n\sim^{6}\tau1\sim^{8}\tau01\sim^{4}\tau2\sim^{10}\\
&-n\sim^{6}\tau2\sim^{12}\tau01\sim^{4}\tau1\sim^{6}+4n\sim^{5}\tau1\sim^{10}\tau2\sim^{6}\tau01\sim^{4}\tau02\sim^{2}+n\sim^{5}\tau1\sim^{10}\tau2\sim^{6}\tau01\sim^{6}\\
&+32n\sim^{5}\tau1\sim^{10}\tau2\sim^{6}\tau02\sim^{4}\tau01\sim^{2}+36n\sim^{5}\tau1\sim^{10}\tau2\sim^{6}\tau02\sim^{6}+34n\sim^{5}\tau1\sim^{8}\tau2\sim^{8}\tau01\sim^{6}\\
&+8n\sim^{5}\tau1\sim^{8}\tau2\sim^{8}\tau02\sim^{4}\tau01\sim^{2}+56n\sim^{5}\tau1\sim^{8}\tau01\sim^{4}\tau2\sim^{8}\tau02\sim^{2}-15n\sim^{5}\tau1\sim^{6}\tau01\sim^{6}\tau2\sim^{10}\\
&-8n\sim^{5}\tau1\sim^{6}\tau2\sim^{10}\tau01\sim^{4}\tau02\sim^{2}+5n\sim^{4}\tau1\sim^{10}\tau2\sim^{4}\tau02\sim^{4}\tau01\sim^{4}+36n\sim^{4}\tau1\sim^{10}\tau2\sim^{4}\tau02\sim^{8}\\
&+16n\sim^{4}\tau1\sim^{10}\tau2\sim^{4}\tau02\sim^{6}\tau01\sim^{2}+100n\sim^{4}\tau1\sim^{8}\tau2\sim^{6}\tau02\sim^{4}\tau01\sim^{4}+5n\sim^{4}\tau1\sim^{8}\tau2\sim^{6}\tau01\sim^{8}\\
&+48n\sim^{4}\tau1\sim^{8}\tau2\sim^{6}\tau02\sim^{6}\tau01\sim^{2}+16n\sim^{4}\tau1\sim^{8}\tau2\sim^{6}\tau02\sim^{2}\tau01\sim^{6}-9n\sim^{4}\tau2\sim^{10}\tau01\sim^{8}\tau1\sim^{4}
\end{aligned}
$$

$$
\begin{aligned}
&-9\,n\sim^{4}\tau1\sim^{6}\tau2\sim^{8}\tau02\sim^{4}\tau01\sim^{4}+48\,n\sim^{4}\tau1\sim^{6}\tau2\sim^{8}\tau02\sim^{2}\tau01\sim^{6}+64\,n\sim^{4}\tau1\sim^{6}\tau2\sim^{8}\tau01\sim^{8}\\
&+12\,n\sim^{3}\tau1\sim^{10}\tau2\sim^{2}\tau02\sim^{10}+4\,n\sim^{3}\tau1\sim^{10}\tau2\sim^{2}\tau02\sim^{8}\tau01\sim^{2}+56\,n\sim^{3}\tau1\sim^{8}\tau2\sim^{4}\tau02\sim^{8}\tau01\sim^{2}\\
&+44\,n\sim^{3}\tau1\sim^{8}\tau2\sim^{4}\tau01\sim^{4}\tau02\sim^{6}+14\,n\sim^{3}\tau1\sim^{8}\tau2\sim^{4}\tau01\sim^{6}\tau02\sim^{4}+54\,n\sim^{3}\tau1\sim^{4}\tau2\sim^{8}\tau01\sim^{10}\\
&+32\,n\sim^{3}\tau1\sim^{6}\tau2\sim^{6}\tau02\sim^{2}\tau01\sim^{8}+110\,n\sim^{3}\tau1\sim^{6}\tau2\sim^{6}\tau01\sim^{6}\tau02\sim^{4}+10\,n\sim^{3}\tau1\sim^{6}\tau01\sim^{10}\tau2\sim^{6}\\
&+12\,n\sim^{3}\tau1\sim^{6}\tau2\sim^{6}\tau01\sim^{4}\tau02\sim^{6}+12\,n\sim^{3}\tau1\sim^{4}\tau2\sim^{8}\tau02\sim^{2}\tau01\sim^{8}\\
&+20\,n\sim^{2}\tau1\sim^{8}\tau2\sim^{2}\tau01\sim^{2}\tau02\sim^{10}+6\,n\sim^{2}\tau1\sim^{8}\tau2\sim^{2}\tau01\sim^{4}\tau02\sim^{8}\\
&+16\,n\sim^{2}\tau1\sim^{6}\tau2\sim^{4}\tau02\sim^{4}\tau01\sim^{8}+25\,n\sim^{2}\tau1\sim^{6}\tau2\sim^{4}\tau01\sim^{4}\tau02\sim^{8}\\
&+52\,n\sim^{2}\tau1\sim^{6}\tau2\sim^{4}\tau01\sim^{6}\tau02\sim^{6}+10\,n\sim^{2}\tau1\sim^{4}\tau2\sim^{6}\tau01\sim^{12}+42\,n\sim^{2}\tau1\sim^{4}\tau2\sim^{6}\tau02\sim^{4}\tau01\sim^{8}\\
&+32\,n\sim^{2}\tau1\sim^{4}\tau2\sim^{6}\tau01\sim^{10}\tau02\sim^{2}+17\,n\sim^{2}\tau2\sim^{8}\tau01\sim^{12}\tau1\sim^{2}+5\,n\sim\tau1\sim^{6}\tau2\sim^{2}\tau01\sim^{6}\tau02\sim^{8}\\
&+24\,n\sim\tau1\sim^{4}\tau2\sim^{4}\tau02\sim^{6}\tau01\sim^{8}+5\,n\sim\tau1\sim^{2}\tau2\sim^{6}\tau01\sim^{14}+12\,n\sim\tau1\sim^{6}\tau2\sim^{2}\tau01\sim^{4}\tau02\sim^{10}\\
&+10\,n\sim\tau1\sim^{4}\tau2\sim^{4}\tau01\sim^{10}\tau02\sim^{4}+12\,n\sim\tau1\sim^{2}\tau2\sim^{6}\tau01\sim^{12}\tau02\sim^{2})\,\tau02\sim\sqrt{2}\Big)\Big/\Big(8\,(\tau1\sim^{2}n\sim^{2}\\
&\tau2\sim^{4}+2\,\tau1\sim^{2}\tau02\sim^{2}n\sim\tau2\sim^{2}+\tau2\sim^{2}\tau01\sim^{2}n\sim\tau1\sim^{2}+\tau1\sim^{2}\tau02\sim^{4}+\tau2\sim^{2}\tau01\sim^{4})\,(\tau01\sim^{2}
\end{aligned}
$$

$$+ n\sim \tau 1\sim^{2}) \operatorname{sqrt}($$

$$n\sim \tau 1\sim^{4} \tau 02\sim^{4} + 2\, n\sim^{2} \tau 1\sim^{4} \tau 02\sim^{2} \tau 2\sim^{2} + n\sim^{3} \tau 1\sim^{4} \tau 2\sim^{4} + \tau 01\sim^{2} \tau 1\sim^{2} \tau 02\sim^{4}$$

$$+ 2\, \tau 01\sim^{2} \tau 1\sim^{2} \tau 02\sim^{2} n\sim \tau 2\sim^{2} + n\sim^{2} \tau 2\sim^{4} \tau 01\sim^{2} \tau 1\sim^{2} + \tau 01\sim^{6} \tau 2\sim^{2} + 2\, \tau 01\sim^{4} \tau 2\sim^{2} n\sim \tau 1\sim^{2}$$

$$+ \tau 01\sim^{2} \tau 2\sim^{2} n\sim^{2} \tau 1\sim^{4}) \sqrt{\pi}\, n\sim^{(3/2)} \operatorname{sqrt}($$

$$(n\sim \tau 2\sim^{4} \tau 01\sim^{4} + n\sim^{2} \tau 2\sim^{4} \tau 01\sim^{2} \tau 1\sim^{2} + \tau 02\sim^{2} \tau 1\sim^{2} n\sim^{2} \tau 2\sim^{4} + 2\, \tau 02\sim^{4} \tau 1\sim^{2} n\sim \tau 2\sim^{2}$$

$$+ \tau 01\sim^{2} \tau 1\sim^{2} \tau 02\sim^{2} n\sim \tau 2\sim^{2} + \tau 02\sim^{6} \tau 1\sim^{2} + \tau 02\sim^{2} \tau 2\sim^{2} \tau 01\sim^{4}) (\tau 1\sim^{2} n\sim^{2} \tau 2\sim^{4}$$

$$+ 2\, \tau 1\sim^{2} \tau 02\sim^{2} n\sim \tau 2\sim^{2} + \tau 2\sim^{2} \tau 01\sim^{2} n\sim \tau 1\sim^{2} + \tau 1\sim^{2} \tau 02\sim^{4} + \tau 2\sim^{2} \tau 01\sim^{4})) (n\sim \tau 2\sim^{4} \tau 01\sim^{4}$$

$$+ n\sim^{2} \tau 2\sim^{4} \tau 01\sim^{2} \tau 1\sim^{2} + \tau 02\sim^{2} \tau 1\sim^{2} n\sim^{2} \tau 2\sim^{4} + 2\, \tau 02\sim^{4} \tau 1\sim^{2} n\sim \tau 2\sim^{2}$$

$$\left. + \tau 01\sim^{2} \tau 1\sim^{2} \tau 02\sim^{2} n\sim \tau 2\sim^{2} + \tau 02\sim^{6} \tau 1\sim^{2} + \tau 02\sim^{2} \tau 2\sim^{2} \tau 01\sim^{4}) \right)$$

$$^{-} (3 \sqrt{2}\, \tau 02\sim (2\, \tau 02\sim^{4} \tau 01\sim^{8} \tau 2\sim^{2} \tau 1\sim^{2} + \tau 01\sim^{4} \tau 02\sim^{8} \tau 1\sim^{4} + n\sim^{4} \tau 1\sim^{8} \tau 2\sim^{4} \tau 01\sim^{4}$$

$$+ 6\, n\sim^{2} \tau 1\sim^{4} \tau 01\sim^{8} \tau 2\sim^{4} + 8\, n\sim^{3} \tau 01\sim^{4} \tau 1\sim^{6} \tau 2\sim^{4} \tau 02\sim^{2} + 4\, n\sim^{2} \tau 1\sim^{8} \tau 02\sim^{8}$$

$$+ 4\, n\sim^{3} \tau 1\sim^{6} \tau 2\sim^{4} \tau 01\sim^{6} + 4\, n\sim^{3} \tau 1\sim^{8} \tau 2\sim^{2} \tau 02\sim^{4} \tau 01\sim^{2} + 8\, n\sim^{3} \tau 1\sim^{8} \tau 2\sim^{2} \tau 02\sim^{6}$$

$$+10\,n\sim^{2}\,\tau1\sim^{6}\,\tau2\sim^{2}\,\tau02\sim^{4}\,\tau01\sim^{4}-2\,n\sim^{2}\,\tau2\sim^{6}\,\tau01\sim^{8}\,\tau1\sim^{2}+4\,n\sim\tau02\sim^{8}\,\tau01\sim^{2}\,\tau1\sim^{6}$$

$$+8\,n\sim\tau01\sim^{6}\,\tau2\sim^{2}\,\tau02\sim^{4}\,\tau1\sim^{4}+4\,n\sim\tau2\sim^{4}\,\tau01\sim^{10}\,\tau1\sim^{2}+\tau01\sim^{12}\,\tau2\sim^{4}$$

$$+4\,n\sim^{4}\,\tau1\sim^{8}\,\tau2\sim^{4}\,\tau02\sim^{4}-2\,n\sim^{4}\,\tau1\sim^{6}\,\tau2\sim^{6}\,\tau01\sim^{4}-4\,n\sim^{3}\,\tau1\sim^{4}\,\tau2\sim^{6}\,\tau01\sim^{6}$$

$$+4\,n\sim^{4}\,\tau1\sim^{8}\,\tau2\sim^{4}\,\tau01\sim^{2}\,\tau02\sim^{2}-4\,n\sim^{4}\,\tau1\sim^{6}\,\tau2\sim^{6}\,\tau01\sim^{2}\,\tau02\sim^{2}+n\sim^{4}\,\tau1\sim^{4}\,\tau2\sim^{8}\,\tau01\sim^{4}$$

$$-2\,n\sim^{2}\,\tau1\sim^{4}\,\tau2\sim^{4}\,\tau02\sim^{4}\,\tau01\sim^{4}+4\,n\sim^{2}\,\tau1\sim^{4}\,\tau2\sim^{4}\,\tau02\sim^{2}\,\tau01\sim^{6}-4\,n\sim^{3}\,\tau1\sim^{6}\,\tau2\sim^{4}\,\tau02\sim^{4}\,\tau01\sim^{2}$$

$$+4\,n\sim^{2}\,\tau1\sim^{6}\,\tau2\sim^{2}\,\tau02\sim^{6}\,\tau01\sim^{2})\,\tau2\sim^{6}\,\tau01\sim^{3})\Big/(8\,(\tau1\sim^{2}\,n\sim^{2}\,\tau2\sim^{4}+2\,\tau1\sim^{2}\,\tau02\sim^{2}\,n\sim\tau2\sim^{2}$$

$$+\tau2\sim^{2}\,\tau01\sim^{2}\,n\sim\tau1\sim^{2}+\tau1\sim^{2}\,\tau02\sim^{4}+\tau2\sim^{2}\,\tau01\sim^{4})\,\mathrm{sqrt}($$

$$n\sim\tau1\sim^{4}\,\tau02\sim^{4}+2\,n\sim^{2}\,\tau1\sim^{4}\,\tau02\sim^{2}\,\tau2\sim^{2}+n\sim^{3}\,\tau1\sim^{4}\,\tau2\sim^{4}+\tau01\sim^{2}\,\tau1\sim^{2}\,\tau02\sim^{4}$$

$$+2\,\tau01\sim^{2}\,\tau1\sim^{2}\,\tau02\sim^{2}\,n\sim\tau2\sim^{2}+n\sim^{2}\,\tau2\sim^{4}\,\tau01\sim^{2}\,\tau1\sim^{2}+\tau01\sim^{6}\,\tau2\sim^{2}+2\,\tau01\sim^{4}\,\tau2\sim^{2}\,n\sim\tau1\sim^{2}$$

$$+\tau01\sim^{2}\,\tau2\sim^{2}\,n\sim^{2}\,\tau1\sim^{4})\,\sqrt{\pi}\,\mathrm{sqrt}($$

$$(n\sim\tau2\sim^{4}\,\tau01\sim^{4}+n\sim^{2}\,\tau2\sim^{4}\,\tau01\sim^{2}\,\tau1\sim^{2}+\tau02\sim^{2}\,\tau1\sim^{2}\,n\sim^{2}\,\tau2\sim^{4}+2\,\tau02\sim^{4}\,\tau1\sim^{2}\,n\sim\tau2\sim^{2}$$

$$+\tau01\sim^{2}\,\tau1\sim^{2}\,\tau02\sim^{2}\,n\sim\tau2\sim^{2}+\tau02\sim^{6}\,\tau1\sim^{2}+\tau02\sim^{2}\,\tau2\sim^{2}\,\tau01\sim^{4})\,(\tau1\sim^{2}\,n\sim^{2}\,\tau2\sim^{4}$$

$$+2\,\tau1\sim^{2}\,\tau02\sim^{2}\,n\sim\tau2\sim^{2}+\tau2\sim^{2}\,\tau01\sim^{2}\,n\sim\tau1\sim^{2}+\tau1\sim^{2}\,\tau02\sim^{4}+\tau2\sim^{2}\,\tau01\sim^{4}))\,(\tau02\sim^{2}+n\sim\tau2\sim^{2})$$

$$(n\sim\tau 2\sim^{4}\tau 01\sim^{4}+n\sim^{2}\tau 2\sim^{4}\tau 01\sim^{2}\tau 1\sim^{2}+\tau 02\sim^{2}\tau 1\sim^{2}n\sim^{2}\tau 2\sim^{4}+2\,\tau 02\sim^{4}\tau 1\sim^{2}n\sim\tau 2\sim^{2}$$

$$+\tau 01\sim^{2}\tau 1\sim^{2}\tau 02\sim^{2}n\sim\tau 2\sim^{2}+\tau 02\sim^{6}\tau 1\sim^{2}+\tau 02\sim^{2}\tau 2\sim^{2}\tau 01\sim^{4})\sqrt{n\sim})$$

```
> simplify(answ);
```

$$-(\tau 01\sim\tau 02\sim\sqrt{2}\,(4\,n\sim\tau 02\sim^{14}\tau 1\sim^{8}+4\,n\sim^{6}\tau 1\sim^{10}\tau 2\sim^{8}\tau 01\sim^{4}+12\,n\sim^{6}\tau 1\sim^{10}\tau 2\sim^{8}\tau 02\sim^{4}$$

$$+13\,n\sim^{6}\tau 1\sim^{10}\tau 2\sim^{8}\tau 01\sim^{2}\tau 02\sim^{2}+38\,n\sim^{6}\tau 1\sim^{8}\tau 2\sim^{10}\tau 01\sim^{2}\tau 02\sim^{2}+20\,n\sim^{6}\tau 1\sim^{8}\tau 01\sim^{4}\tau 2\sim^{10}$$

$$+4\,n\sim^{6}\tau 2\sim^{12}\tau 01\sim^{4}\tau 1\sim^{6}+n\sim^{5}\tau 1\sim^{10}\tau 2\sim^{6}\tau 01\sim^{4}\tau 02\sim^{2}+10\,n\sim^{5}\tau 1\sim^{10}\tau 2\sim^{6}\tau 02\sim^{4}\tau 01\sim^{2}$$

$$+12\,n\sim^{5}\tau 1\sim^{10}\tau 2\sim^{6}\tau 02\sim^{6}+20\,n\sim^{5}\tau 1\sim^{8}\tau 2\sim^{8}\tau 01\sim^{6}+100\,n\sim^{5}\tau 1\sim^{8}\tau 2\sim^{8}\tau 02\sim^{4}\tau 01\sim^{2}$$

$$+66\,n\sim^{5}\tau 1\sim^{8}\tau 01\sim^{4}\tau 2\sim^{8}\tau 02\sim^{2}+32\,n\sim^{5}\tau 1\sim^{6}\tau 01\sim^{6}\tau 2\sim^{10}+3\,\tau 02\sim^{10}\tau 01\sim^{6}\tau 2\sim^{2}\tau 1\sim^{4}$$

$$+41\,n\sim^{5}\tau 1\sim^{6}\tau 2\sim^{10}\tau 01\sim^{4}\tau 02\sim^{2}+4\,n\sim^{4}\tau 1\sim^{10}\tau 2\sim^{4}\tau 02\sim^{8}+3\,\tau 02\sim^{6}\tau 01\sim^{10}\tau 2\sim^{4}\tau 1\sim^{2}$$

$$+4\,n\sim\tau 01\sim^{14}\tau 2\sim^{8}+n\sim^{4}\tau 1\sim^{10}\tau 2\sim^{4}\tau 02\sim^{6}\tau 01\sim^{2}+56\,n\sim^{4}\tau 1\sim^{8}\tau 2\sim^{6}\tau 02\sim^{4}\tau 01\sim^{4}$$

$$+112\,n\sim^{4}\tau 1\sim^{8}\tau 2\sim^{6}\tau 02\sim^{6}\tau 01\sim^{2}+5\,n\sim^{4}\tau 1\sim^{8}\tau 2\sim^{6}\tau 02\sim^{2}\tau 01\sim^{6}+20\,n\sim^{4}\tau 2\sim^{10}\tau 01\sim^{8}\tau 1\sim^{4}$$

$$+112\,n\sim^{4}\tau 1\sim^{6}\tau 2\sim^{8}\tau 02\sim^{4}\tau 01\sim^{4}+108\,n\sim^{4}\tau 1\sim^{6}\tau 2\sim^{8}\tau 02\sim^{2}\tau 01\sim^{6}+40\,n\sim^{4}\tau 1\sim^{6}\tau 2\sim^{8}\tau 01\sim^{8}$$

$$+56\,n\sim^{3}\tau 1\sim^{8}\tau 2\sim^{4}\tau 02\sim^{8}\tau 01\sim^{2}+10\,n\sim^{3}\tau 1\sim^{8}\tau 2\sim^{4}\tau 01\sim^{4}\tau 02\sim^{6}+40\,n\sim^{3}\tau 1\sim^{4}\tau 2\sim^{8}\tau 01\sim^{10}$$

$$+10\,n\sim^{3}\,\tau1\sim^{6}\,\tau2\sim^{6}\,\tau02\sim^{2}\,\tau01\sim^{8}+96\,n\sim^{3}\,\tau1\sim^{6}\,\tau2\sim^{6}\,\tau01\sim^{6}\,\tau02\sim^{4}$$

$$+130\,n\sim^{3}\,\tau1\sim^{6}\,\tau2\sim^{6}\,\tau01\sim^{4}\,\tau02\sim^{6}+70\,n\sim^{3}\,\tau1\sim^{4}\,\tau2\sim^{8}\,\tau02\sim^{2}\,\tau01\sim^{8}$$

$$+10\,n\sim^{2}\,\tau1\sim^{8}\,\tau2\sim^{2}\,\tau01\sim^{2}\,\tau02\sim^{10}+68\,n\sim^{2}\,\tau1\sim^{6}\,\tau2\sim^{4}\,\tau01\sim^{4}\,\tau02\sim^{8}$$

$$+20\,n\sim^{2}\,\tau1\sim^{6}\,\tau2\sim^{4}\,\tau01\sim^{6}\,\tau02\sim^{6}+64\,n\sim^{2}\,\tau1\sim^{4}\,\tau2\sim^{6}\,\tau02\sim^{4}\,\tau01\sim^{8}$$

$$+10\,n\sim^{2}\,\tau1\sim^{4}\,\tau2\sim^{6}\,\tau01\sim^{10}\,\tau02\sim^{2}+20\,n\sim^{2}\,\tau2\sim^{8}\,\tau01\sim^{12}\,\tau1\sim^{2}+14\,n\sim\tau1\sim^{4}\,\tau2\sim^{4}\,\tau02\sim^{6}\,\tau01\sim^{8}$$

$$+13\,n\sim\tau1\sim^{6}\,\tau2\sim^{2}\,\tau01\sim^{4}\,\tau02\sim^{10}+5\,n\sim\tau1\sim^{2}\,\tau2\sim^{6}\,\tau01\sim^{12}\,\tau02\sim^{2}+4\,n\sim^{7}\,\tau1\sim^{10}\,\tau2\sim^{10}\,\tau02\sim^{2}$$

$$+4\,n\sim^{7}\,\tau1\sim^{10}\,\tau2\sim^{10}\,\tau01\sim^{2}+4\,n\sim^{7}\,\tau1\sim^{8}\,\tau2\sim^{12}\,\tau02\sim^{2}+4\,n\sim^{7}\,\tau1\sim^{8}\,\tau2\sim^{12}\,\tau01\sim^{2}$$

$$+24\,n\sim^{6}\,\tau1\sim^{8}\,\tau2\sim^{10}\,\tau02\sim^{4}+n\sim^{6}\,\tau1\sim^{6}\,\tau2\sim^{12}\,\tau01\sim^{2}\,\tau02\sim^{2}+60\,n\sim^{5}\,\tau1\sim^{8}\,\tau2\sim^{8}\,\tau02\sim^{6}$$

$$+80\,n\sim^{4}\,\tau1\sim^{8}\,\tau2\sim^{6}\,\tau02\sim^{8}+6\,n\sim^{5}\,\tau1\sim^{6}\,\tau2\sim^{10}\,\tau02\sim^{4}\,\tau01\sim^{2}+15\,n\sim^{4}\,\tau1\sim^{6}\,\tau2\sim^{8}\,\tau02\sim^{6}\,\tau01\sim^{2}$$

$$+60\,n\sim^{3}\,\tau1\sim^{8}\,\tau2\sim^{4}\,\tau02\sim^{10}+7\,n\sim^{4}\,\tau1\sim^{4}\,\tau2\sim^{10}\,\tau02\sim^{2}\,\tau01\sim^{6}+20\,n\sim^{3}\,\tau1\sim^{6}\,\tau2\sim^{6}\,\tau02\sim^{8}\,\tau01\sim^{2}$$

$$+24\,n\sim^{3}\,\tau1\sim^{4}\,\tau2\sim^{8}\,\tau02\sim^{4}\,\tau01\sim^{6}+4\,n\sim^{3}\,\tau01\sim^{10}\,\tau2\sim^{10}\,\tau1\sim^{2}+24\,n\sim^{2}\,\tau1\sim^{8}\,\tau2\sim^{2}\,\tau02\sim^{12}$$

$$+15\,n\sim^{2}\,\tau1\sim^{6}\,\tau2\sim^{4}\,\tau01\sim^{2}\,\tau02\sim^{10}+30\,n\sim^{2}\,\tau1\sim^{4}\,\tau2\sim^{6}\,\tau01\sim^{6}\,\tau02\sim^{6}$$

$$+15\,n\sim^{2}\,\tau1\sim^{2}\,\tau2\sim^{8}\,\tau01\sim^{10}\,\tau02\sim^{2}+6\,n\sim\tau1\sim^{6}\,\tau2\sim^{2}\,\tau02\sim^{12}\,\tau01\sim^{2}+\tau02\sim^{14}\,\tau01\sim^{2}\,\tau1\sim^{6}$$

$$+14\,n\sim\tau1\sim^{2}\tau2\sim^{6}\tau02\sim^{4}\tau01\sim^{10}+16\,n\sim\tau1\sim^{4}\tau2\sim^{4}\tau02\sim^{8}\tau01\sim^{6}+\tau01\sim^{14}\tau02\sim^{2}\tau2\sim^{6}))\Big/\Big(8$$

$$(n\sim\tau2\sim^{4}\tau01\sim^{4}+n\sim^{2}\tau2\sim^{4}\tau01\sim^{2}\tau1\sim^{2}+\tau02\sim^{2}\tau1\sim^{2}n\sim^{2}\tau2\sim^{4}+2\,\tau02\sim^{4}\tau1\sim^{2}n\sim\tau2\sim^{2}$$

$$+\tau01\sim^{2}\tau1\sim^{2}\tau02\sim^{2}n\sim\tau2\sim^{2}+\tau02\sim^{6}\tau1\sim^{2}+\tau02\sim^{2}\tau2\sim^{2}\tau01\sim^{4})\text{^}(3/2)$$

$$\sqrt{\tau1\sim^{2}n\sim^{2}\tau2\sim^{4}+2\,\tau1\sim^{2}\tau02\sim^{2}n\sim\tau2\sim^{2}+\tau2\sim^{2}\tau01\sim^{2}n\sim\tau1\sim^{2}+\tau1\sim^{2}\tau02\sim^{4}+\tau2\sim^{2}\tau01\sim^{4}}$$

$$n\sim^{(3/2)}\sqrt{\pi}\,(\tau01\sim^{2}+n\sim\tau1\sim^{2})^{2}\ \mathrm{sqrt}($$

$$n\sim\tau1\sim^{4}\tau02\sim^{4}+2\,n\sim^{2}\tau1\sim^{4}\tau02\sim^{2}\tau2\sim^{2}+n\sim^{3}\tau1\sim^{4}\tau2\sim^{4}+\tau01\sim^{2}\tau1\sim^{2}\tau02\sim^{4}$$

$$+2\,\tau01\sim^{2}\tau1\sim^{2}\tau02\sim^{2}n\sim\tau2\sim^{2}+n\sim^{2}\tau2\sim^{4}\tau01\sim^{2}\tau1\sim^{2}+\tau01\sim^{6}\tau2\sim^{2}+2\,\tau01\sim^{4}\tau2\sim^{2}n\sim\tau1\sim^{2}$$

$$+\tau01\sim^{2}\tau2\sim^{2}n\sim^{2}\tau1\sim^{4})(\tau02\sim^{2}+n\sim\tau2\sim^{2})\Big)$$

```
> #check sign of integral
signum(answ);
```

$$-1$$

In the above simplified expression, a minus sign appears at the beginning and tilde (~) indicates that the preceding variable is restricted to be positive. The negative sign can be easily verified by noting that all terms in the expression are positive and added, so the initial negation determines the sign. Because this is always negative, the outer two integrations cannot change the sign, so the second derivative of (A5) with respect to *n* is negative, confirming concavity and condition B_{min}.

Web Appendix D: Code used to evaluate conditions B_{min} and B_{root} for Bayesian decision theory comparing success rates with conjugate priors

This summarizes numerical investigations of when value defined by (A5) under Bayesian decision theory comparing mean success rates under two treatments meets conditions B_{min} and B_{root}.

The S-plus program used to approximately evaluate (A5) over a range of sample sizes and scenarios is below:

```
options(object.Size=1e9)
value <- function(p12, mu01=0.5, mu02=0.5, n01=1, n02=1, n=10) {  # Integrand of (A3)
      p1 <- p12[1]
      p2 <- p12[2]
      dn <- (n01*mu01+n*p1)/(n01+n) - (n02*mu02+n*p2)/(n02+n)
      sn <- sqrt( n/((n01+n)^2)*p1*(1-p1) + n/((n02+n)^2)*p2*(1-p2) )
      if (mu01<mu02) vn <- (p1-p2)*pnorm(dn/sn)
            else vn <- (p1-p2)*(pnorm(dn/sn)-1)
      vn
}

scenario <- numeric()    # this block generates many scenarios for evaluation
ab <- numeric()
mu01 <- 0.05
for (del in c(-0.025,0,0.025)) for (sd01 in c(0.01,0.03,0.05)) for (sdfac in c(0.5,1,2)) {
      mu02 <- mu01+del
      sd02 <- sd01*sdfac
      a1 <- (1-mu01)*mu01*mu01/sd01/sd01-mu01
      b1 <- a1*(1-mu01)/mu01
      n01<-a1+b1
      a2 <- (1-mu02)*mu02*mu02/sd02/sd02-mu02
      b2 <- a2*(1-mu02)/mu02
      n02<-a2+b2
      if (min(c(a1,a2,b1,b2))>0) {                            # screen out impossible configurations
            scenario <- rbind(scenario,c(mu01,sd01,mu02,sd02))   # accumulate scenarios of interest
            ab <- rbind(ab,c(a1,b1,a2,b2,mu01,n01,mu02,n02))     # record parameters of priors and scenario
      }
}
mu01 <- 0.10
for (del in c(-0.05,-0.02,0,0.02,0.05)) for (sd01 in c(0.01,0.03,0.05,0.10)) for (sdfac in c(0.5,1,2)) {
      mu02 <- mu01+del
```

```
      sd02 <- sd01*sdfac
      a1 <- (1-mu01)*mu01*mu01/sd01/sd01-mu01
      b1 <- a1*(1-mu01)/mu01
      n01<-a1+b1
      a2 <- (1-mu02)*mu02*mu02/sd02/sd02-mu02
      b2 <- a2*(1-mu02)/mu02
      n02<-a2+b2
      if (min(c(a1,a2,b1,b2))>0) {
            scenario <- rbind(scenario,c(mu01,sd01,mu02,sd02))
            ab <- rbind(ab,c(a1,b1,a2,b2,mu01,n01,mu02,n02))
      }
}
mu01 <- 0.30
for (del in c(-0.2,-0.1,0,0.1,0.2)) for (sd01 in c(0.05,0.1,0.3)) for (sdfac in c(0.5,1,2)) {
      mu02 <- mu01+del
      sd02 <- sd01*sdfac
      a1 <- (1-mu01)*mu01*mu01/sd01/sd01-mu01
      b1 <- a1*(1-mu01)/mu01
      n01<-a1+b1
      a2 <- (1-mu02)*mu02*mu02/sd02/sd02-mu02
      b2 <- a2*(1-mu02)/mu02
      n02<-a2+b2
      if (min(c(a1,a2,b1,b2))>0) {
            scenario <- rbind(scenario,c(mu01,sd01,mu02,sd02))
            ab <- rbind(ab,c(a1,b1,a2,b2,mu01,n01,mu02,n02))
      }
}
mu01 <- 0.5
for (del in c(-0.3,-0.2,-0.1,0)) for (sd01 in c(0.05,0.1,0.3,0.4)) for (sdfac in c(0.5,1,2)) {
      mu02 <- mu01+del
      sd02 <- sd01*sdfac
      a1 <- (1-mu01)*mu01*mu01/sd01/sd01-mu01
      b1 <- a1*(1-mu01)/mu01
      n01<-a1+b1
      a2 <- (1-mu02)*mu02*mu02/sd02/sd02-mu02
      b2 <- a2*(1-mu02)/mu02
      n02<-a2+b2
      if (min(c(a1,a2,b1,b2))>0) {
            scenario <- rbind(scenario,c(mu01,sd01,mu02,sd02))
            ab <- rbind(ab,c(a1,b1,a2,b2,mu01,n01,mu02,n02))
      }
```

```
}
nscen <- length(ab[,1])  # number of scenarios
res <- numeric()         # results for n's meeting B_min and B_root
for (i in (1:nscen)) {

a1 <- ab[i,1]; b1 <- ab[i,2]; a2 <- ab[i,3]; b2 <- ab[i,4]; mu01 <- ab[i,5]; n01 <- ab[i,6];
  mu02 <- ab[i,7]; n02 <- ab[i,8]
p1 <- rbeta(100000,a1,b1)               # Simulated pairs of outcome probabilities
p2 <- rbeta(100000,a2,b2)               #  generated under the independent priors
pmat <- cbind(p1,p2)
nup <- c(1:100,100+5*(1:20),200+10*(1:20),400+20*(1:30))
vn <- apply(pmat,1,value,mu01=mu01,mu02=mu02,n01=n01,n02=n02,n=nup) # evaluate integrand for each pair
vn <- apply(vn,1,mean)  # average, approximating integral
vnb1<-vn/nup
vnb2<-vn/sqrt(nup)
nb1 <- max(nup[vnb1==max(vnb1)])  # Find n meeting B_min
nb2 <- max(nup[vnb2==max(vnb2)])  # Find n meeting B_root
nlen <- length(nup)
vnb1diff <- c(0,vnb1[2:nlen]-vnb1[1:(nlen-1)])
nb1alt <- max(nup[vnb1diff>0])                  # this checks for multiple modes (never occurred)
vnb2diff <- c(0,vnb2[2:nlen]-vnb2[1:(nlen-1)])
nb2alt <- max(nup[vnb2diff>0])                  # this checks for multiple modes (never occurred)
res <- rbind(res,c(mean(p2>p1),nb1,nb2,nb1alt,nb2alt))  # Accumulate results

}

cbind(scenario,res)   # show results
```

Because some results, particularly concerning condition B_{root}, appeared to be unstable, all equipoise cases (mu01=mu02) with any $n \geq 4$ appearing to fail to meet B_{root} were re-calculated using 1,000,000 simulated pairs (p1, p2) instead of 100,000. This was repeated 10 times, and confirmed in each case that B_{root} was met for all $n \geq 4$.

Results are in Web Table 1. These are sorted by the degree of departure from equipoise, as measured by the relative difference in prior means. Results for just the non-equipoise cases are arranged differently in Web Table 2, to facilitate assessment of how high prior uncertainty impacts the conditions. These are sorted by the maximum (over the two priors) of the prior SD/the prior mean

Web Table 1. Numerical results for when conditions B_{min} and B_{root} hold for Bayesian decision between success rates.

Treatment 1 Prior		Treatment 2 Prior		Prior Probability	Abs diff	Minimum n* meeting		1+ Maximum n* not meeting		Relative
Mean	SD	Mean	SD	2 better	from 0.5	B_{min}	B_{root}	B_{min}	B_{root}	Diff in means
0.5	0.3	0.5	0.15	0.500	0.000	1	<5	1	<5	0.00
0.3	0.1	0.3	0.1	0.500	0.000	1	<5	1	<5	0.00
0.1	0.01	0.1	0.01	0.500	0.000	1	<5	1	<5	0.00
0.5	0.1	0.5	0.1	0.500	0.000	1	<5	1	<5	0.00
0.5	0.1	0.5	0.2	0.500	0.000	1	<5	1	<5	0.00
0.5	0.1	0.5	0.05	0.500	0.000	1	<5	1	<5	0.00
0.05	0.05	0.05	0.05	0.500	0.000	1	<5	1	<5	0.00
0.05	0.01	0.05	0.01	0.500	0.001	1	<5	1	<5	0.00
0.5	0.05	0.5	0.1	0.499	0.001	1	<5	1	<5	0.00
0.05	0.03	0.05	0.03	0.501	0.001	1	<5	1	<5	0.00
0.5	0.3	0.5	0.3	0.499	0.001	1	<5	1	<5	0.00
0.1	0.03	0.1	0.03	0.499	0.001	1	<5	1	<5	0.00
0.1	0.05	0.1	0.05	0.501	0.001	1	<5	1	<5	0.00
0.5	0.4	0.5	0.4	0.501	0.001	1	<5	1	<5	0.00
0.5	0.4	0.5	0.2	0.499	0.001	1	<5	1	<5	0.00
0.5	0.05	0.5	0.025	0.502	0.002	1	<5	1	<5	0.00
0.1	0.1	0.1	0.1	0.498	0.002	1	<5	1	<5	0.00
0.5	0.05	0.5	0.05	0.502	0.002	1	<5	1	<5	0.00
0.3	0.05	0.3	0.05	0.497	0.003	1	<5	1	<5	0.00
0.3	0.3	0.3	0.3	0.496	0.004	1	<5	1	<5	0.00
0.3	0.05	0.3	0.025	0.507	0.007	1	<5	1	<5	0.00
0.1	0.01	0.1	0.005	0.507	0.007	1	<5	1	<5	0.00
0.1	0.01	0.1	0.02	0.484	0.016	1	<5	1	<5	0.00
0.05	0.01	0.05	0.005	0.518	0.018	1	<5	1	<5	0.00
0.3	0.05	0.3	0.1	0.481	0.019	1	<5	1	<5	0.00
0.3	0.1	0.3	0.05	0.519	0.019	1	<5	1	<5	0.00
0.1	0.03	0.1	0.015	0.524	0.024	1	<5	1	<5	0.00
0.05	0.01	0.05	0.02	0.466	0.034	1	<5	1	<5	0.00
0.3	0.1	0.3	0.2	0.460	0.040	1	<5	1	<5	0.00
0.1	0.05	0.1	0.025	0.542	0.042	1	<5	1	<5	0.00
0.1	0.03	0.1	0.06	0.450	0.050	1	<5	1	<5	0.00
0.05	0.03	0.05	0.015	0.552	0.052	1	<5	1	<5	0.00
0.3	0.3	0.3	0.15	0.579	0.079	1	<5	1	<5	0.00
0.05	0.05	0.05	0.025	0.590	0.090	1	<5	1	<5	0.00

Treatment 1 Prior		Treatment 2 Prior		Prior Probability	Abs diff	Minimum n* meeting		1+ Maximum n* not meeting		Relative
Mean	SD	Mean	SD	2 better	from 0.5	B_{min}	B_{root}	B_{min}	B_{root}	Diff in means
0.1	0.05	0.1	0.1	0.410	0.090	1	<5	1	<5	0.00
0.1	0.1	0.1	0.05	0.593	0.093	1	<5	1	<5	0.00
0.05	0.03	0.05	0.06	0.391	0.109	1	<5	1	<5	0.00
0.05	0.05	0.05	0.1	0.312	0.188	1	<5	1	<5	0.00
0.1	0.1	0.1	0.2	0.288	0.212	1	<5	1	<5	0.00
0.1	0.05	0.12	0.1	0.507	0.007	4	8	4	8	0.20
0.1	0.1	0.12	0.1	0.578	0.078	1	<5	1	<5	0.20
0.1	0.03	0.12	0.06	0.582	0.082	15	33	15	33	0.20
0.1	0.05	0.12	0.05	0.621	0.121	4	25	4	25	0.20
0.1	0.1	0.12	0.2	0.370	0.130	1	<5	1	<5	0.20
0.1	0.1	0.12	0.05	0.660	0.160	1	<5	1	<5	0.20
0.1	0.05	0.12	0.025	0.675	0.175	2	11	2	11	0.20
0.1	0.03	0.12	0.03	0.684	0.184	32	110	32	110	0.20
0.1	0.03	0.12	0.015	0.737	0.237	20	110	20	110	0.20
0.1	0.01	0.12	0.02	0.815	0.315	390	840	390	840	0.20
0.1	0.01	0.12	0.01	0.921	0.421	>1000	>1000	>1000	>1000	0.20
0.1	0.01	0.12	0.005	0.960	0.460	>1000	>1000	>1000	>1000	0.20
0.5	0.4	0.4	0.2	0.452	0.048	1	<5	1	<5	0.25
0.5	0.4	0.4	0.4	0.418	0.082	1	<5	1	<5	0.25
0.5	0.3	0.4	0.15	0.405	0.095	1	<5	1	<5	0.25
0.5	0.3	0.4	0.3	0.405	0.095	1	<5	1	<5	0.25
0.5	0.1	0.4	0.2	0.327	0.173	2	5	2	5	0.25
0.5	0.1	0.4	0.1	0.241	0.259	14	37	14	37	0.25
0.5	0.1	0.4	0.05	0.190	0.310	27	64	27	64	0.25
0.5	0.05	0.4	0.1	0.185	0.315	22	56	22	56	0.25
0.5	0.05	0.4	0.05	0.079	0.421	165	360	165	360	0.25
0.5	0.05	0.4	0.025	0.037	0.463	330	720	330	720	0.25
0.1	0.1	0.08	0.05	0.509	0.009	4	8	4	8	0.25
0.1	0.1	0.08	0.1	0.407	0.093	1	<5	1	<5	0.25
0.1	0.05	0.08	0.025	0.392	0.108	23	50	23	50	0.25
0.1	0.05	0.08	0.05	0.373	0.127	2	20	2	20	0.25
0.1	0.03	0.08	0.06	0.324	0.176	1	<5	1	<5	0.25
0.1	0.05	0.08	0.1	0.313	0.187	1	<5	1	<5	0.25
0.1	0.03	0.08	0.03	0.310	0.190	24	92	24	92	0.25
0.1	0.03	0.08	0.015	0.285	0.215	100	220	100	220	0.25
0.1	0.1	0.08	0.2	0.199	0.301	1	<5	1	<5	0.25
0.1	0.01	0.08	0.02	0.181	0.319	93	350	93	350	0.25

Treatment 1 Prior		Treatment 2 Prior		Prior Probability	Abs diff	Minimum n* meeting		1+ Maximum n* not meeting		Relative
Mean	SD	Mean	SD	2 better	from 0.5	B_{min}	B_{root}	B_{min}	B_{root}	Diff in means
0.1	0.01	0.08	0.01	0.080	0.420	>1000	>1000	>1000	>1000	0.25
0.1	0.01	0.08	0.005	0.034	0.466	>1000	>1000	>1000	>1000	0.25
0.3	0.3	0.4	0.3	0.612	0.112	1	<5	1	<5	0.33
0.3	0.1	0.4	0.2	0.645	0.145	3	7	3	7	0.33
0.3	0.3	0.4	0.15	0.663	0.163	1	<5	1	<5	0.33
0.3	0.1	0.4	0.1	0.760	0.260	13	34	13	34	0.33
0.3	0.05	0.4	0.1	0.811	0.311	32	69	32	69	0.33
0.3	0.1	0.4	0.05	0.815	0.315	14	42	14	42	0.33
0.3	0.05	0.4	0.05	0.921	0.421	150	330	150	330	0.33
0.3	0.05	0.4	0.025	0.959	0.459	200	480	200	480	0.33
0.05	0.05	0.075	0.1	0.501	0.001	5	9	5	9	0.50
0.1	0.1	0.15	0.2	0.481	0.019	2	<5	2	<5	0.50
0.3	0.3	0.2	0.15	0.474	0.026	1	<5	1	<5	0.50
0.05	0.03	0.075	0.06	0.605	0.105	18	35	18	35	0.50
0.1	0.05	0.15	0.1	0.643	0.143	12	24	12	24	0.50
0.3	0.3	0.2	0.3	0.338	0.162	1	<5	1	<5	0.50
0.1	0.1	0.15	0.1	0.677	0.177	1	6	1	6	0.50
0.05	0.05	0.075	0.05	0.680	0.180	1	14	1	14	0.50
0.3	0.1	0.2	0.2	0.274	0.226	1	<5	1	<5	0.50
0.1	0.1	0.15	0.05	0.739	0.239	1	<5	1	<5	0.50
0.05	0.03	0.075	0.03	0.742	0.242	12	70	12	70	0.50
0.05	0.05	0.075	0.025	0.744	0.244	1	<5	1	<5	0.50
0.3	0.1	0.2	0.1	0.235	0.265	8	25	8	25	0.50
0.1	0.03	0.15	0.06	0.769	0.269	53	105	53	105	0.50
0.1	0.05	0.15	0.05	0.770	0.270	16	58	16	58	0.50
0.05	0.03	0.075	0.015	0.793	0.293	10	41	10	41	0.50
0.3	0.1	0.2	0.05	0.190	0.310	33	68	33	68	0.50
0.3	0.05	0.2	0.1	0.183	0.317	7	24	7	24	0.50
0.1	0.05	0.15	0.025	0.824	0.324	12	47	12	47	0.50
0.05	0.01	0.075	0.02	0.875	0.375	400	820	400	820	0.50
0.1	0.03	0.15	0.03	0.883	0.383	130	320	130	320	0.50
0.3	0.05	0.2	0.05	0.081	0.419	110	250	110	250	0.50
0.1	0.03	0.15	0.015	0.924	0.424	115	360	115	360	0.50
0.05	0.01	0.075	0.01	0.959	0.459	>1000	>1000	>1000	>1000	0.50
0.3	0.05	0.2	0.025	0.033	0.467	330	680	330	680	0.50
0.05	0.01	0.075	0.005	0.982	0.482	>1000	>1000	>1000	>1000	0.50
0.5	0.4	0.3	0.2	0.397	0.103	1	<5	1	<5	0.67

Treatment 1 Prior		Treatment 2 Prior		Prior Probability	Abs diff	Minimum n* meeting		1+ Maximum n* not meeting		Relative
Mean	SD	Mean	SD	2 better	from 0.5	B_{min}	B_{root}	B_{min}	B_{root}	Diff in means
0.5	0.4	0.3	0.4	0.318	0.182	1	<5	1	<5	0.67
0.5	0.3	0.3	0.15	0.311	0.189	2	<5	2	<5	0.67
0.5	0.3	0.3	0.3	0.306	0.194	1	<5	1	<5	0.67
0.3	0.3	0.5	0.3	0.696	0.196	1	<5	1	<5	0.67
0.3	0.3	0.5	0.15	0.731	0.231	1	<5	1	<5	0.67
0.3	0.1	0.5	0.2	0.803	0.303	8	16	8	16	0.67
0.5	0.1	0.3	0.2	0.192	0.308	3	8	3	8	0.67
0.5	0.1	0.3	0.1	0.083	0.417	33	75	33	75	0.67
0.3	0.1	0.5	0.1	0.918	0.418	35	77	35	77	0.67
0.5	0.05	0.3	0.1	0.045	0.455	51	120	51	120	0.67
0.3	0.1	0.5	0.05	0.955	0.455	51	120	51	120	0.67
0.3	0.05	0.5	0.1	0.962	0.462	90	190	90	190	0.67
0.5	0.1	0.3	0.05	0.037	0.463	88	185	88	185	0.67
0.1	0.1	0.05	0.05	0.345	0.155	11	21	11	21	1.00
0.05	0.05	0.025	0.025	0.340	0.160	25	49	25	49	1.00
0.05	0.05	0.025	0.05	0.239	0.261	1	<5	1	<5	1.00
0.1	0.1	0.05	0.1	0.236	0.264	1	<5	1	<5	1.00
0.05	0.03	0.025	0.015	0.223	0.277	96	180	96	180	1.00
0.05	0.03	0.025	0.03	0.222	0.278	3	17	3	17	1.00
0.1	0.05	0.05	0.05	0.201	0.299	3	20	3	20	1.00
0.1	0.05	0.05	0.025	0.179	0.321	73	140	73	140	1.00
0.1	0.05	0.05	0.1	0.179	0.321	1	<5	1	<5	1.00
0.1	0.03	0.05	0.06	0.175	0.325	2	<5	2	<5	1.00
0.05	0.03	0.025	0.06	0.164	0.336	1	<5	1	<5	1.00
0.05	0.01	0.025	0.02	0.128	0.372	19	62	19	62	1.00
0.1	0.03	0.05	0.03	0.113	0.387	42	150	42	150	1.00
0.05	0.05	0.025	0.1	0.107	0.393	1	<5	1	<5	1.00
0.1	0.1	0.05	0.2	0.072	0.428	1	<5	1	<5	1.00
0.1	0.03	0.05	0.015	0.057	0.443	340	660	340	660	1.00
0.05	0.01	0.025	0.01	0.042	0.458	460	>1000	460	>1000	1.00
0.1	0.01	0.05	0.02	0.024	0.476	210	600	210	600	1.00
0.5	0.4	0.2	0.2	0.320	0.180	1	<5	1	<5	1.50
0.5	0.3	0.2	0.15	0.212	0.288	4	7	4	7	1.50
0.5	0.3	0.2	0.3	0.203	0.297	1	<5	1	<5	1.50
0.5	0.1	0.2	0.2	0.112	0.388	3	7	3	7	1.50
0.5	0.1	0.2	0.1	0.023	0.477	46	100	46	100	1.50
0.5	0.05	0.2	0.1	0.010	0.490	56	130	56	130	1.50

Treatment 1 Prior Mean	Treatment 1 Prior SD	Treatment 2 Prior Mean	Treatment 2 Prior SD	Prior Probability 2 better	Abs diff from 0.5	Minimum n* meeting B_{min}	Minimum n* meeting B_{root}	1+ Maximum n* not meeting B_{min}	1+ Maximum n* not meeting B_{root}	Relative Diff in means
0.3	0.3	0.1	0.15	0.283	0.217	2	<5	2	<5	2.00
0.3	0.1	0.1	0.2	0.130	0.370	1	<5	1	<5	2.00
0.3	0.1	0.1	0.1	0.083	0.417	7	22	7	22	2.00
0.3	0.05	0.1	0.1	0.061	0.439	7	18	7	18	2.00
0.3	0.1	0.1	0.05	0.029	0.471	78	155	78	155	2.00

Web Table 2. Results for just the non-equipoise cases are arranged differently below, to facilitate assessment of how high prior uncertainty impacts the conditions. These are sorted by the maximum (over the two priors) of the prior SD/the prior mean.

Treatment 1 Prior Mean	Treatment 1 Prior SD	Treatment 2 Prior Mean	Treatment 2 Prior SD	Max SD/mean	Prior Probability 2 better	Abs diff from 0.5	Minimum n* meeting B_{min}	Minimum n* meeting B_{root}	Relative Diff in means
0.05	0.05	0.025	0.1	4.00	0.107	0.393	1	<5	1.00
0.1	0.1	0.05	0.2	4.00	0.072	0.428	1	<5	1.00
0.1	0.1	0.08	0.2	2.50	0.199	0.301	1	<5	0.25
0.05	0.03	0.025	0.06	2.40	0.164	0.336	1	<5	1.00
0.3	0.1	0.1	0.2	2.00	0.130	0.370	1	<5	2.00
0.05	0.05	0.025	0.05	2.00	0.239	0.261	1	<5	1.00
0.1	0.1	0.05	0.1	2.00	0.236	0.264	1	<5	1.00
0.1	0.05	0.05	0.1	2.00	0.179	0.321	1	<5	1.00
0.1	0.1	0.12	0.2	1.67	0.370	0.130	1	<5	0.20
0.3	0.3	0.1	0.15	1.50	0.283	0.217	2	<5	2.00
0.5	0.3	0.2	0.3	1.50	0.203	0.297	1	<5	1.50
0.3	0.3	0.2	0.3	1.50	0.338	0.162	1	<5	0.50
0.05	0.05	0.075	0.1	1.33	0.501	0.001	5	9	0.50
0.1	0.1	0.15	0.2	1.33	0.481	0.019	2	<5	0.50
0.5	0.4	0.3	0.4	1.33	0.318	0.182	1	<5	0.67
0.1	0.1	0.08	0.1	1.25	0.407	0.093	1	<5	0.25
0.1	0.05	0.08	0.1	1.25	0.313	0.187	1	<5	0.25
0.05	0.03	0.025	0.03	1.20	0.222	0.278	3	17	1.00
0.1	0.03	0.05	0.06	1.20	0.175	0.325	2	<5	1.00
0.05	0.05	0.025	0.025	1.00	0.340	0.160	25	49	1.00
0.3	0.1	0.1	0.1	1.00	0.083	0.417	7	22	2.00
0.1	0.1	0.05	0.05	1.00	0.345	0.155	11	21	1.00
0.1	0.05	0.05	0.05	1.00	0.201	0.299	3	20	1.00
0.3	0.05	0.1	0.1	1.00	0.061	0.439	7	18	2.00

Treatment 1 Prior		Treatment 2 Prior		Max	Prior Probability	Abs diff	Minimum n* meeting		Relative
Mean	SD	Mean	SD	SD/mean	2 better	from 0.5	B_{min}	B_{root}	Diff in means
0.05	0.05	0.075	0.05	1.00	0.680	0.180	1	14	0.50
0.1	0.1	0.08	0.05	1.00	0.509	0.009	4	8	0.25
0.5	0.1	0.2	0.2	1.00	0.112	0.388	3	7	1.50
0.1	0.1	0.15	0.1	1.00	0.677	0.177	1	6	0.50
0.05	0.05	0.075	0.025	1.00	0.744	0.244	1	<5	0.50
0.1	0.1	0.12	0.1	1.00	0.578	0.078	1	<5	0.20
0.5	0.4	0.2	0.2	1.00	0.320	0.180	1	<5	1.50
0.3	0.3	0.2	0.15	1.00	0.474	0.026	1	<5	0.50
0.1	0.1	0.15	0.05	1.00	0.739	0.239	1	<5	0.50
0.5	0.3	0.3	0.3	1.00	0.306	0.194	1	<5	0.67
0.3	0.3	0.5	0.3	1.00	0.696	0.196	1	<5	0.67
0.3	0.3	0.5	0.15	1.00	0.731	0.231	1	<5	0.67
0.3	0.1	0.2	0.2	1.00	0.274	0.226	1	<5	0.50
0.3	0.3	0.4	0.3	1.00	0.612	0.112	1	<5	0.33
0.3	0.3	0.4	0.15	1.00	0.663	0.163	1	<5	0.33
0.5	0.4	0.4	0.4	1.00	0.418	0.082	1	<5	0.25
0.1	0.1	0.12	0.05	1.00	0.660	0.160	1	<5	0.20
0.1	0.05	0.12	0.1	0.83	0.507	0.007	4	8	0.20
0.05	0.03	0.075	0.06	0.80	0.605	0.105	18	35	0.50
0.5	0.4	0.3	0.2	0.80	0.397	0.103	1	<5	0.67
0.5	0.4	0.4	0.2	0.80	0.452	0.048	1	<5	0.25
0.05	0.01	0.025	0.02	0.80	0.128	0.372	19	62	1.00
0.1	0.03	0.08	0.06	0.75	0.324	0.176	1	<5	0.25
0.5	0.3	0.2	0.15	0.75	0.212	0.288	4	7	1.50
0.5	0.3	0.4	0.3	0.75	0.405	0.095	1	<5	0.25
0.1	0.05	0.15	0.1	0.67	0.643	0.143	12	24	0.50
0.5	0.1	0.3	0.2	0.67	0.192	0.308	3	8	0.67
0.1	0.05	0.08	0.05	0.63	0.373	0.127	2	20	0.25
0.05	0.03	0.025	0.015	0.60	0.223	0.277	96	180	1.00
0.1	0.03	0.05	0.03	0.60	0.113	0.387	42	150	1.00
0.05	0.03	0.075	0.03	0.60	0.742	0.242	12	70	0.50
0.05	0.03	0.075	0.015	0.60	0.793	0.293	10	41	0.50
0.5	0.3	0.3	0.15	0.60	0.311	0.189	2	<5	0.67
0.5	0.3	0.4	0.15	0.60	0.405	0.095	1	<5	0.25
0.3	0.1	0.1	0.05	0.50	0.029	0.471	78	155	2.00
0.1	0.05	0.05	0.025	0.50	0.179	0.321	73	140	1.00
0.5	0.05	0.2	0.1	0.50	0.010	0.490	56	130	1.50

Treatment 1 Prior Mean	Treatment 1 Prior SD	Treatment 2 Prior Mean	Treatment 2 Prior SD	Max SD/mean	Prior Probability 2 better	Abs diff from 0.5	Minimum n* meeting B_{min}	Minimum n* meeting B_{root}	Relative Diff in means
0.5	0.1	0.2	0.1	0.50	0.023	0.477	46	100	1.50
0.1	0.05	0.15	0.05	0.50	0.770	0.270	16	58	0.50
0.1	0.05	0.08	0.025	0.50	0.392	0.108	23	50	0.25
0.1	0.05	0.15	0.025	0.50	0.824	0.324	12	47	0.50
0.1	0.03	0.12	0.06	0.50	0.582	0.082	15	33	0.20
0.3	0.1	0.2	0.1	0.50	0.235	0.265	8	25	0.50
0.1	0.05	0.12	0.05	0.50	0.621	0.121	4	25	0.20
0.3	0.05	0.2	0.1	0.50	0.183	0.317	7	24	0.50
0.1	0.05	0.12	0.025	0.50	0.675	0.175	2	11	0.20
0.3	0.1	0.4	0.2	0.50	0.645	0.145	3	7	0.33
0.5	0.1	0.4	0.2	0.50	0.327	0.173	2	5	0.25
0.1	0.03	0.15	0.06	0.40	0.769	0.269	53	105	0.50
0.3	0.1	0.5	0.2	0.40	0.803	0.303	8	16	0.67
0.05	0.01	0.025	0.01	0.40	0.042	0.458	460	>1000	1.00
0.1	0.01	0.05	0.02	0.40	0.024	0.476	210	600	1.00
0.1	0.03	0.08	0.03	0.38	0.310	0.190	24	92	0.25
0.5	0.05	0.3	0.1	0.33	0.045	0.455	51	120	0.67
0.3	0.1	0.5	0.05	0.33	0.955	0.455	51	120	0.67
0.3	0.1	0.5	0.1	0.33	0.918	0.418	35	77	0.67
0.5	0.1	0.3	0.1	0.33	0.083	0.417	33	75	0.67
0.3	0.1	0.2	0.05	0.33	0.190	0.310	33	68	0.50
0.3	0.1	0.4	0.05	0.33	0.815	0.315	14	42	0.33
0.3	0.1	0.4	0.1	0.33	0.760	0.260	13	34	0.33
0.1	0.03	0.05	0.015	0.30	0.057	0.443	340	660	1.00
0.1	0.03	0.15	0.015	0.30	0.924	0.424	115	360	0.50
0.1	0.03	0.15	0.03	0.30	0.883	0.383	130	320	0.50
0.1	0.03	0.08	0.015	0.30	0.285	0.215	100	220	0.25
0.1	0.03	0.12	0.03	0.30	0.684	0.184	32	110	0.20
0.1	0.03	0.12	0.015	0.30	0.737	0.237	20	110	0.20
0.05	0.01	0.075	0.02	0.27	0.875	0.375	400	820	0.50
0.1	0.01	0.08	0.02	0.25	0.181	0.319	93	350	0.25
0.3	0.05	0.2	0.05	0.25	0.081	0.419	110	250	0.50
0.3	0.05	0.4	0.1	0.25	0.811	0.311	32	69	0.33
0.5	0.05	0.4	0.1	0.25	0.185	0.315	22	56	0.25
0.5	0.1	0.4	0.1	0.25	0.241	0.259	14	37	0.25
0.3	0.05	0.5	0.1	0.20	0.962	0.462	90	190	0.67
0.5	0.1	0.3	0.05	0.20	0.037	0.463	88	185	0.67

Treatment 1 Prior		Treatment 2 Prior		Max	Prior Probability	Abs diff	Minimum n* meeting		Relative
Mean	SD	Mean	SD	SD/mean	2 better	from 0.5	B_{min}	B_{root}	Diff in means
0.5	0.1	0.4	0.05	0.20	0.190	0.310	27	64	0.25
0.05	0.01	0.075	0.01	0.20	0.959	0.459	>1000	>1000	0.50
0.05	0.01	0.075	0.005	0.20	0.982	0.482	>1000	>1000	0.50
0.1	0.01	0.12	0.02	0.17	0.815	0.315	390	840	0.20
0.3	0.05	0.2	0.025	0.17	0.033	0.467	330	680	0.50
0.3	0.05	0.4	0.025	0.17	0.959	0.459	200	480	0.33
0.3	0.05	0.4	0.05	0.17	0.921	0.421	150	330	0.33
0.1	0.01	0.08	0.01	0.13	0.080	0.420	>1000	>1000	0.25
0.5	0.05	0.4	0.05	0.13	0.079	0.421	165	360	0.25
0.5	0.05	0.4	0.025	0.10	0.037	0.463	330	720	0.25
0.1	0.01	0.08	0.005	0.10	0.034	0.466	>1000	>1000	0.25
0.1	0.01	0.12	0.01	0.10	0.921	0.421	>1000	>1000	0.20
0.1	0.01	0.12	0.005	0.10	0.960	0.460	>1000	>1000	0.20