Change-point Problem and Regression: An Annotated Bibliography

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Ahmad Khodadadi and Masoud Asgharian

Abstract

The problems of identifying changes at unknown times and of estimating the location of changes in stochastic processes are referred to as "the change-point problem" or, in the Eastern literature, as "disorder".

The change-point problem, first introduced in the quality control context, has since developed into a fundamental problem in the areas of statistical control theory, stationarity of a stochastic process, estimation of the current position of a time series, testing and estimation of change in the patterns of a regression model, and most recently in the comparison and matching of DNA sequences in microarray data analysis.

Numerous methodological approaches have been implemented in examining change-point models. Maximum-likelihood estimation, Bayesian estimation, isotonic regression, piecewise regression, quasi-likelihood and non-parametric regression are among the methods which have been applied to resolving challenges in change-point problems. Grid-searching approaches have also been used to examine the change-point problem.

Statistical analysis of change-point problems depends on the method of data collection. If the data collection is ongoing until some random time, then the appropriate statistical procedure is called sequential. If, however, a large finite set of data is collected with the purpose of determining if at least one change-point occurred, then this may be referred to as non-sequential. Not surprisingly, both the former and the latter have a rich literature with much of the earlier work focusing on sequential methods inspired by applications in quality control for industrial processes. In the regression literature, the change-point model is also referred to

as two- or multiple-phase regression, switching regression, segmented regression, two-stage least squares (Shaban, 1980), or broken-line regression.

The area of the change-point problem has been the subject of intensive research in the past half-century. The subject has evolved considerably and found applications in many different areas. It seems rather impossible to summarize all of the research carried out over the past 50 years on the change-point problem. We have therefore confined ourselves to those articles on change-point problems which pertain to regression.

The important branch of sequential procedures in change-point problems has been left out entirely. We refer the readers to the seminal review papers by Lai (1995, 2001). The so called structural change models, which occupy a considerable portion of the research in the area of change-point, particularly among econometricians, have not been fully considered. We refer the reader to Perron (2005) for an updated review in this area. Articles on change-point in time series are considered only if the methodologies presented in the paper pertain to regression analysis.

1 Regression Analysis and Change-point

The general problem of regression can perhaps be best described as fitting a function to a set of random pairs $\{(Y_t, X_t)\}_{t \in T}$, where T is a subset of R^k, though most applications are only concerned with $k = 1$. Having such description in mind, one may consider the following model,

$$Y_t = f(X_t) + \varepsilon_t,$$

where f is a link function and Y_t and ε_t are random vectors, while X_t can be a random or deterministic vector, or it might have both random and deterministic components.

The simplest possible scenario is when the link function f is linear and X_t is a deterministic vector. A case of special interest in many applications is the so-called linear regression with random slope which pertains to the case where X_t has a mixture of stochastic and deterministic components. It often happens that a linear link function cannot adequately explain the possible relationship between X_t and Y_t. In such cases, one may choose nonlinear link functions should a specific choice of link function be plausible. When such a choice is not readily available, one may resort to nonparametric regression. There is a vast literature pertaining to nonparametric regression.

Choosing an appropriate link function that can encompass the most important features of the data is at the heart of regression analysis. In many applications, a smooth link function by which we mean a C^1 map, cannot describe the possible relationship between X_t and Y_t, and one has to fit different models in different subregions. The points at which the link function is not smooth are of special interest, since they often represent a change in the pattern of data. Such points are often called *change-points*.

There has been a surge of research over the past several decades on locating and making inferences about the change-points as well as the pattern of the data before and after the change-points. We have collected an annotated list of articles written on change-points and related subjects. We have mostly confined ourselves to the literature on change-point problems pertaining to regression analysis. The list is, by no means, an exhaustive list. It reflects only those articles which happen to be closer to some applications we have had in mind.

The standard change-point problem in regression models consists of (1) testing the null hypothesis that no change in regimes has taken place against the alternative that observations were generated by two (or possibly more) distinct regression equations, and (2) estimating the two regimes that gave rise to the data. Literature on this topic is divided between models in which continuity is assumed and those which allow a discontinuity at the point of change. The regression model without the restriction of continuity is, in fact, the generalization of the mean-shift problem in which the interest is testing and estimating the shift in mean in a sequence of random variables.

Generally speaking, change-point regression is a regression problem in which the expected

value of the dependent variable or response is assumed to have a different functional form in several neighborhoods of the explanatory variable space. Estimation of the change-point in the so-called broken-line regression models, where the regression function is assumed to be continuous at the point of change, with the assumption of normality, has been developed by various authors. Sprent (1961) was among the first to discuss the estimation of piecewise linear models. Hinkley (1969) considered the same two-phase straight-line model and derived the maximum likelihood estimator (MLE) of the change-point by its marginal likelihood function and presented the asymptotic distribution of the estimator. Feder (1975) studied the model in a more general framework and proved the consistency of the least squares estimators of the regression coefficients and the change-point. The estimators are asymptotically normal for some special cases, including models with all linear segments. The asymptotic properties of the change-point and regression coefficient estimation using a local log likelihood process approach has been derived by Bhattacharya (1991, 1994). Through this approach, he has shown the distinctive features of the asymptotic properties of the change-point with and without the continuity at the point of change. In change-point literature, models with continuity restriction are called continuously (or gradually) changing models and without this restriction are called abruptly changing models.

A major difficulty in estimating the change-point for regression models is the non-smoothness of the likelihood function with respect to the change-point considered as a parameter. Many authors have tried to circumvent this problem by using various smooth transitions between the two linear regimes, or by using other types of functions such as the quadric function in one of the segments separated by the change-point. This technique was mostly used for models with normally distributed response variables. Gallant and Fuller (1973) discussed such a model and used a modified Gauss-Newton method to obtain the least squared estimates. Bacon and Watts (1971) proposed a model which can accommodate a smooth transition as well as the abrupt change with a Bayesian estimation procedure to determine the parameter values. Bai and Perron (2003) proposed an estimation procedure for the change-point and the corresponding regression coefficients in the framework of generalized linear models, and presented the asymptotic properties of the proposed estimators.

There are many applications which involve unknown change-points when the data are correlated. Examples arise in medicine when the data is longitudinal in nature or in statistical genetics when the members of the same families are needed to be involved in the studies. Liang and Zeger (1986) introduced a novel approach to the correlated data using generalized linear models. The generalized estimating equation (GEE) in essence is a multivariate quasi-likelihood approach. A quasi-likelihood function by definition has the important properties of a score function (log-likelihood derivative), which is essential to the asymptotic theory associated with likelihood functions. It does not, however, require a complete specification of the underlying distribution (McCullagh and Nelder, 1989).

A relatively large class of longitudinal data with change-points, termed the multipath change-point problem, has been recently studied (see Asgharian and Wolfson (2001) and references cited therein). A distinguishing feature of the multipath change-point problem is

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that different subjects may experience a possible change at different time points. This extra twist no doubt imposes further challenges in statistical analyses of multipath change-point problems. It is also worth mentioning that this feature renders any direct applications of GEE implausible.

The assumption that the regressors are fixed and non-random is appropriate in the analysis of experimental data, since the explanatory variables will generally represent conditions of the experiment that are fixed by the experimenter, while the nature of observational studies often necessitates application of *mixed models*, in which the randomness of the regressors is explicitly recognized. For events like the great depression, oil price shocks, and so on, models with constant coefficients have been found to perform poorly for prediction purposes or for analysing the effects of policy changes. The alternative to parameter constancy can be either a discrete change in parameters or a continuous one over time. Smooth continuous change over time can be a more realistic assumption in many applications, though the discrete change can be more suitable in others. Another strand of literature is concerned with the case in which the alternative to constancy is that the parameters are stochastic and fluctuate according to a stochastic model.

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2 Annotated Bibliography

Claudia Kirch and Josef Steinebach(2006-1)

Permutation principles for the change analysis of stochastic processes under strong invariance

Journal of Computational and Applied Mathematics, 186, 64-88

Keywords: Permutation principle; Bootstrap; Change-point; Invariance principle; Abrupt change; Gradual change; Rank statistic; Limiting extreme value distribution

Abstract: Approximations of the critical values for change-point tests are obtained through permutation methods. Both, abrupt and gradual changes are studied in models of possibly dependent observations satisfying a strong invariance principle, as well as gradual changes in an i.i.d. model. The theoretical results show that the original test statistics and their corresponding permutation counterparts follow the same distributional asymptotic. Some simulation studies illustrate that the permutation tests behave better than the original tests if performance is measured by the a- and -error, respectively.

Goldenshluger, A.; Tsbakov, A. and Zeevi, A.(2006-2)

Optimal change-point estimation from indirect observations

The Annals of Statistics, 34, 350-372

Abstract: We study nonparametric change-point estimation from indirect noisy observations. Focusing on the white noise convolution model, we consider two classes of functions that smooth apart from the change-point. We establish lower bounds on the minimax risk in estimating the change-point and develop rate optimal estimation procedures. The results demonstrate that the best achievable rates of convergence are determined both by smoothness of the function away from the change-point and by the degree of ill-posedness of the convolution operator. Optimality is obtained by introducing a new technique that involves, as a key element, detection of zero crossing of an estimate of the properly smoothed second derivative of the underlying function.

Gregory Gurevich A(2006-3)

Nonparametric AMOC Changepoint Tests for Stochastically Ordered Alternatives

Communications in Statistics: Theory and Methods, 35, 887-903

Keywords: AMOC procedure; Change-point; Mann-Whitney statistic; Nonparametric statistics

Abstract: The problem considered is that of testing on the basis of a finite sequence of independent observations if all the observations have the same distribution versus the alternative

that there is a unique change in the distribution and i.i.d. observations after the change are stochastically larger. The distributions before and after the possible change are continuous but not fully specified. We suggest a family of nonparametric tests based on ranks. Asymptotic approximations for the significance level of the test are obtained analytically. Monte Carlo experiments show that the rate of convergence of our asymptotics is fast.

Osorio, Felipe and Galea, Manuel (2006-4)
Detection of a change-point in student-t linear regression models
Statistical Papers, 47, 31-48
Keywords: student-t model; Schwarz information criterion

Abstract: The Schwarz Information Criterion (SIC) is used in order to locate a change-point in linear regression models with independent errors distributed according to the Student-t distribution. The methodology is applied to data sets from the financial area.

Singer, Julio da Motta and Cúri, Mariana (2006-5)
Modelling regression and dispersion parameters in a complex repeated measures experiment
Environmental and Ecological Statistics, 13, 53-68
Keywords: Covariance structure; Segmented regression; Random effects model

Abstract :We analyze data from a complex repeated measures experiment directed at the evaluation of the response to an electric stimulus applied to mussel nerves under 5 different salinity levels. We discuss the form of the relation between the response and the different salinity levels, as well as the choice of an adequate within subjects covariance structure that includes random effects and autoregressive models.

Parka, Cheolwoo and Kimb, Woo-Chul(2006-6)
Wavelet estimation of a regression function with a sharp change point in a random design
Journal of Statistical Planning and Inference, 136, 7, 2381-2394
Keywords: Block thresholding; Continuous wavelet transform; Design transformation and binning; Random design; Rate of convergence; Sharp change point problem; Wavelet function estimation

Abstract : In a random design nonparametric regression model, this paper deals with the detection of a sharp change point and the estimation of a regression function with a single jump point. A method based on design transformation and binning is used in order to convert a random design into an equispaced design whose number of points is a power of 2. Using the

continuous wavelet transform of the data, we construct a sharp change point estimator and obtain its rate of convergence. Wavelet methods are well known for their good adaptivity around sudden local changes; however, in practice, the Gibbs phenomenon still exists. This difficulty is overcome by suitably adjusting the data with preliminary estimators for the location and the size of discontinuity. Global and local asymptotic results of the proposed method are obtained. The method is also tested on simulated examples and the results show that the proposed method alleviates the Gibbs phenomenon.

Guaranteed testing for epidemic changes of a linear regression model

Abstract: The objective of this paper is to propose and examine a class of generalized maximum likelihood asymptotic power one tests for detection of various types of changes in a linear regression model. The proposed retrospective tests are based on martingales structures ShiryayevRoberts statistics. This approach is widely known in a sequential analysis of change point problems as an optimal method of detecting a change in distribution. Guaranteed non-asymptotic upper bounds for the significance levels of the considered tests are presented. Simulated data sets are used to demonstrate that the proposed tests can give good results in practice.

Guaranteed Local Maximum Likelihood Detection of a Change Point in Nonparametric Logistic Regression

Abstract: We consider nonparametric logistic regression and propose a generalized likelihood test for detecting a threshold effect that indicates a relationship between some risk factor and a defined outcome above the threshold but none below it. One important field of application is occupational medicine and in particular, epidemiological studies. In epidemiological studies, segmented fully parametric logistic regression models are often threshold models, where it is assumed that the exposure has no influence on a response up to a possible unknown threshold, and has an effect beyond that threshold. Finding efficient methods for detection and estimation of a threshold is a very important task in these studies. This article proposes such methods in a context of nonparametric logistic regression. We use a local version of

unknown likelihood functions and show that under rather common assumptions the asymptotic power of our test is one. We present a guaranteed non asymptotic upper bound for the significance level of the proposed test. If applying the test yields the acceptance of the conclusion that there was a change point (and hence a threshold limit value), we suggest using the local maximum likelihood estimator of the change point and consider the asymptotic properties of this estimator.

Wen, Chi-Chung, Wu, Yuh-Jenn, Huang, Yung-Hsiang, Chen, Wei-Chen, Liu, Shu-Chen, Jiang, Shih Sheng, Juang, Jyh-Lyh, Lin, Chung-Yen, Fang, Wen-Tsen, Hsiung, Chao and Chang, I-Shou (2006-9)
A Bayes regression approach to array-CGH data

Abstract: This paper develops a Bayes regression model having change points for the analysis of array-CGH data by utilizing not only the underlying spatial structure of the genomic alterations but also the observation that the noise associated with the ratio of the fluorescence intensities is bigger when the intensities get smaller. We show that this Bayes regression approach is particularly suitable for the analysis of cDNA microarray-CGH data, which are generally noisier than those using genomic clones. A simulation study and a real data analysis are included to illustrate this approach.

Bernard Garel(2005-1)
Asymptotic theory of the likelihood ratio test for the identification of a mixture

Abstract: The problems that arise when using the likelihood ratio test for the identification of a mixture distribution are well known: non-identifiability of the parameters and null hypothesis corresponding to a boundary point of the parameter space. In their approach to the problem of testing homogeneity against a mixture with two components, Ghosh and Sen took into account these specific problems. Under general assumptions, they obtained the asymptotic distribution of the likelihood ratio test statistic. However, their result requires a separation condition which is not completely satisfactory. We show that it is possible to remove this condition with assumptions which involve the second derivatives of the density only.

Brodskya, Boris and Darkhovsky, Boris(2005-2)
Asymptotically optimal methods of change-point detection for composite hypotheses

Abstract:In this paper the problem of change-point detection for the case of composite hypotheses is considered. We assume that the distribution functions of observations before and after an unknown change-point belong to some parametric family. The true value of the parameter of this family is unknown but belongs to two disjoint sets for observations before and after the change-point, respectively. A new criterion for the quality of change-point detection is introduced. Modifications of generalized CUSUM and GRSh (GirshickRubin-Shiryaev) methods are considered and their characteristics are analyzed. Comparing these characteristics with an a priori boundary for the quality of change-point detection we establish asymptotic optimality of these methods when the family of distributions before the change-point consists of one element.

Fernando A. Quintana; Pilar L. and Heleno Bolfarine(2005-3)

Bayesian identification of outliers and change-points in measurement error models

Abstract: The problem of outlier and change-point identification has received considerable attention in traditional linear regression models from both, classical and Bayesian standpoints. In contrast, for the case of regression models with measurement errors, also known as error-in-variables models, the corresponding literature is scarce and largely focused on classical solutions for the normal case. The main object of this paper is to propose clustering algorithms for outlier detection and change-point identification in scale mixture of error-in-variables models. We propose an approach based on product partition models (PPMs) which allows one to study clustering for the models under consideration. This includes the change-point problem and outlier detection as special cases. The outlier identification problem is approached by adapting the algorithms developed by Quintana and Iglesias [Journal of the Royal Statistical Society: Series B (Statistical Methodology) Volume 65 Page 557 - May 2003] for simple linear regression models. A special algorithm is developed for the change-point problem which can be applied in a more general setup. The methods are illustrated with two applications: (i) outlier identification in a problem involving the relationship between two methods for measuring serum kanamycin in blood samples from babies, and (ii) change-point identification in the relationship between the monthly dollar volume of sales on the Boston Stock Exchange and the combined monthly dollar volumes for the New York and American Stock Exchanges.

Gadeikis, K. and Paulauskas, V.(2005-4)

On the Estimation of a Changepoint in a Tail Index

Abstract: We investigate the application of a new estimator for the tail index proposed in [Yu. Davydov, V. Paulauskas, and A. Rackauskas, More on p-stable convex sets in Banach spaces, J. Theoret. Probab., 13, 39-64 (2001).] and [V. Paulauskas, A New Estimator for Tail Index, Acta Appl. Math., 79, 55-67 (2003).]. Testing hypothesis of change at unknown place and detecting change in mean allow us to provide theoretical results on estimation of the changepoint in the tail index. We demonstrate the applicability of these results in practice.

Grace Chiu, Richard Lockhart, and Richard Routledge (2005-6)

Asymptotic theory for bent-cable regression the basic case

Abstract: We use what we call the bent-cable model to describe potential change-point phenomena. The class of bent cables includes the commonly used broken stick (a bent cable without a bend segment). Theory for least-squares (LS) estimation is developed for the basic bent cable, whose incoming and outgoing linear phases have slopes 0 and 1, respectively, and are joined smoothly by a quadratic bend. Conditions on the design are given to ensure regularity of the estimation problem, despite non-differentiability of the model's first partial derivatives (with respect to the covariate and model parameters). Under such conditions, we show that the LS estimators (i) are consistent, regardless of a zero or positive true bend width; and (ii) asymptotically follow a bivariate normal distribution, if the underlying cable has all three segments. In the latter case, we show that the deviance statistic has an asymptotic chi-squared distribution with two degrees of freedom.

Gregory Gurevicha and Albert Vexler(2005-7)

Change point problems in the model of logistic regression

Abstract: The paper considers generalized maximum likelihood asymptotic power one tests which aim to detect a change point in logistic regression when the alternative specifies that a change occurred in parameters of the model. A guaranteed non-asymptotic upper bound for the significance level of each of the tests is presented. For cases in which the test supports the conclusion that there was a change point, we propose a maximum likelihood estimator

of that point and present results regarding the asymptotic properties of the estimator. An important field of application of this approach is occupational medicine, where for a lot chemical compounds and other agents, so-called threshold limit values (or TLVs) are specified. We demonstrate applications of the test and the maximum likelihood estimation of the change point using an actual problem that was encountered with real data.

Bootstrap in Detection of Changes in Linear Regression

Abstract: Applications of bootstrap with and without replacement in change point analysis in linear regression models are discussed. Particularly, bootstrap based approximations for critical values for two classes of M-type test procedures are treated. As a particular case, we obtain L_1 procedures and regression quantile procedures. Their asymptotic performance is investigated and finite sample properties are checked in a simulation study.

Jing-rung Yu, Gwo-hshiung Tzeng and Han-Lin Li(2005-9)
Interval piecewise regression model with automatic change-point detection by quadratic programming

Abstract: To handle large variation data, an interval piecewise regression method with automatic change-point detection by quadratic programming is proposed as an alternative to Tanaka and Lee's method. Their unified quadratic programming approach can alleviate the phenomenon where some coefficients tend to become crisp in possibilistic regression by linear programming and also obtain the possibility and necessity models at one time. However, that method can not guarantee the existence of a necessity model if a proper regression model is not assumed especially with large variations in data. Using automatic change-point detection, the proposed method guarantees obtaining the necessity model with better measure of fitness by considering variability in data. Without piecewise terms in estimated model, the proposed method is the same as Tanaka and Lee's model. Therefore, the proposed method is an alternative method to handle data with the large variations, which not only reduces the number of crisp coefficients of the possibility model in linear programming, but also simultaneously obtains the fuzzy regression models, including possibility and necessity models with better fitness. Two examples are presented to demonstrate the proposed method.

Lai, Tze Leung, Liu, Haiyan and Xing,Haipeng (2005-10)
Autoregressive models with piecewise constant volatility and regression parameters

Abstract : We introduce herein a new class of autoregressive models in which the regression parameters and error variances may undergo changes at unknown time points while staying constant between adjacent change-points. Assuming conjugate priors, we derive closed-form recursive Bayes estimates of the regression parameters and error variances. Approximations to the Bayes estimates are developed that have much lower computational complexity and yet are comparable to the Bayes estimates in statistical efficiency. We also address the problem of unknown hyperparameters and propose two practical methods for simultaneous estimation of the hyperparameters, regression parameters and error variances.

Lee, Ji-Hyun; Qaqish, Bahjat F(2005-11)
A Latent Changepoint Model Using A Generalized Estimating Equations Approach

Abstract: We propose a latent changepoint model for the analysis of longitudinal biomarker data in relation to progression or recurrence of disease. A parametric model that contains a random changepoint in the expected biomarker values is considered. In this article, estimation through generalized estimating equations is proposed. The procedure allows estimation of the biomarker trend over time and the changepoint distribution. We provide the details of the estimation procedure. Through a Monte Carlo simulation study, several aspects of the small sample performance of the estimates are investigated.

Loschi, R. H. and Cruz, F. R. B.(2005-12)
Bayesian identification of multiple change points in poisson data.

Abstract: The identification of multiple change points is a problem shared by many subject areas, including disease and criminality mapping, medical diagnosis, industrial control, and finance. An algorithm based on the Product Partition Model (PPM) is developed to solve the multiple change point identification problem in Poisson data sequences. In order to address the PPM, a simple and easy way to implement Gibbs sampling scheme is derived.

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A sensitivity analysis is performed, for different prior specifications. The algorithm is then applied to the analysis of a real data sequence. The results show that the method is quite effective and provides useful inferences

Myung Hwan Naa, Jongwoo Jeonb and Dong Ho Park(2005-13)
Testing whether failure rate changes its trend with unknown change points
Journal of Statistical Planning and Inference, 129, 317-325
Keywords: Testing; Failure rate; Trend; Monte Carlo simulations

Abstract: The problem of testing the trend change of failure rate is of great interest in the reliability and survival analysis. In this paper, we develop a new test procedure for testing whether or not the failure rate changes its trend. One big advantage of this test is that neither the change points nor the proportions at which the trend changes occur need to be known. We establish the asymptotic null distribution of the proposed test statistic to obtain the asymptotic null critical values for the test to be applied. To study the performance of the new test procedure, we conduct Monte Carlo simulations to compute the powers of the test against the lognormal alternatives and the Hjorth alternatives and to compare these powers with those of other existing tests. An example is presented to illustrate the application of the test.

Preminger, Arie and Wettstein, David (2005-14)
Using the penalized likelihood method for model selection with nuisance parameters present only under the alternative: An application to switching regression models
Journal of Time Series Analysis, 26, 715-741
Keywords: Model selection; switching regression models; penalized likelihood method; Law of the iterated logarithm; C12; C32; C52

Abstract: We study the problem of model selection with nuisance parameters present only under the alternative. The common approach for testing in this case is to determine the true model through the use of some functionals over the nuisance parameters space. Since in such cases the distribution of these statistics is not known, critical values had to be approximated usually through computationally intensive simulations. Furthermore, the computed critical values are data and model dependent and hence cannot be tabulated. We address this problem by using the penalized likelihood method to choose the correct model. We start by viewing the likelihood ratio as a function of the unidentified parameters. By using the empirical process theory and the uniform law of the iterated logarithm (LIL) together with sufficient conditions on the penalty term, we derive the consistency properties of this method. Our approach generates a simple and consistent procedure for model selection. This methodology is presented in the context of switching regression models. We also provide some Monte Carlo simulations to analyze the finite sample performance of our procedure.

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Salazar, Diego, Venkatesan, G. and Moen, David (2005-15)
Switching linear models: A general approach
Communications in Statistics: Simulation and Computation, 34, 309-320
Keywords: Bayes estimators; Simulation; Switching intervals; Switching linear models; switching regression models; Switching sequences of random variables

Abstract: With reference to switching linear models, using the notion of a switching interval, the posterior distributions of all the parameters in the model are obtained. This includes the beginning and end points of the switching interval and the parameters determining the nature of the switch. This is done by studying three cases of the problem: a permanent switch in a finite interval, a permanent switch in an infinite interval, and a temporary switch in a finite interval. The analysis is general in the sense that it can be applied to any problem that can be formulated as a linear model. A numerical study illustrates the methodology.

Shurenkov, G.(2005-16)
Asymptotic behavior of median estimators of multiple change points
Theory Probability and Mathematical Statistics, 70 , 167-176.
Keywords: Estimation of change points, limit distribution, dynamic programming algorithm, sampling median

Abstract: We consider the problem of posterior estimation of multiple change points in the case of only two distributions. We find the asymptotic distribution of the difference between the median estimator of a single change point and the true change point and show that the distribution does not change if the unknown parameter is estimated by a median of the sample. We generalize the results to the case of multiple change points.

Sofronov, G. Yu.(2005-17)
An asymptotically d-optimal test of a posteriori change-point detection
Theory of probability and its application, 49, 367-371
Keywords: Change-point detection; Hypothesis discrimination; Close hypothesis: d–a posteriori approach; d-optimality; Weak convergence; Wiener process functional

Abstract: We consider the problem of a posteriori change-point detection for a sequence of independent identically distributed random variables. We propose to use d-risks instead of error of the first type and error of the second type. We construct an asymptotically optimal test minimizing one d-risk and guaranteeing another

Tartakovsky, A. G. and Veeravalli, V. V.(2005-18)
General asymptotic bayesian theory of quickest change detection.
Theory of Probability and Its Applications; 2005-, 49, 458-497

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Keywords: Change-point detection; Sequential detection; Asymptotic Optimality; Nonlinear renewal theory

Abstract: The optimal detection procedure for detecting changes in independent and identically distributed (i.i.d.) sequences in a Bayesian setting was derived by Shiryaev in the 1960s. However, the analysis of the performance of this procedure in terms of the average detection delay and false alarm probability has been an open problem. In this paper, we develop a general asymptotic change-point detection theory that is not limited to a restrictive i.i.d. assumption. In particular, we investigate the performance of the Shiryaev procedure for general discrete-time stochastic models in the asymptotic setting, where the false alarm probability approaches zero. We show that the Shiryaev procedure is asymptotically optimal in the general non-i.i.d. case under mild conditions. We also show that the two popular non-Bayesian detection procedures, namely the Page and the Shiryaev-Roberts-Pollak procedures, are generally not optimal (even asymptotically) under the Bayesian criterion. The results of this study are shown to be especially important in studying the asymptotic of decentralized change detection procedures.

Tsai-Hung Fan and Wei-chen Chen(2005-19)
Bayesian change points analysis on the seismic activity in northeastern Taiwan
Journal of Statistical Computation and Simulation, 75, 857-868
Keywords: Seismic activity, Bayesian change points analysis, Reversible jump MCMC method, BIC criteria

Abstract: Bayesian change points analysis on the seismic activity in northeastern Taiwan is studied via the reversible jump Markov chain Monte Carlo simulation. An epidemic model is considered with Gamma prior distributions for the parameters. The prior distributions are essentially determined based on an earlier period of the seismic data in the same region. It is investigated that there exist two change points during the time period considered. This result is also confirmed by the BIC criteria.

Wang, Zhiguo and Wang, Jinde (2005-20)
Parameter estimation of some NHPP software reliability models with change-point
Communications in Statistics: Simulation and Computation, 34, 121-134
Keywords: Change-point; Maximum likelihood estimation; NHPP model; Nonparametric estimation; Software reliability

Abstract: The nonhomogeneous Poisson process (NHPP) model is an important class of software reliability models and is widely used in software reliability engineering. The failure intensity function is usually assumed to be continuous and smooth. However, in many realistic situations, the failure intensity may be not continuous for many possible causes, such

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as the change in running environment, testing strategy, or resource allocation. The change-point and other parameters are often unknown and to be estimated from the observed failure data. In this article we constructed a method of the type of maximum likelihood estimation, which can be applied in the case that the change-point is not necessarily the observation time point and in the case that the data is grouped. Furthermore, if the failure intensity function is completely unknown, we designed a nonparametric method for estimating the change-point.

Wu, Yanhong(2005-21)
Inference for change-point and post-change means after a CUSUM test
Lecture Notes in Statistics, 180. Springer, New York, 2005-.

Abstract: This monograph is the first to systematically study the bias of estimators and the construction of corrected confidence intervals for change-point and post-change parameters after a change is detected by using a CUSUM procedure. Researchers in change-point problems and sequential analysis, time series and dynamical systems, and statistical quality control will find that the book's methods and techniques are mostly new and can be extended to more general dynamic models where the structural and distributional parameters are monitored. Practitioners who are interested in applications to quality control, dynamical systems, financial markets, clinical trials and other areas will benefit from case studies based on data sets from river flow, accident interval, stock prices, and global warming. Readers with an elementary probability and statistics background and some knowledge of CUSUM procedures will be able to understand most results, as the material is relatively self-contained. The exponential family distribution is used as the basic model that includes changes in mean, variance, and hazard rate as special cases. There are fundamental differences between the sequential sampling plan and fixed sample size case. Although the results are given under the CUSUM procedure, the methods and techniques discussed provide new approaches to dealing with inference problems after sequential change-point detection, and they also contribute to the theoretical aspects of sequential analysis. Many results are of independent interest and can be used to study random walk related stochastic models.

This monograph contains ten chapters. Chapter 1 introduces the regular CUSUM procedure and gives a simple approximation for the average run lengths for design purposes. Chapter 2 considers the bias and the absolute bias for the estimator of the change-point conditioning in a change detected. Chapter 3 constructs a lower confidence limit for the change-point and derives its asymptotic results. Chapter 4 concentrates on the inference problem for the post-change mean in the normal case. Chapter 5 studies the behavior of the post-change means estimator when the signal is false. Chapter 6 extends the results to a specific problem in the normal case when the variance is subject to possible changes as well. Chapter 7 considers a sequential classification and segmentation procedure for alternatively changing means as an application of the results in Chapter 2. Chapter 8 proposes an adaptive CUSUM procedure to extend the results to linear post-change means in the normal case. Chapter 9 generalizes the methods to a correlated data case, and Chapter 10

compares the estimates obtained using the CUSUM procedure and the estimates obtained by the Shiryayev-Roberts procedure. (Reviewed by Lian Fen Qian)

Wu, Yanhong(2005-22)
Inference for Change-Point and Post-Change Mean with Possible Change in Variance
Sequential Analysis; Aug 2005-, 24, 279-302
Keywords: Biased estimation; Change-point problem; Corrected confidence interval; CUSUM procedure; Random walk theory; Renewal theorem .

Abstract: For a sequence of independent normal random variables, we consider the estimation of the change-point and the post-change mean after a change in the mean is detected by a CUSUM procedure, subject to a possible change in variance. Conditional on the event that a change is detected and it occurred far away from the starting point and the threshold is large, the (absolute) bias of the maximum likelihood estimator for the change-point (obtained at the reference value) is found. The first-order biases for the post-change mean and variance estimators are also obtained by using Wald's Likelihood Ratio Identity and the renewal theorem. In the local case when the reference value and the post-change mean are both small, accurate approximations are derived. A confidence interval for post-change mean based on a corrected normal pivot is then discussed.

Yoshiyuki Ninomiya(2005-23)
Information criterion for Gaussian change-point model
Statistics and Probability Letters, 72, 237-247
Keywords: Akaike's information criterion; Brownian motion with drift; Maximum of random walk; Number of change-points

Abstract: AIC-type information criterion is generally estimated by the bias-corrected maximum log-likelihood. In regular models, the bias can be estimated by p, where p is the number of parameters. The present paper considers the AIC-type information criterion for change-point models which are not regular, the bias of which will not be the same as for regular models. The bias is shown to depend on the expected maximum of a random walk with negative drift. Furthermore, it is shown that by using an approximation to a Brownian motion, the evaluated bias is given by $3m + p_m$ (not $m + p_m$), where m is the number of change-points and p_m is the number of regular parameters, which differs from regular models.

Young Sook Son Kim, Seong W.(2005-24)
Bayesian single change point detection in a sequence of multivariate normal observations

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Abstract: A Bayesian method is used to see whether there are changes of mean, covariance, or both at an unknown time point in a sequence of independent multivariate normal observations. Noninformative priors are used for all competing models: no-change model, mean change model, covariance change model, and mean and covariance change model. We use several versions of the intrinsic Bayes factor of Berger and Pericchi (Berger, J.O. and Pericchi, L.R., 1996, The intrinsic Bayes factor for model selection and prediction. Journal of the American Statistical Association, 91, 109-122; Berger, J.O. and Pericchi, L.R., 1998, Accurate and stable Bayesian model selection: the median intrinsic Bayes factor. Sankhya Series B , 60, 1-18.) to detect a change point. We demonstrate our results with some simulated data sets and a real data set.

Andersen, Lars Bo (2004-1)
Relative risk of mortality in the physically inactive is underestimated because of real changes in exposure level during follow-up

Abstract: Relative risk among exposure groups in prospective cohort studies is based on the assumption that all subjects are exposed at the level recorded at baseline throughout the study. Changes in risk behavior during follow-up will dilute the relative risk. This prospective cohort study in Copenhagen, Denmark, between 1964 and 1994 included 30,640 men and women; 19,149 were examined twice, with an interval of 6.7 (standard deviation, 3.4) years. Relative risks calculated from baseline measurements for moderately active and sedentary groups compared with the highly active group were 1.11 (95% confidence interval: 1.05, 1.18) and 1.64 (95%confidence interval: 1.53, 1.75), respectively. The relative risk between the highly active group and the sedentary group decreased with increasing follow-up time. When intraindividual changes in physical activity level during follow-up were taken into account, the relative risk of physical inactivity was 24-59% higher compared with the relative risk estimated from baseline measurements. The risk of a sedentary lifestyle is underestimated when it is calculated from one baseline measurement in prospective studies, because subjects change behavior during follow-up.

Borovkov, A. A. and Linke, Yu. Yu.(2004-2)
Asymptotically optimal estimates in the smooth change-point problem.

Abstract: A sample X containing independent observations $x_1, \ldots, x_n$ taking values in an arbitrary measurable space has the following structure: The first $(\theta - 1)$ observations have distribution $F = F_0$, the next T observations have distributions $F_1, \ldots, F_T$, respectively, and the remaining ones have distribution $G \equiv F_T$ (different from F). Regarding the distribution $F_j, j = 0, \ldots, T$, they are assumed to change linearly from F to G: $F_j = F + (j/T)(G - F)$. Based on the sample X the authors consider the estimation of the finite-valued parameter θ of the beginning time of the smooth change under the assumption that this parameter unboundedly increases. This parameter is called the smooth change-point. In the particular case of $T \equiv 1$ (i.e. in case of abrupt change from the first distribution F to the second one G), the problem is well known as the classical change-point problem, which has been discussed extensively in the literature.

F and G are assumed to be absolutely continuous with respect to some measure. Three cases are considered regarding the supports of F and G, i.e. the ranges where their densities, $f(x)$ and $g(x)$, respectively, are positive. In the first case they have common support, in the second (the so-called partly singular case) their supports only partly overlap, and in the third case (the singular case) their supports are disjoint.

The authors construct asymptotically optimal estimates of θ. Three types of estimates are considered: the maximum likelihood estimate, the mean likelihood estimate, and the Bayes estimate. Their asymptotic distributions are derived, and their properties studied.

The present study is based largely on results obtained for the classical change-point problem in two recent publications by the first author(Teor. Veroyatnost. i Primenen. 43 (1998), no. 4, 625-654; Mathematical statistics (Russian), "Nauka" Sibirsk. Predpr. RAN, Novosibirsk, 1997). (Reviewed by Jon Stene)

Caussinus, H. and Mestre, O.(2004-3)

Detection and correction of artificial shifts in climate series

Journal of Royal Statistical Society, Series C:Applied Statistics, 53, 405-425

Keywords: Changepoint; Climate series; Linear model; Model choice; Outlier; Penalized likelihood.

Abstract: Many long instrumental climate records are available and might provide useful information in climate research. These series are usually affected by artificial shifts, due to changes in the conditional of measurement and various kinds of spurious data. A comparison with surrounding weather-stations by means of a suitable two-factor model allows us to check the reliability of the series. An adapted penalized log-likelihood procedure is used to detect an unknown number of breaks and outliers. An example concerning temperature series from France confirms that a systematic comparison of the series together is valuable and allows us to correct the data even no reliable series can be taken as a reference.

Cheol-Woo Park and Woo-Chul Kim(2004-4)

Estimation of a regression function with a sharp change point using boundary wavelets

Abstract: We propose a sharp change point estimator based on the differences between right and left boundary wavelet smoothers. It is constructed by applying a two-step procedure to the observed data and has the minimax convergence rate. Next, we estimate the regression function with boundary wavelets in the left and right regions of the estimated jump point separately. This method helps us to capture the feature of a discontinuity in practice. Both mean integrated squared error and mean squared error of the estimated function are derived and we then show that these rates of convergence are the same as the case in which a jump point does not exist. Simulated examples demonstrate the improved performance of the proposed methods.

Fabio Busetti, and A. M. Robert Taylor(2004-5)

Tests of stationarity against a change in persistence

Abstract: This paper considers testing against a change in the order of integration of a time series, either from I(0) to I(1) or from I(1) to I(0), at some known or unknown point in the sample. The null hypothesis is that the series is stochastically stationary around a deterministic trend function. For the case of a known change-point the locally best invariant (LBI) tests against the above changes in the order of integration are derived under the assumption of Gaussianity. When the change-point is not known we construct our tests taking functions of the LBI statistics over all possible break-dates. Sub-sample implementations of existing stationarity tests are also considered. We demonstrate by a series of simulation experiments that, for a given direction of change, the LBI-based approach can deliver considerably more powerful tests than both the sub-sample stationarity tests and the ratio-based tests of Kim et al. (J. Econom. 109 2002 389) and Busetti and Taylor (Tests of stationarity against a change in persistence, University of Birmingham, Department of Economics, Discussion Paper 01-13, 2001). Moreover, the power losses from an unknown breakpoint do not appear to be large. We also find that standard stationarity tests have good power against both changes from I(0) to I(1) and vice versa, while the ratio-based tests are consistent only against a known direction of change. A further test constructed in terms of the LBI-based statistics for the two possible directions of change is shown to perform generally better than the standard stationarity tests when the direction of change under the alternative is not known. Finally, we apply the tests discussed in the paper to the US inflation rate and find evidence for a change in persistence from I(1) to I(0) behaviour although, significantly, the

timing of this change varies according to whether or not a simultaneous change in the level of the series is allowed.

Gabriela Ciuperca(2004-6)
Maximum likelihood estimator in a two-phase nonlinear random regression model
Statistics and Decisions, 22, 335-349

Abstract: We consider a two-phase random design nonlinear regression model, the regression function is discontinuous at the change-point. The errors are arbitrary, with $E(\varepsilon) = 0$ and $E(\varepsilon^2) < \infty$. We prove that Koul and Qians results [J. Statist. Plann. Inference 108 (2002), no. 1-2, 99–119] for linear regression still hold true for the nonlinear case. Thus the maximum likelihood estimator $\hat{r_n}$ of the change-point γ is n-consistent and the estimator $\hat{\theta_n}$ of the regression parameters θ_1 is $n^{1/2}$-consistent. The asymptotic distribution of $n^{1/2}(\hat{\theta_{1n}} - \theta_1^0)$ is Gaussian and $n(\hat{\gamma_n} - \gamma)$ converges to the left end point of the maximizing interval with respect to the change point. The likelihood process is asymptotically equivalent to a compound Poisson process.

Gijbels, Irne and Goderniaux, A-C.(2004-7)
Bandwidth selection for changepoint estimation in nonparametric regression
Technometrics, 46, 76-86
Keywords: Bandwidth; Bootstrap; Cross-validation; Discontinuity points; Least squares fitting.

Abstract: Nonparametric estimation of abrupt changes in a regression function involves choosing smoothing (bandwidth) parameters. The performance of estimation procedures depends heavily on this choice. So far, little attention has been paid to the crucial issue of choosing appropriate bandwidth parameters in practice. In this article we propose a bootstrap procedure for selecting the bandwidth parameters in a nonparametric two-step estimation method. This method results in a fully data-driven procedure for estimating a finite (but possibly unknown) number of changepoints in a regression function. We evaluate the performance of the data driven procedure via a simulation study, which reveals that the fully automatic procedure performs quite well. As an illustration, we apply the procedure to some real data.

Gijbels, I. and Goderniaux, A-C.(2004-8)
Bootstrap test for change-points in nonparametric regression.
Journal of Nonparametric Statistics; 16, 591-611
Keywords: Bandwidth; Bootstrap; Cross-validation; Discontinuity points; Derivative; Least-squares fitting; Local polynomial approximation

Abstract: The objective of this article is to test whether or not there is an abrupt change in the regression function itself or in its first derivative at certain (prespecified or not) locations. The test does not rely on asymptotics but approximates the sample distribution of the test statistic using a bootstrap procedure. The proposed testing method involves a data-driven choice of the smoothing parameters. The performance of the testing procedures is evaluated via a simulation study. Some comparison with an asymptotic test by Hamrouni (1999, Doctrol Thesis, université de Joseph Fourier, Grenoble, France.) and Grégoire and Hamrouni (2002b, J. of Nonparametric Statist.,14,87-112) and asymptotic tests by Müller and Stadtmüller (1999, Ann. of Statist.,21,299-337) and Dubowik and Stadtmüller (2000, Asymtotics in statistics and prob. Ed. Puri, M.L. pp 171-184) is provided. We also demonstrate the use of the testing procedures on some real data.

Gill, Ryan (2004-9)
Maximum likelihood estimation in generalized broken-line regression
The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 32, 227-238
Keywords: Change-point model; Existence; Exponential family; Generalized linear model; Identifiability; Maximum likelihood; Uniqueness

Abstract: The author examines the existence, uniqueness, and identifiability of estimators produced by maximum likelihood for a model where the canonical parameter of an exponential family gradually begins to drift from its initial value at an unknown change point. He illustrates these properties with theoretical examples and applies his results to global warming data and failure data for emergency diesel generators.

Hušková, Marie(2004-10)
Weak invariance principles for regression rank statistics
Sequential Analysis, 23, 121-140
Keywords: simple linear rank statistics; change point analysis; nonparametric asymptotic efficiency; Asymptotic distribution; functional limit theorems.

Abstract: Weak invariance principles are proved for regression rank statistics. As a consequence limit theorems for max- and Lp-functionals of partial sums of vectors of simple linear rank statistics are obtained. The results are useful in change point analysis, particularly in justification of application of permutation arguments, see Antoch and Hušková [Antoch, J.; Hušková, M. Detection of Structural Changes in Regression. Tatra Mountains Publications, 2003, 26, 1-15] and Hušková and Picek [Hušková, M.; Picek, J. M-tests for detection of structural changes in regression. In Statistical Data Analysis Based on the L1-Norm and Related Methods; Dodge, Y., Ed.; Birhäuser: Basel, 2002; 213-229]. The results of Hušková [Hušková, M. Limit theorems for rank statistics. Statist. Probab. Letters 1997, 32, 45-55] are generalized.

Huškoá, Marie and Neuhaus, Georg(2004-11)

Change point analysis for censored data

Journal of Statistical Planning and Inference Volume 126, 207-223

Keywords: Change(s) in location model; Censored data

Abstract: A class of rank-based procedures for testing a change in distribution when observations are independent but possibly censored is introduced and studied. The censoring variables are assumed to be independent but not necessarily identically distributed. The test procedures are developed along the line of two-sample rank tests under random censoring (e.g. Ann. Statist. 21 (1993) 1760 by Neuhaus). The limit behavior under the null hypothesis (no change in distribution of censored variables) is derived.

Huh, J. and Park, B.U. (2004-12)

Detection of a change point with local polynomial fits for the random design case

Australian and New Zealand Journal of Statistics, 46, 425-441

Keywords: discontinuity point; non-parametric regression; one-sided kernel; Rate of convergence; two-sided Brownian motion; Weak convergence

Abstract: Regression functions may have a change or discontinuity point in the νth derivative function at an unknown location. This paper considers a method of estimating the location and the jump size of the change point based on the local polynomial fits with one-sided kernels when the design points are random. It shows that the estimator of the location of the change point achieves the rate $n^{-1/(2n\nu+1)}$ when ν is even. On the other hand, when is odd, it converges faster than the rate $n^{-1/(2n\nu+1)}$ due to a property of one-sided kernels. Computer simulation demonstrates the improved performance of the method over the existing ones.

Jackson, Christopher H. and Sharples, Linda D.(2004-13)

Models for longitudinal data with censored changepoints.

Journal of the Royal Statistical Society: Series C (Applied Statistics); Vol. 53 Issue 1, 149-162

Keywords: Changepoint model; Longitudinal data; Lung transplantation; Mixture models; Reversible.

Abstract: In longitudinal studies of biological markers, different individuals may have different underlying patterns of response. In some applications, a subset of individuals experiences latent events, causing an instantaneous change in the level or slope of the marker trajectory. The paper presents a general mixture of hierarchical longitudinal models for serial biomarkers. Interest centres both on the time of the event and on levels of the biomarker before and after the event. In observational studies where marker series are incomplete, the latent event can be modelled by a survival distribution. Risk factors for the occurrence of the event

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can be investigated by including covariates in the survival distribution. A combination of Gibbs, MetropolisHastings and reversible jump Markov chain Monte Carlo sampling is used to fit the models to serial measurements of forced expiratory volume from lung transplant recipients.

Detection of structural changes in generalized linear models
Statistics and Probability Letters. 69, 315-332

Abstract: Some results on testing for changes in generalized linear models are presented and approximations to the critical values developed. Procedure is illustrated on simulated data.

Statistical inference of covariance change points in gaussian model.
Statistics; 38, 17-28

Abstract: In this paper, we study the testing and estimation of multiple covariance change points for a sequence of m -dimensional ($m > 1$) Gaussian random vectors by using the Schwarz information criterion (SIC). The unbiased SIC is also obtained. The asymptotic null distribution of the test statistic is derived. The result is applied to a simulated bivariate normal vector sequence ($m = 2$), and changes are successfully detected.

Evolutionary computation of a deterministic switching regressions estimator
Computational Statistics, 19, 211-225

Comparability of segmented line regression models
Biometrics, 60, 1005-1014

Abstract: Segmented line regression models, which are composed of continuous linear phases, have been applied to describe changes in rate trend patterns. In this article, we propose a procedure to compare two segmented line regression functions, specifically to test (i) whether the two segmented line regression functions are identical or (ii) whether the two mean functions are parallel allowing different intercepts. A general form of the test statistic is described and then the permutation procedure is proposed to estimate the p-value of the test. The permutation test is compared to an approximate F-test in terms of the p-value estimation and the performance of the permutation test is studied via simulations. The tests are applied to compare female lung cancer mortality rates between two registry areas and also to compare female breast cancer mortality rates between two states.

Lee, Sangyeol and Lee, Taewook (2004-18)
Cusum test for parameter change based on the maximum likelihood estimator
Sequential Analysis, 23, 239-256
Keywords: cusum test based on the MLE; Hidden Markov model; Weak convergence; Brownian bridge

Abstract: In this paper we consider the problem of testing for a parameter change based on the cusum test (cf. Lee, S.; Ha, J.; Na, O.; Na, S. The cusum test for parameter change in time series models. Scand. J. Statist. 2003, 30, 651-739) utilizing the maximum likelihood estimator. The issue is handled in iid random samples, and then special attention is paid to hidden Markov models. It is shown that the limiting distribution of the cusum test statistic is the sup of a standard Brownian bridge under regularity conditions. A simulation result is provided for illustration.

Lee, Sangyeol, Tokutsu, Yasuyoshi and Maekawa, Koichi (2004-19)
The cusum test for parameter change in regression models with ARCH errors
Journal of the Japan Statistical Society, 34, 173-188
Keywords:Brownian bridge, regression models with ARCH errors, residual cusum test, test for parameter change, weak convergence

Abstract: In this paper we consider the problem of testing for a parameter change in regression models with ARCH errors based on the residual cusum test. It is shown that the limiting distribution of the residual cusum test statistic is the sup of a Brownian bridge. Through a simulation study, it is demonstrated that the proposed test circumvents the drawbacks of Kim et al.'s (2000) cusum test. For illustration, we apply the residual cusum test to the return of yen/dollar exchange rate data."

O'Brien, Sean M. (2004-20)
Cutpoint selection for categorizing a continuous predictor

keywords: Piecewise linear logistic regression; Recursive partitioning; Trend-adjusted
chi-square test; Unbiased variable selection.

Abstract : This article presents a new approach for choosing the number of categories and the location of category cutpoints when a continuous exposure variable needs to be categorized to obtain tabular summaries of the exposure effect. The optimum categorization is defined as the partition that minimizes a measure of distance between the true expected value of the outcome for each subject and the estimated average outcome among subjects in the same exposure category. To estimate the optimum partition, an efficient nonparametric estimate of the unknown regression function is substituted into a formula for the asymptotically optimum categorization. This new approach is easy to implement and it outperforms existing cutpoint selection methods.

O'Quigley, John and Natarajan, Loki (2004-21)
Erosion of regression effect in a survival study
Keywords: changepoint models; Cox model; Estimating equation; Kaplan-Meier estimate;
Partial likelihood; Stochastic processes; time-varying effects; Weighted score

Abstract: Lack of persistence, or erosion, of the regression effect is an alternative to proportional hazards of particular interest in many medical applications. Such a departure from proportional hazards is often the most likely direction in which the model may be inadequate. Questions such as, is the effect of treatment only transitory or to what extent does an initially measured prognostic variable maintain its impact, frequently arise. In the context of a simple changepoint model, we propose a test of the null hypothesis of proportional hazards against the specific alternative of erosion of the regression effect. The particular changepoint model used can be viewed as a first approximation to a more complex reality, an approximation that enables us to avoid specifically modeling the functional form that any erosion might take. Practical guidelines for carrying out the test are provided. The approach is illustrated in the context of a study on risk factors for breast cancer survival.

Park, Junoh and Park, Sunghyun(2004-22)
Estimation of the Change Point in the $\overline{X}$ and S Control Charts.
Keywords: Control charts; Detecting changes; Joint estimation of a change point

Abstract: The $\overline{X}$ and S (or R) control charts are most commonly used in practice for monitoring the process mean and variance, respectively. A method for detecting changes in the process mean or variance is to obtain a stopping time at which the control charts issue a signal. The potential delay in generating a signal from the control charts calls for the change

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point estimation combined with control charts. In this paper, when $\overline{X}$ and S control charts issue a signal, a maximum likelihood joint estimator of the change point is suggested considering simultaneous change in the process mean and variance. The use of the proposed estimator is illustrated with an example.

Pitarakis, Jean-Yves(2004-23)

Least squares estimation and tests of breaks in mean and variance under misspecification.

Econometrics Journal; 7, 32-54

Keywords: Misspecification; Variance shifts; Bootstrapping; Structural breaks

Abstract: In this paper we investigate the consequences of misspecification on the large sample properties of change-point estimators and the validity of tests of the null hypothesis of linearity versus the alternative of a structural break. Specifically this paper concentrates on the interaction of structural breaks in the mean and variance of a time series when either of the two is omitted from the estimation and inference procedures. Our analysis considers the case of a break in mean under omitted-regime-dependent heteroscedasticity and that of a break in variance under an omitted mean shift. The large and finite sample properties of the resulting least-squares-based estimators are investigated and the impact of the two types of misspecification on inferences about the presence or absence of a structural break subsequently analyzed

Ramanayake, Asoka and Gupta, Arjun K.(2004-24)

Epidemic Change Model for the Exponential Family.

Communications in Statistics: Theory and Methods; 33, 2175-2198

Keywords: Exponential family; Maximum likelihood Likelihood ratio test; Change point

Abstract: We assume that a sequence of independent observations are given from an exponential family. It is hypothesized that the sequence has the same natural parameter λ_0. We would like to test if this natural parameter has been subjected to an epidemic change after an unknown point, for an unknown duration in the sequence. The likelihood ratio statistic for testing such an hypothesis is derived and then it's asymptotic null distribution is derived. We discuss the asymptotic behavior of the maximum likelihood estimates of the change points. We prove that the asymptotic non-null distribution is of the likelihood ratio statistic is normal. Some special cases of the exponential family are considered in detail.

Raimondo, Marc and Tajvidi, Nader(2004-25)

A peaks over threshold model for change-point detection by wavelets

Statistica Sinica 14, 395-412

Keywords: Change point; General Pareto distribution; Nonparametric regression; Peaks over threshold; Tail exponent wavelets.

Abstract: Newly available wavelet bases on multi-resolution analysis have exciting implications for detection of change-points. By checking the absolute value of wavelet coefficients one can detect discontinuities in an otherwise smooth curve even in the presence of additive noise. In this paper, we combine wavelet methods and extreme value theory to test the presence of an arbitrary number of discontinuities in an unknown function observed with noise. Our approach is based on a Peaks Over Threshold modelling of noisy wavelet transforms. Particular features of our method include the estimation of the extreme value index in the tail of the noise distribution. The critical region of our test is derived using a Generalised Pareto Distribution approximation to normalised sums. Asymptotic results show that our method is powerful in a wide range of medium size wavelet frequencies. We compare our test with competing approaches on simulated examples and illustrate the method on Dow-Jones data.

Ramanayake, Asoka(2004-26)

Tests for a Change Point in the Shape Parameter of Gamma Random Variables.

Communications in Statistics: Theory and Methods; 33, 821-833

Keywords: Change-point; Likelihood; Gamma distribution; Shape parameter

Abstract: Tests for detecting a change in the shape parameter of a sequence of gamma random variables are introduced. The asymptotic properties of the tests under the null hypothesis are studied. A Monte Carlo study is used to compare the critical values for moderate to large sample sizes. The tests are applied to data sets, one on time intervals between coal mine explosions and the other on inter arrival times of aircrafts, to detect possible changes in the shape parameter.

Richardson, Mary and Basu, Asit P. (2004-27)

Inferences on the parameters and system reliability for a failure-truncated power law process: A Bayesian approach using a change-point

ASA Proceedings of the Joint Statistical Meetings, 2164-2169 American Statistical Association (Alexandria, VA)

Shiryayev, Albert N. (2004-28)

A remark on the quickest detection problems

Statistics and Decisions, 22, 79-82

Keywords: disorder; disruption; change point problems; Optimal stopping; martingales; Stochastic differential equations; Stopping times; martingales; 60G40; 60G44

Wolfgang Bischoffa, Enkelejd Hashorvab, Jrg Hüsler, and Frank Millera(2004-29)
On the power of the Kolmogorov test to detect the trend of a Brownian bridge with
applications to a change-point problem in regression models

Abstract: Given a Brownian bridge B_0 with trend $g : [0,1] \to [0,\infty)$,

$$Y(z) = g(z) + B_0(z), z \in [0,1], \quad (1)$$

we are interested in testing $H_0 : g \equiv 0$ against the alternative $K : g > 0$. For this test problem
we study weighted Kolmogorov tests reject $H_0 \Leftrightarrow sup_{z \in [0,1]} w(z) Y(z) > c$, where $c > 0$ is
a suitable constant and $w : [0,1] \to [0,\infty)$ is a weight function. To do such an investigation
a recent result of the authors on a boundary crossing probability of the Brownian bridge
is useful. In case the trend is large enough we show an optimality property for weighted
Kolmogorov tests. Furthermore, an additional property for weighted Kolmogorov tests is
shown which is useful to find the more favourable weight for specific test problems. Finally,
we transfer our results to the change-point problem whether a regression function is or is
not constant during a certain period.

Yin-cai Tang and He-liang Fei(2004-30)
**Detecting Change Points in Polynomial Regression Models with an Application
to Cable Data Sets**

Abstract: In this paper, the Schwarz Information Criterion (SIC) is used to detect the
change points in polynomial regression models. Switching quadratic regression models with
same amount of model deviation and switching polynomial regression models with different
amount of model deviation for different segments of regression are considered. The number
of separate regimes and their corresponding regression orders are assume to be known. The
method is then applied to cable data sets and the change points are successfully detected.
Keywords Information criterion - change points - polynomial regression models

Yoshiyuki Ninomiya(2004-31)
**Construction of conservative test for change-point problem in two-dimensional
random fields**

Abstract: This paper is concerned with a change-point problem in random fields. Tests for detecting a change-structure in a two-way table are constructed. Solutions to this problem require calculations of multiple integrals, but the calculations become difficult as the table gets large. To overcome this difficulty, the tube method is developed in differential geometry. It is shown by simulations that the proposed method is superior to other available methods.

Zhong Guan(2004-32)
A semiparametric changepoint model

Abstract: A semiparametric changepoint model is considered and the empirical likelihood method is applied to detect the change from a distribution to a weighted distribution in a sequence of independent random variables. The maximum likelihood changepoint estimator is shown to be consistent. The empirical likelihood ratio test statistic is proved to have the same limit null distribution as that with parametric models. A data-based test for the validity of the models is also proposed. Simulation shows the sensitivity and robustness of the semiparametric approach. The methods are applied to some classical datasets such as the Nile River data and stock price data.

Achim Zeileis, Christian Kleiber, Walter Krmer and Kurt Hornik (2003-1)
Testing and dating of structural changes in practice

Abstract:An approach to the analysis of data that contains (multiple) structural changes in a linear regression setup is presented. Various strategies which have been suggested in the literature for testing against structural changes as well as a dynamic programming algorithm for the dating of the breakpoints are implemented in the statistical software package. Using historical data on Nile river discharges, road casualties in Great Britain and oil prices in Germany, it is shown that statistically detected changes in the mean of a time series as well as in the coefficients of a linear regression coincide with identifiable historical, political or economic events which might have caused these breaks.

Albin, J.M.P. and Jarukova, D.(2003-2)
On a Test Statistic for Linear Trend

Keywords: Change point detection; χ^2-process; Extremes; Gaussian process; Linear trend; OrnsteinUhlenbeck process; Test of linear trend

Abstract: Let $\{W(s)\}_{s\geq 0}$ be a standard Wiener process. The supremum of the squared Euclidian norm $\mid Y(t) \mid^2$, of the R^2-valued process $Y(t) = (\sqrt{1/t}W(t), \sqrt{12/t^3}\int_0^t s\, dW(s) - \sqrt{3/t}W(t))$, $t \in [\alpha, 1]$, is the asymptotic, large sample distribution, of a test statistic for a change point detection problem, of appearance of linear trend. We determine the asymptotic behavior $\mathbf{P}\{\sup_{t\in[\alpha,1]}\mid Y(t) \mid^2 > u\}$ as $u \to \infty$, of this statistic, for a fixed $\alpha \in (0,1)$, and for a moving $\alpha = \alpha(u) \downarrow 0$ at a suitable rate as $u \to \infty$. The statistical interest of our results lie in their use as approximate test levels.

Ali S. Dabye, Christian Farinetto, and Yury A. Kutoyants(2003-3)
On Bayesian estimators in misspecified change-point problems for Poisson process
Keywords: Inhomogeneous Poisson process; Change-point type problem; Parameter estimation; Misspecified model; Bayesian estimator; Consistency; Limit distribution

Abstract: Consider an inhomogeneous Poisson process X on $[0,T]$ whose unknown intensity function switches from a lower function g_* to an upper function h_* at some unknown point ϑ_*. Here, ϑ_* is a random variable. What is known are continuous bounding functions g and h such that $g_*(t) \leq g(t) < h(t) \leq h_*(t)$ for $0 \leq t \leq T$ and the prior density function of ϑ. It is shown that on the basis of n observations of the process X the Bayesian estimator $\overline{\vartheta_n}$ of ϑ_* is consistent for $n \to \infty$, and also that $n(\overline{\vartheta_n} - \vartheta_*)$ converges in law and in p th moment to limits described in terms of the unknown functions g_* and h_*.

Andrews, D. W. K. (2003-4)
End-of-sample instability tests
Econometrica, 71, 1661-1694
Keywords: instrumental variables estimator; generalized method of moments estimator; Least squares estimator; parameter change; structural instability test; Structural change

Abstract: This paper considers tests for structural instability of short duration, such as at the end of the sample. The key feature of the testing problem is that the number, m, of observations in the period of potential change is relatively small - possibly as small as one. The well-known F test of Chow (1960, Econometrica,28,591-605) for this problem only applies in a linear regression model with normally distributed iid errors and strictly exogenous regressors, even when the total number of observations, $n + m$, is large. We generalize the F test to cover regression models with much more general error processes, regressors that are not strictly exogenous, and estimation by instrumental variables as well as least squares. In addition, we extend the F test to nonlinear models estimated by generalized method of

moments and maximum likelihood. Asymptotic critical values that are valid as $n \to [\infty]$ with m fixed are provided using a subsampling-like method. The results apply quite generally to processes that are strictly stationary and ergodic under the null hypothesis of no structural instability.

Bai, J. and Perron, P.(2003-5)
Computation and analysis of multiple structural change models
Journal of Applied Econometrics 18, 1-22

Abstract: In a recent paper, Bai and Perron ([1998, Econometrica, 66, 47-78]) considered theoretical issues related to the limiting distribution of estimators and test statistics in the linear model with multiple structural changes. In this companion paper, we consider practical issues for the empirical applications of the procedures. We first address the problem of estimation of the break dates and present an efficient algorithm to obtain global minimizers of the sum of squared residuals. This algorithm is based on the principle of dynamic programming and requires at most least-squares operations of order $O(T^2)$ for any number of breaks. Our method can be applied to both pure and partial structural change models. Second, we consider the problem of forming confidence intervals for the break dates under various hypotheses about the structure of the data and the errors across segments. Third, we address the issue of testing for structural changes under very general conditions on the data and the errors. Fourth, we address the issue of estimating the number of breaks. Finally, a few empirical applications are presented to illustrate the usefulness of the procedures. All methods discussed are implemented in a GAUSS program

Dechang Chen, Michael Fries, and John M. Lyon(2003-6)
A Statistical Method of Detecting Bioremediation
Journal of Data Science, v.1, no.1, 27-41.
Keywords: Change point; Linear model; Posterior distribution; Bayesisn approach.

Abstract: Hydrocarbon contaminated soils result from pipeline ruptures, petroleum manufacture spills, as well as storage and transportation accidents (Bossert and Bartha (1984) Petroleum Microbiology, R.M. Atlas(Ed.), MacMillan, Ney York, 453-473). The cost of removal of the contaminated solids followed by incineration or by disposal in a landfill is prohibitive. Bioremediation – the use of microorganism populations to eliminate hydrocarbon contaminations from the environment - is the most acceptable technology for hydrocarbon cleanup (Bossert and Bartha (1984)). It can be argued that a decrease of the oil concentration in soil is not due to biodegradation but due to sorption. If this were the case, since mass transfer of sorption is a gradual process, a slow decrease in the oil recovery rate may be observed after a spill. However, a rapid or sudden decrease in the oil concentration during the incubation should exclude sorption as the primary mechanism contributing to the observed hydrocarbon loss. A Bayesian procedure is given to detect a change of the linear

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relationship between the oil concentration (the dependent variable) and the time in days since the addition of the oil (the independent variable). The advantage of this procedure is that it does not need to assume that the variance of the error before the change is equal to that after the change. The implementation of this procedure is straightforward.

Emad-Eldin A. A. Aly , Abd-Elnaser S. Abd-Rabou and Noriah M. Al-Kandari(2003-7)
Tests for multiple change points under ordered alternatives

Abstract: We consider the problem of testing the null hypothesis of no change against the alternative of multiple change points in a series of independent observations when the changes are in the same direction. We extend the tests of Terpstra (1952, Statistical department of the mathematical center, Arosterdom. Rep. S(92)(VP2)), Jonckheere (1954, Biometrika 41, 135-145) and Puri (1965, Commun. Pure Appl. Math. 18,51-63) to the change point setup. We obtain the asymptotic null distribution of the proposed tests. We also give approximations for their limiting critical values and tables of their finite sample Monte Carlo critical values. The results of Monte Carlo power studies conducted to compare the proposed tests with some competitors are reported.

Esa Kokki and Antti Penttinen(2003-8)
Poisson Regression with Change-Point Prior in the Modelling of Disease Risk around a Point Source

Abstract: Bayesian estimation of the risk of a disease around a known point source of exposure is considered. The minimal requirements for data are that cases and populations at risk are known for a fixed set of concentric annuli around the point source, and each annulus has a uniquely defined distance from the source. The conventional Poisson likelihood is assumed for the counts of disease cases in each annular zone with zone-specific relative risk and parameters and, conditional on the risks, the counts are considered to be independent. The prior for the relative risk parameters is assumed to be piecewise constant at the distance having a known number of components. This prior is the well-known change-point model. Monte Carlo sampling from the posterior results in zone-specific posterior summaries, which can be applied for the calculation of a smooth curve describing the variation in disease risk as a function of the distance from the putative source. In addition, the posterior can be used in the calculation of posterior probabilities for interesting hypothesis. The suggested model is suitable for use in geographical information systems (GIS) aimed for monitoring

disease risks. As an application, a case study on the incidence of lung cancer around a former asbestos mine in eastern Finland is presented. Further extensions of the model are discussed.

Hall, Peter and Molchanov, Ilya (2003-9)
Sequential methods for design-adaptive estimation of discontinuities in regression curves and surfaces

Abstract: In fault-line estimation in spatial problems it is sometimes possible to choose design points sequentially, by working one's way gradually through the 'response plane', rather than distributing design points across the plane prior to conducting statistical analysis. For example, when estimating a change line in the concentration of resources on or under the sea bed, individual measurements can be particularly expensive to make. In such cases, sequential, design-adaptive methods might be attractive. Appropriate methodology is largely lacking, however, and the potential advantages of taking a sequential approach are unclear. In the present paper we address both these problems. We suggest a methodology based on 'sequential refinement with reassessment' that relies upon assessing the correctness of each sequential result, and reappraising previous results if significance tests show that there is reason for concern. We focus part of our attention on univariate problems, and we show how methods for the spatial case can be constructed from univariate ones." Reviewed by Valeri Stefanov

Hall, Peter and Müller, Hans-Georg (2003-10)
Order-preserving nonparametric regression, with applications to conditional distribution and quantile function estimation

Abstract: In some regression problems we observe a "response" Y_{ti} to level t of a "treatment" applied to an individual with level X_i of a given characteristic, where it has been established that response is monotone increasing in the level of the treatment. A related problem arises when estimating conditional distributions, where the raw data are typically independent and identically distributed pairs (X_i, Z_i), and Y_{ti} denotes the proportion of Z_i's that do not exceed t. We expect the regression means $g_t(x) = E(Y_{ti}|X_i = x)$ to enjoy the same order relation as the responses, that is, $g_t \leq g_s$ whenever $s \leq t$. This requirement is necessary to obtain bona fide conditional distribution functions, for example. If we estimate g_t by passing a linear smoother through each dataset $X_t = \{(X_i, Y_{ti}) : 1 \leq i \leq n\}$, then the

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order-preserving property is guaranteed if and only if the smoother has nonnegative weights. However, in such cases the estimators generally have high levels of boundary bias. On the other hand, the order-preserving property usually fails for linear estimators with low boundary bias, such as local linear estimators, or kernel estimators employing boundary kernels. This failure is generally most serious at boundaries of the distribution of the explanatory variables, and ironically it is often in just those places that estimation is of greatest interest, because responses there imply constraints on the larger population. In this article we suggest nonlinear, order-invariant estimators for nonparametric regression, and discuss their properties. The resulting estimators are applied to the estimation of conditional distribution functions at endpoints and also changepoints. The availability of bona fide distribution function estimators at endpoints also enables the computation of changepoint diagnostics that are based on differences in a suitable norm between two estimated conditional distribution functions, obtained from data that fall into one-sided bins.

Abstract: This paper discusses the consistency and limiting distributions of a class of M-estimators in two-phase random design linear regression models where the regression function is discontinuous at the change-point with a fixed jump size. The consistency rate of an M-estimator for the change-point parameter r is shown to be n while it is $n_{1/2}$ for the coefficient parameter estimators, where n denotes the sample size. The normalized M-process is shown to be uniformly locally asymptotically equivalent to the sum of a quadratic form in the coefficient parameter vector and a jump point process in the change-point parameter, in probability. These results are then used to obtain the joint weak convergence of the M-estimators. In particular, is shown to converge weakly to a random variable which minimizes a compound Poisson process, a suitably standardized coefficient parameter M-estimator vector is shown to be asymptotically normal, and independent of .

Abstract: We introduce a procedure for generalized monotonic curve fitting that is based on a Bayesian analysis of the isotonic regression model. Conventional isotonic regression fits monotonically increasing step functions to data. In our approach we treat the number and location of the steps as random. For each step level we adopt the conjugate prior to the

sampling distribution of the data as if the curve was unconstrained. We then propose to use Markov chain Monte Carlo simulation to draw samples from the unconstrained model space and retain only those samples for which the monotonic constraint holds. The proportion of the samples collected for which the constraint holds can be used to provide a value for the weight of evidence in terms of Bayes factors for monotonicity given the data. Using the samples, probability statements can be made about other quantities of interest such as the number of change points in the data and posterior distributions on the location of the change points can be provided. The method is illustrated throughout by a reanalysis of the leukaemia data studied by Schell and Singh.

Hurn, Merrilee, Justel, Ana and Robert, Christian P. (2003-13)
Estimating mixtures of regressions
Journal of Computational and Graphical Statistics, 12, 55-79
Keywords: Bayesian inference; Birth-and-death process; Label switching; Logistic regression; Loss functions; MCMC algorithms; Poisson regression; switching regression.

Abstract: This article shows how Bayesian inference for switching regression models and their generalizations can be achieved by the specification of loss functions which overcome the label switching problem common to all mixture models. We also derive an extension to models where the number of components in the mixture is unknown, based on the birth-and-death technique developed in recent literature. The methods are illustrated on various real data sets.

Huškova, M.(2003-14)
Serial rank statistics for detection of changes
Statistics and Probability Letters Volume 61, 199-213
Keywords: Independence; AR-sequences; Change point detection; Serial rank statistics

Abstract: A class of ranks based test statistics for testing hypothesis of randomness (observations are independent and identically distributed) against the alternative that the observations become dependent at some unknown time point is introduced and its limit properties are studied. The considered problem belongs to the area of the change-point analysis.

Irène Gijbels and Ülkü Gürler(2003-15)
Estimation of a Change Point in a Hazard Function Based on Censored Data
Lifetime Data Analysis, 9, 395-411
Keyword: Change point hazard model; Cumulative hazard function; Least squares estimation; Random censoring

Abstract: The hazard function plays an important role in reliability or survival studies since it describes the instantaneous risk of failure of items at a time point, given that they have not failed before. In some real life applications, abrupt changes in the hazard function are observed due to overhauls, major operations or specific maintenance activities. In such situations it is of interest to detect the location where such a change occurs and estimate the size of the change. In this paper we consider the problem of estimating a single change point in a piecewise constant hazard function when the observed variables are subject to random censoring. We suggest an estimation procedure that is based on certain structural properties and on least squares ideas. A simulation study is carried out to compare the performance of this estimator with two estimators available in the literature: an estimator based on a functional of the Nelson-Aalen estimator and a maximum likelihood estimator. The proposed least squares estimator turns out to be less biased than the other two estimators, but has a larger variance. We illustrate the estimation method on some real data sets.

Jaruková, Daniela(2003-16)
Asymptotic distribution of a statistic testing a change in simple linear regression with equidistant design
Statistics and Probability Letters, 64, 89-95
Keywords: Change in regression; Maximum-type test statistic; Asymptotic distribution; Extremes of χ^2 processes

Abstract: Testing procedure for testing a change in a linear regression with equidistant design is considered. Limit distribution for "over-all" maximum-type test statistics under assumption of no change is given.

Koul, Hira L., Qian, Lianfen and Surgailis, Donatas (2003-17)
Asymptotics of M-estimators in two-phase linear regression models
Stochastic Processes and their Applications, 103, 123-154
Keywords: Change-point estimator; Fixed jump size; Compound Poisson process

Abstract: This paper discusses the consistency and limiting distributions of a class of M-estimators in two-phase random design linear regression models where the regression function is discontinuous at the change-point with a fixed jump size. The consistency rate of an M-estimator Image for the change-point parameter r is shown to be n while it is $n^{1/2}$ for the coefficient parameter estimators, where n denotes the sample size. The normalized M-process is shown to be uniformly locally asymptotically equivalent to the sum of a quadratic form in the coefficient parameter vector and a jump point process in the change-point parameter, in probability. These results are then used to obtain the joint weak convergence of the M-estimators. In particular, Image is shown to converge weakly to a random variable which minimizes a compound Poisson process, a suitably standardized coefficient parameter M-estimator vector is shown to be asymptotically normal, and independent of Image.

Lee, Sangyeol, Ha, Jeongcheol, Na, Okyoung and Na, Seongryong (2003-18)
The cusum test for parameter change in time series models
Scandinavian Journal of Statistics, 30, 781-796
Keywords: Autocovariance function; CUSUM test; Invariance principle; Linear process;
Martingale difference; RCA model; Testing for parameter change; Weak convergence

Abstract: The paper deals with the problem of testing the parameter changes in time series based on a so-called cusum test. The contribution here is to develop a more general formulation which allows for testing the change of any parameter (while actually it is well established only for mean and variance). So, this new algorithm is constructed, and it is applied to random coefficient autoregressive series (RCA) based on a least-squares estimator. In addition one considers the problem of testing the autocovariance change in infinite order moving average processes based on the sample autocovariance functions. Some simulation results are displayed.

Marc A. Suchard, Robert E. Weiss; Karin S. Dorman and Janet S. Sinsheimer(2003-19)
Inferring spatial phylogenetic variation along nucleotide sequences: a multiple changepoint model.
Journal of the American Statistical Association, 98, 427-437
Keywords: Bayesian; Changepoints; Human immunodeficiency virus; Phylogeny;
Recombination; Reversible-jump Markov chain Monte Carlo.

Abstract: We develop a Bayesian multiple changepoint model to infer spatial phylogenetic variation (SPV) along aligned molecular sequence data. SPV occurs in sequences from organisms that have undergone biological recombination or when evolutionary rates and selective pressures vary along the sequences. This Bayesian approach permits estimation of uncertainty regarding recombination, the crossing-over locations, and all other model parameters. The model assumes that the sites along the data separate into an unknown number of contiguous segments, each with possibly different evolutionary relationships between organisms, evolutionary rates, and transition: transversion ratios. We develop a transition kernel, use reversible-jump Markov chain Monte Carlo to fit our model, and draw inference from both simulated and real data. Through simulation, we examine the minimal length recombinant segment that our model can detect for several levels of evolutionary divergence. We examine the entire genome of a reported human immunodef iciency virus (HIV)-1 isolate, related to a purported recombinant virus thought to be the causative agent of an epidemic outbreak of HIV-1 infection among intravenous drug users in Russia. We find that regions of the genome differ in their evolutionary history and selective pressures. There is strong evidence for multiple crossovers along the genome and frequent shifts in selective pressure changes throughout the vif through env genes.

Paulauskas, V.(2003-20)

A New Estimator for a Tail Index

Abstract: We investigate properties of a new estimator for a tail index introduced by Davydov and co-workers. The main advantage of this estimator is the simplicity of the statistic used for the estimator. We provide results of simulation by comparing plots of our's and Hill's estimators.

Pons, Odile(2003-21)

Estimation in a Cox regression model with a change-point according to a threshold in a covariate

Abstract: We consider a nonregular Cox model for independent and identically distributed right censored survival times, with a change-point according to the unknown threshold of a covariate. The maximum partial likelihood estimators of the parameters and the estimator of the baseline cumulative hazard are studied. We prove that the estimator of the change-point is n-consistent and the estimator of the regression parameters are $n^{1/2}$-consistent, and we establish the asymptotic distributions of the estimators. The estimators of the regression parameters and of the baseline cumulative hazard are adaptive in the sense that they do not depend on the knowledge of the change-point.

Salanti, Georgia and Ulm, Kurt(2003-22)

A nonparametric changepoint model for stratifying continuous variables under order restrictions and binary outcome.

Abstract: Modeling using monotonic regression can be a useful alternative to parametric approaches when optimal stratification for continuous predictors is of interest. This method is described here in the context of binary response. Within this framework we aim to address two points. First, we propose a method to enhance the parsimony of the model, by applying a reducing procedure based on a sequence of Fisher exact tests and a bootstrap method to select between full monotonic and reduced model. Secondly, we discuss the case of multiple predictors: an iterative algorithm (an extension of the Pool Adjacent Violators Algorithm) can be applied when more than one predictor variable is taken into account. The resulting model is a monotonic surface and can be applied alternatively to the additive monotonic models as described by Morton-Jones and colleagues when the explanatory variables are

assumed to interact. The monotonic-surface model provides also a multivariate extension of the monotonic likelihood ratio test. This test is discussed here and an approach based on permutations to assess the p-value is proposed. Finally, we combine both ideas (reduced monotonic regression and monotonic-surface estimation) to a simple and easy to interpret model, which leads to a combination of the predictors in a few constant risk groups. Despite the fact that the proposed approach becomes somewhat cumbersome due to the lack of asymptotic methods to infer, it is attractive because of its simplicity and stability. An application will outline the benefit of using bivariate step functions in modeling.

Sertkaya, Durdu and Sözer, M. Tekin(2003-23)

A bayesian approach to the constant hazard model with a change point and an application to breast cancer data

Abstract: In this paper, a Bayesian approach to the problem of constant hazard with a change point is considered using noninformative priors. We apply the model to a data set gathered from a group of patients with breast cancer.

Shuai, Xiufu, Zhou, Zhijun and Yost, Russell S.(2003-24)

Using segmented regression models to fit soil nutrient and soybean grain yield changes due to liming

Abstract: Frequently soil-plant relationships and responses are complex combinations of increases-level-decreases consisting of linear segments of differing slope. Segmented regression is very useful to express and quantify such relationships and responses. Fitting segmented regression models to such data, however, remains a challenge. The problem is in estimating the join points and coefficients. We use median functions to express segmented regression models, and estimate the join points by standard estimation routines such as Marquardt, Newton, and doesn't use derivatives (DUD) methods that are available in statistical software such as SAS. Segmented straight-line models are fit to data reflecting soil Manganese (Mn), Calcium (Ca), Phosphorus (P), and soybean yield changes under different soil pH conditions due to liming. A systematic comparison of the slopes and join points suggests that different mechanisms are limiting soybean yield at different intervals as soil pH increased.

Tomá Víek(2003-25)

The likelihood ratio method for testing changes in the parameters of double exponential observations

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Abstract: Several papers have been published on the change point problem in location models. Majority of them uses least squares type methods. The least absolute deviation estimator of change point was studied in Bai (Econom. Theory 11 (1995) 403). Horváth (Ann. Statist. 21 (1993) 671) derived the least squares statistic for location and scale models. In the present paper, we focus on location and scale models and we use the least absolutes deviation methods. We construct a test statistic and a change point estimator and establish their limit behaviour

Toms, Judith D. and Lesperance, Mary L. (2003-26)
Piecewise regression: A tool for identifying ecological thresholds

Abstract : We demonstrate the use of piecewise regression as a statistical technique to model ecological thresholds. Recommended procedures for analysis are illustrated with a case study examining the width of edge effects in two understory plant communities. Piece-wise regression models are "broken-stick" models, where two or more lines are joined at unknown points, called breakpoints. "Breakpoints" can be used as estimates of thresholds and are used here to determine the width of edge effects. We compare a sharp-transition model with three models incorporating smooth transitions: the hyperbolic-tangent, bent-hyperbola, and bent-cable models. We also calculate three types of confidence intervals for the breakpoint estimate: an interval based on the computed standard error of the estimate from the fitting procedure, an empirical bootstrap confidence interval, and a confidence interval derived from an inverted F test. We recommend use of the inverted F test confidence interval when sample sizes are large, and cautious use of bootstrapped confidence intervals when sample sizes are smaller. Our analysis demonstrates the need for a careful study of the likelihood surface when fitting and interpreting the results from piecewise-regression models.

Wu, C. Q. , Zhao, L. C. and Wu, Y. H.(2003-27)
Estimation in change-point hazard function models

Abstract: In this paper, we consider hazard function models with a change-point allowing for random censoring when the base-line hazard function is unknown with some parameters. A

non-parametric estimator of the change-point is proposed in the context of counting process. The estimators of change-point and other parameters are shown to be consistent

Zhang, Heping, Yu, Chang-Yung, Zhu, Hongtu and Shi, Jian (2003-28)
Identification of linear directions in multivariate adaptive spline models
Journal of the American Statistical Association, 98, 369-376
Keywords: Additive models; Change-point regression; Linear discriminant analysis; Projection pursuit

Abstract: Identifying linear directions in multivariate regression has been a statistical challenge and has attracted particular attention since projection pursuit was developed. To this end, we propose and investigate the use of linear discriminant analysis and projection Hessian directions in conjunction with multivariate adaptive regression splines. Simulation studies in a variety of settings demonstrate the usefulness of our approach in revealing both the functional forms and the linear substructures based on the observed data. Mathematical results are also provided to support our approach. Comparisons are made between our approach and existing approaches, and the improvements are evident. Depending on the circumstance, the extent of improvement can be substantial.

Alexander Aue and Josef Steinebach(2002-1)
A note on estimating the change-point of a gradually changing stochastic process
Statistics and Probability Letters, 56, 177-191
Keywords: Gradual change; Change-point; Location model; Weak invariance principle; Wiener process; Limiting distribution; Asymptotics

Abstract: We consider an estimator of the change-point of a stochastic process satisfying some weak invariance principles. Making use of the known asymptotics of the approximating Wiener processes we derive various limiting distributions according to different orders of magnitude of the underlying change. The results take into account, but also extend those of Huškova (J. Statist. Plann. Infer. 76 (1999) 109125), who studied a location model for gradual changes with independent, identically distributed (iid) errors. Aim of this note is to show that corresponding results hold also true in our more general setting.

Antoniadis, Anestis and Gijbels, Irène(2002-2)
Detecting Abrupt Changes by Wavelet Methods.
Journal of Nonparametric Statistics; 14, 7-29
Keywords:Change-point detection; Continuous discrete wavelet transform; Rate of convergence; Segmented multiresolution analysis

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Abstract: The objective of this paper is to contribute to the methodology available for dealing with the detection and the estimation of the location of discontinuities in one-dimensional piecewise smooth regression functions observed in white Gaussian noise over an interval. Our approach is nonparametric in nature because the unknown function is not assumed to have any specific form. Our method relies upon a wavelet analysis of the observed signal and belongs to the class of "indirect" methods, where one detects and locates the change points prior to fitting the curve, and then uses one's favorite function estimation technique on each segment to recover the curve. We show that, provided discontinuities can be detected and located with sufficient accuracy, detection followed by wavelet smoothing enjoys optimal rates of convergence.

Blisle, Patrick; Joseph, Lawrence ; Wolfson,David B. and Zhou, Xiaojie(2002-3)

Bayesian estimation of cognitive decline in patients with Alzheimer's disease

The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 30, 37-54

Keywords: Alzheimer's disease; Bayese factors; Biphasic regression; Change point; Gibbs sampler; Mini-Mental State Exam.

Abstract: Recently, there has been great interest in estimating the decline in cognitive ability in patients with Alzheimer's disease. Measuring decline is not straightforward, since one must consider the choice of scale to measure cognitive ability, possible floor and ceiling effects, between-patient variability, and the unobserved age of onset. The authors demonstrate how to account for the above features by modeling decline in scores on the Mini-Mental State Exam in two different data sets. To this end, they use hierarchical Bayesian models with change points, for which posterior distributions are calculated using the Gibbs sampler. They make comparisons between several such models using both prior and posterior Bayes factors, and compare the results from the models suggested by these two model selection criteria.

Balakumar, Sivanandan(2002-4)

Multivariate changepoint problem

Missouri Journal of Mathematical Sciences Articles, 14, 186-195

Abstract: Procedures for detecting a changepoint in a sequence of N random p-vectors, when there is a location or scale change are considered. An extension of such procedures for the case of simultaneous occurrences of location and scale changes is carried out. The asymptotic distributions of the proposed statistics under the null hypothesis, in two different changepoint models are obtained.

Barry, Daniel(2002-5)

A bayesian analysis for derivative change points

Communications in Statistics: Theory and Methods; 31, 1335-1348

Keywords: Change point; Bayesian analysis; Smoothing splines

Abstract: Let $y_1, y_2, \cdots, y_n$, be a sequence of observations satisfying $Y_i = \theta(x_i) + e_i$ where $x_1 < x_2 < \cdots < x_n$ are real numbers, θ is a fixed but unknown regression function, and the errors $e_1, e_2, cdots, e_n$, are independent and identically distributed observations from a $N(0, v)$ density. We develop a prior probability model for the regression function θ which allows for the possibility of jump discontinuities in the mth derivative of θ. Neither the number of jumps nor their locations are assumed known. A Bayesian analysis is implemented using Markov Chain Monte Carlo methods. The new method is compared to smoothing splines in a simulation study and in the analysis of a set of data representing the average weight to height ratio of a group of boys recorded at one month intervals.

Davies, Robert B. (2002-6)
Hypothesis testing when a nuisance parameter is present only under the alternative: Linear model case
Biometrika, 89, 484-489
Keywords: Changepoint; process; Frequency component; Hypothesis test; Nuisance parameter; process; Two-phase regression; Upcrossing

Abstract: From the introduction: R. B. Davies(Biometrika 64 (1977), no. 2, 247-254); (Biometrika 74 (1987), no. 1, 33-43) introduced a method for testing a hypothesis in the presence of a nuisance parameter, θ, which enters into the model only under the alternative. Examples 1 and 3 in Davies(1987) were concerned with detecting a discrete frequency component and detecting a change in slope. Both are of the form in which we observe $Y_1, \ldots, Y_n$, denoted by a column vector, Y, which are independently normally distributed with variance σ^2 and mean $E(Y) = X\gamma + W(\theta)\xi$, where X is an $n \times s$ matrix and $W(\theta)$ is an $n \times p$ matrix function of the scalar nuisance parameter $\theta \in [L, U]$. The column vectors γ and ξ are unknown parameters. The objective is to test the hypothesis $\xi = 0$ against the alternative that at least one element of ξ is nonzero. If $p = 1$ we may be interested in testing $\xi = 0$ against the one-sided alternative $\xi > 0$. If $\xi = 0$ then θ does not enter into the model, so the model is of the general form considered by Davies(1977).

This paper extends the results of Davies(1987) to the situation when the residual variance σ^2 is unknown and n is too small for it to be estimated accurately. This requires us, in particular, to replace the Gaussian and χ^2 processes by t and F processes.

Dempfle, Astrid and Stute, Winfried (2002-7)
Nonparametric estimation of a discontinuity in regression
Statistica Neerlandica, 56, 233-242
Keywords: Changepoint; Rate of convergence

Abstract: We propose and study a new method to nonparametrically estimate a discontinuity of a regression function. The optimal rate of convergence n-1 is obtained under minimal assumptions. No smoothing is required.

El Barmi, Hammou(2002-8)
On detecting change in likelihood ratio ordering
Journal of Nonparametric Statistics; 14, 555-568

Abstract: This article studies the problem of testing and locating changepoints in likelihood
ratios of two multinomial probability vectors. We propose a binary search procedure to de-
tect the changepoints in the sequence of the ratios of probabilities and obtain the maximum
likelihood estimators of two multinomial probability vectors under the assumption that the
probability ratio sequence has a changepoint. We also give a strongly consistent estimator
for the changepoint location. An information theoretic approach is used to test the equal-
ity of two discrete probability distributions against the alternative that their ratios have a
changepoint. Approximate critical values of the test statistics are provided by simulation
for several choices of model parameters. Finally, we examine a real life data set pertaining
to average daily insulin dose from the Boston Collaborative Drug Surveillance Program and
locate the changepoints in the probability ratios.

Grégoire, Géard and Hamrouni, Zouhir(2002-9)
**Two Non-Parametric Tests For Change-Point Problems. IDOPT Project: It is
a joint project of CNRS, INRIA, UJF and INPG**
Journal of Nonparametric Statistics; 14, 87-112

Abstract: Two non-parametric tests for existence of change-points in a regression function
are introduced. The first test is based on the jump estimate of the regression function
at a known point t, whereas the second one, named local test in the text, is based on
the rescaled process of local variation in the neighborhood of t. This process is proved to
be asymptotically gaussian. We derive the asymptotic distributions for the test statistics
under the null hypothesis and show that their power under local alternatives tends to unity.
Numerical experiments are provided to give evidence for performances of these tests.

Gérard Grégoire and Zouhir Hamrouni(2002-10)
Change Point Estimation by Local Linear Smoothing
Journal of Multivariate Analysis, 83, 56-83

Abstract: We consider the problem of estimating jump points in smooth curves. Observations
(X_i, Y_i) $i = 1,,n$ from a random design regression function are given. We focus essentially on

the basic situation where a unique change point is present in the regression function. Based on local linear regression, a jump estimate process $t \to \gamma(t)$ is constructed. Our main result is the convergence to a compound Poisson process with drift, of a local dilated-rescaled version of $\gamma(t)$, under a positivity condition regarding the asymmetric kernel involved. This result enables us to prove that our estimate of the jump location converges with exact rate n^{-1} without any particular assumption regarding the bandwidth h_n. Other consequences such as asymptotic normality are investigated and some proposals are provided for an extension of this work to more general situations. Finally we present Monte-Carlo simulations which give evidence for good numerical performance of our procedure.

Estimating a changepoint, boundary, or frontier in the presence of observation error.

Abstract: A range of problems in economics and statistics involve calculation of the boundary, or frontier, of the support of a distribution. Several practical and attractive solutions exist if the sampled distribution has a sharp discontinuity at the frontier, but accuracy can be greatly diminished if the data are observed with error. Indeed, if the error is additive and has variance σ^2 then inaccuracies are usually of order σ, for small σ. In this article we suggest an elementary method for reducing the effect of error to $O(\sigma^2)$. and show that refinements can improve accuracy still further, to $O(\sigma^3)$ or less. The problem is inherently ill-posed, however, to such an extent that the frontier is generally not even identifiable unless the error distribution is known. The latter assumption is unreasonable in most practical settings, not in the least because the error is often asymmetrically distributed. For example, in the context of productivity analysis the error distribution tends to have a longer tail in the direction of underestimation of production. Nevertheless, even when the error distribution is unknown, it is often true that error variance is relatively low, and so methods for reducing systematic error in that case arc useful in practice.

Changepoint Analysis as a Method for Isotonic Inference

Abstract: Concavity and sigmoidicity hypotheses are developed as a natural extension of the simple ordered hypothesis in normal means. Those hypotheses give reasonable shape constraints for obtaining a smooth response curve in the non-parametric inputoutput analysis.

The slope change and inflection point models are introduced correspondingly as the corners of the polyhedral cones defined by those isotonic hypotheses. Then a maximal contrast type test is derived systematically as the likelihood ratio test for each of those changepoint hypotheses. The test is also justified for the original isotonic hypothesis by a complete class lemma. The component variables of the resulting test statistic have second or third order Markov property which, together with an appropriate nonlinear transformation, leads to an exact and very efficient algorithm for the probability calculation. Some considerations on the power of the test are given showing this to be a very promising way of approaching to the isotonic inference.

Hoeting, Jennifer A., Raftery, Adrian E. and Madigan, David (2002-13)
Bayesian variable and transformation selection in linear regression
Journal of Computational and Graphical Statistics, 11, 485-507
Keywords: Bayesian model averaging; Change-point transformation ; MARKOV Chain
Monte Carlo model composition; Model uncertainty; Posterior model probability

Abstract: This article suggests a method for variable and transformation selection based on posterior probabilities. Our approach allows for consideration of all possible combinations of untransformed and transformed predictors along with transformed and untransformed versions of the response. To transform the predictors in the model, we use a change-point model, or "change-point transformation," which can yield more interpretable models and transformations than the standard Box-Tidwell approach. We also address the problem of model uncertainty in the selection of models. By averaging over models, we account for the uncertainty inherent in inference based on a single model chosen from the set of ...

Huh, J. and Carrière, K. C.(2002-14)
Estimation of regression functions with a discontinuity in a derivative with local polynomial fits
Statistics and Probability Letters, 56, 329-343
Keywords: Change point; Nonparametric regression; Asymptotic minimax rate of
convergence; L^p convergence

Abstract: We consider an estimation strategy for regression functions which may have discontinuity/change point in the derivative functions at an unknown location. First, we propose methods of estimation for the location and the jump size of the change point via the local polynomial fitting based on a kernel weighted method. The estimated location of the change point will be shown to achieve the asymptotic minimax rate of convergence of $n^{-1/(2v+1)}$, where v is the degree of the derivative. Next, using the data sets split by the estimated location of the change point, we estimate their respective regression functions. Global L^p rate of convergence of the estimated regression function is derived. Computer simulation

will demonstrate the improved performance of the proposed methods over the existing ones.

Hušková, M. and J. Steinebach (2002-15)
Asymptotic tests for gradual changes
Statistics and Decisions 20, 137-151

Abstract: A class of tests for detecting gradual changes in a location model is introduced. Their limiting behavior, both under the null hypothesis of no change and under local alternative, is studied.

Jandhyala, V. K.; Fotopoulos, S. B. and Hawkins, D. M.(2002-16)
Detection and estimation of abrupt changes in the variability of a process
Computational Statistics and Data Analysis, 40, 1-19
Keywords: Change-point; Quality improvement; Likelihood ratio test; Estimation

Abstract: Detection of change-points in normal means is a well-studied problem. The parallel problem of detecting changes in variance has had less attention. The form of the generalized likelihood ratio test statistic has long been known, but its null distribution resisted exact analysis. In this paper, we formulate the change-point problem for a sequence of chi-square random variables. We describe a procedure that is exact for the distribution of the likelihood ratio statistic for all even degrees of freedom, and gives upper and lower bounds for odd (and also for non-integer) degrees of freedom. Both the liberal and conservative bounds for χ_1^2 degrees of freedom are shown through simulation to be reasonably tight. The important problem of testing for change in the normal variance of individual observations corresponds to the χ_1^2 case. The non-null case is also covered, and confidence intervals for the true change point are derived. The methodology is illustrated with an application to quality control in a deep level gold mine. Other applications include ambulatory monitoring of medical data and econometrics.

Kaushik Patra1 and Dipak K. Dey(2002-17)
A General Class of Change Point and Change Curve Modeling for Life Time Data
Annals of the Institute of Statistical Mathematics, 54, 517- 530
Keywords: Change point; Gibbs sampling; hazard function; posterior inference; survival function

Abstract: Change point hazard rate models arise in many life time data analysis, for example, in studying times until the undesirable side effects occur in clinical trials. In this paper we propose a general class of change point hazard model for survival data. This class includes

and extends different types of change point models for survival data, e.g. cure rate model and lag model. Most classical approach develops estimates of model parameters, with particular interest in change point parameter and often the whole hazard function, but exclusively in terms of asymptotic properties. We propose a Bayesian approach, avoiding asymptotics and provide inference conditional upon the observed data. The proposed Bayesian models are fitted using Markov chain Monte Carlo method. We illustrate our proposed methodology with an application to modeling life times of the printed circuit board.

Application of tube formula to distributional problems in multiway layouts.

Abstract: Recently, an integral geometric method called the tube method has been actively developed. The tube method gives us a powerful tool to tackle problems where conventional matrix theory such as the singular value decomposition cannot be applied. The aim of this paper is to survey several recent applications of the tube method to distributional problems in multiway layouts which are of practical importance but hardly handled by conventional methods. Null distributions of test statistics of the following three testing problems are discussed: (i) A test for interaction in three-way layout based on the three-way analogue of the largest singular value. (ii) Testing independence in ordered categorical data by maximizing row and column scores under order restriction. (iii) Detecting a change point in two-way layout with ordinal factors

Asymptotics of maximum likelihood estimator in a two-phase linear regression model

Abstract: This paper considers two-phase random design linear regression models with arbitrary error densities and where the regression function has a fixed jump at the true change-point. It obtains the consistency and the limiting distributions of maximum likelihood estimators of the underlying parameters in these models. The left end point of the maximizing interval with respect to the change point, herein called the maximum likelihood estimator $\widehat{\gamma}_n$ of the change-point parameter γ, is shown to be n-consistent and the underlying likelihood process, as a process in the standardized change-point parameter, is shown to converge weakly to a compound Poisson process. This process obtains maximum over a bounded interval and $n(\widehat{\gamma}_n - \gamma)$ converges weakly to the left end point of this interval. These results are

different from those available in the literature for the case of the two-phase linear regression models when jump sizes tend to zero as n tends to infinity.

Loschi, R. H. Cruz, F. R. B.(2002-20)
Applying the Product Partition Model to the Identification of Multiple Change Points.
Advances in Complex Systems; 5, 371-387
Keywords: Beta prior distribution; Relevance; Student-t distribution; Yao's cohesion.

Abstract: The multiple change point identification problem may be encountered in many subject areas, including disease mapping, medical diagnosis, industrial control, and finance. One appealing way of tackling the problem is through the product partition model (PPM), a Bayesian approach. Nowadays, practical applications of Bayesian methods have attracted attention perhaps because of the generalized use of powerful and inexpensive personal computers. A Gibbs sampling scheme, simple and easy to implement, is used to obtain the estimates. We apply the algorithm to the analysis of two important stock market data in Brazil. The results show that the method is efficient and effective in analyzing change point problems.

Loschi, R. H. and Cruz, F. R. B.(2002-21)
An analysis of the influence of some prior specifications in the identification of change points via product partition model
Computational Statistics and Data Analysis, 39, 477-501
Keywords: Gibbs sampling; Relevance; Student-t distribution; Yao's algorithm; Cohesions

Abstract: In this paper, we consider the product partition model for the estimation of normal means and variances of a sequence of observations that experiences changes in these parameters at unknown times. The estimates of the parameters by using product partition model are the weighted average of the estimates based in blocks (groups) of observations by the posterior relevance of these blocks which depends on the prior cohesions. We implement the Barry and Hartigan's method to this change point problem and propose an easy-to-implement modification to their method. We use Yao's prior cohesions and investigate the influence of different prior distributions to the parameter that indexes these cohesions in the product estimates. A comparison between the estimates obtained by using both these methods and those provided by using Yao's method is done considering different settings for its application. We apply the three methods presented in this paper to stock market data. The results seem to indicate that the method proposed is competitive and also that the prior specifications influence in the product estimates.

Orváth, Lajos and Kokoszka, Piotr (2002-22)

Change-point detection with non-parametric regression
Statistics, 36, 9-31
Keywords: Nonparametric change-point tests; Polynomial smoothing

Abstract: Focuses on change-point detection with non-parametric regression. Derivation of the Darling-Erdos type limit theorems for the test statistics under the null hypothesis of no change; Consistency of the related change-point estimators.

Polansky, Alan M. and Check, Catherine E. (2002-23)
Testing trends in environmental compliance
Journal of Agricultural, Biological, and Environmental Statistics, 7, 452-468
Keywords: Change point; Ozone levels; Randomization test; Two-phase regression

Abstract: Many environmental regulations are based on threshold levels. Such a level is generally set such that exposure to a toxin above the level is considered a hazard to health. In many instances, the level may be exceeded a certain number of times before regulatory action is taken. Such is the case with the National Ambient Air Quality Standards (NAAQS) set by the Environmental Projection Agency (EPA) in accordance with 1990 amendment of Clean Air Act of 1970. The set of standards sets threshold levels for several pollutants including carbon monoxide, nitrogen dioxide, ground-level ozone, and lead. Many areas of the United States, which the EPA designates as nonattainment areas, persistently exceed the air pollution levels set by the NAAQS. Progress in these areas is often slow but can be studied through data collected on the elapsed time between violations. The purpose of this article is to present a relatively easy but flexible statistical method for assessing whether an area is making progress with compliance o the NAAQS. The method presented in this article is based on a randomization test of a general model for the mean time between violation of the standard. The modeling-type approach of this method makes it flexible enough to study many types of trends. The use of the randomization test makes the method simple to apply and requires fewer assumptions than many other proposed methods. We discuss some general theoretical conditions for which the method is well suited and study the performance of this method empirically. Finally, we apply the method to a well-known set of interviolation times corresponding to the data collected on ground-level ozone in Houston, Texas.

Pons, Odile (2002-24)
Estimation in a Cox regression model with a change-point at an unknown time
Statistics, 36, 101-124
Keywords: Asymptotic distribution; Censored data; Cox's regression; Maximum likelihood estimation; Point process

Abstract: This paper concerns the asymptotic properties of the maximum likelihood estimators of the parameters in a non regular Cox model involving a change-point in the regression

on time-dependent covariates. The global consistency derives from the uniform convergence of the partial log-likelihood. We prove that the estimator of the change-point is n -consistent and the estimator of the regression parameter $n_{1/2}$ -consistent, and their asymptotic distributions are established.

Ramanayake, Asoka Gupta, Arjun K.(2002-25)
Change points with linear trend followed by abrupt change for the exponential distribution.
Journal of Statistical Computation and Simulation; 72, 263-278 Keywords: Changepoint; Likelihood ratio statistics; Consistency; Asymptotic distribution.

Abstract: Discusses the testing procedures in detecting random sequence of exponentially distributed random variables. Statistics of distribution theories; Application of the tests to Stanford heart transplant data and airport inter arrival data; Change in linear trends at an unknown period of time.

Rotondi, R.(2002-26)
On the influence of the proposal distributions on a reversible jump MCMC algorithm applied to the detection of multiple change-points
Computational Statistics and Data Analysis, 40, 633-653
Keywords: Acceptance rate; Bayesian inference; Hierarchical Bayesian model; Levels of seismicity; Poisson process; Random proposal; Reversible jump Markov chain Monte Carlo

Abstract: In this paper we address some issues arising in the implementation of Markov chain Monte Carlo methods; in particular we analyse whether the choice of transition kernels depending on a specific problem speeds up the convergence of a MetropolisHastings-type algorithm. This approach is applied to the retrospective detection of multiple structural changes in the physical process generating earthquakes. As the number of changes is unknown, the adopted hierarchical Bayesian model has variable-dimension parameters. The sensitivity of the method and issues related to the estimation of both the parameters and the posterior model distributions are also dealt with.

Rukhin, Andrew L.(2002-27)
Asymptotic behavior of posterior distribution of the change-point parameter
Journal of Statistical Planning and Inference, 105, 327-345
Keywords: Adaptation region; Bayes factor; Change-point analysis; Geometric distribution; Maximum likelihood; Mixture parameter; Prior distribution

Abstract: In the asymptotic setting of change-point estimation the behavior of the posterior distribution and of Bayes procedures is studied. The limiting distribution is derived when

the prior probabilities converge to geometric probabilities. This distribution is related to the infinite product of random matrices (or affine transforms of the real line). The situation with partially unknown pre- and after-change distributions is investigated. A condition for the limiting law of posterior probabilities to coincide with that for the known pre- and after-change distributions is derived.

Shao, Quanxi and Campbell, N. A. (2002-28)
Modelling trends in groundwater levels by segmented regression with constraints
Australian and New Zealand Journal of Statistics, 44, 129-141
Keywords: Akaike information criterion; Change point; Model selection

Abstract: This paper provides a statistically unified method for modelling trends in groundwater levels for a national project that aims to predict areas at risk from salinity in 2020. It was necessary to characterize the trends in groundwater levels in thousands of boreholes that have been monitored by Agriculture Western Australia throughout the south-west of Western Australia over the last 10 years. The approach investigated in the present paper uses segmented regression with constraints when the number of change points is unknown. For each segment defined by change points, the trend can be described by a linear trend possibly superimposed on a periodic response. Four different types of change point are defined by constraints on the model parameters to cope with different patterns of change in groundwater levels. For a set of candidate change points provided by the user, a modified Akaike information criterion is used for model selection. Model parameters can be estimated by multiple linear regression. Some typical examples are presented to demonstrate the performance of the approach.

Sofronov, G. Yu(2002-29)
Asymptotically d-optimal test of a change-point detection
Theory of Probability and Its Applications; 46 Issue 3, 547-548
Keywords: Change-point detection; D-A posterior approach; D-warranty; Weak convergence; Functionals of the Wiener process.

Abstract: The paper considers the problem of a change-point detection for a sequence of random variables. We construct a d-optimal test guaranteeing the d-risk. The asymptotic of this test is obtained.

Staudenmayer, John and Spiegelman, Donna (2002-30)
Segmented regression in the presence of covariate measurement error in main study/validation study designs
Biometrics, 58, 871-877

Abstract: This article considers the problem of segmented regression in the presence of covariate measurement error in main study/validation study designs. First, we derive a closed and interpretable form for the full likelihood. After that, we use the likelihood results to compute the bias of the estimated changepoint in the case when the measurement error is ignored. We find the direction of the bias in the estimated changepoint to be determined by the design distribution of the observed covariates, and the bias can be in either direction. We apply the methodology to data from a nutritional study that investigates the relation between dietary folate and blood serum homocysteine levels and find that the analysis that ignores covariate measurement error would have indicated a much higher minimum daily dietary folate intake requirement than is obtained in the analysis that takes covariate measurement error into account.

Wang, Li Ming and Wang, Jing Long(2002-31)
Change-point tests of location parameter and their approximate distributions.
Chinese Ann. Math. Ser. A 23 , no. 2, 229-234.

Abstract: In this paper, the change-point problem for the location parameter is discussed based on two-sample U-statistics. The test statistic is proposed, and its approximate distribution is obtained. The approximate distribution of the test statistic is that of $\sup |B(t)|$, where $(B(t), 0 < t < 1)$ is a Brownian bridge. The authors also discuss testing problems for change-point about the scale parameter in two-parameter exponential distributions and Weibull distributions. (Reviewed by K. F. Yu)

Xu, Ronghui and Adak, Sudeshna (2002-32)
Survival analysis with time-varying regression effects using a tree-based approach
Biometrics, 58, 305-315

Abstract: Nonproportional hazards often arise in survival analysis, as is evident in the data from the International Non-Hodgkin's Lymphoma Prognostic Factors Project. A tree-based method to handle such survival data is developed for the assessment and estimation of time-dependent regression effects under a Cox-type model. The tree method approximates the time-varying regression effects as piecewise constants and is designed to estimate change points in the regression parameters. A fast algorithm that relies on maximized score statistics is used in recursive segmentation of the time axis. Following the segmentation, a pruning algorithm with optimal properties similar to those of classification and regression trees

(CART) is used to determine a sparse segmentation. Bootstrap resampling is used in correcting for overoptimism due to split point optimization. The piecewise constant model is often more suitable for clinical interpretation of the regression parameters than the more flexible spline models. The utility of the algorithm is shown on the lymphoma data, where we further develop the published International Risk Index into a time-varying risk index for non-Hodgkin's lymphoma.

Agulló, José(2001-1)
New algorithms for computing the least trimmed squares regression estimator
Computational Statistics and Data Analysis, 36, 425-439
Keywords: Least trimmed squares; Multiple linear regression; Outliers; High breakdown point; Branch and bound; Exchange algorithm

Abstract: The outlier detection in multiple linear regression is a difficult problem because of the masking effect. A procedure that works successfully uses residuals based on a high breakdown estimator. The least trimmed squares (LTS) estimator, which was proposed by Rousseeuw (J. Amer. Statist. Assoc. 79 (1984)), is a high breakdown estimator. In this paper we propose two algorithms to compute the LTS estimator. The first algorithm is probabilistic and is based on an exchange procedure. The second algorithm is exact and based on a branch-and-bound technique that guarantees global optimality without exhaustive evaluation. We discuss the implementation of these algorithms using orthogonal decomposition procedures and propose several accelerations. The application of the new algorithms to real and simulated data sets shows that they significantly reduce the computational cost with respect to the algorithms previously described in the literature.

Antoch, J. and Hušková, M.(2001-2)
Permutation tests in change point analysis
Statistics and Probability Letters, 53, 37-46
Keywords: Change(s) in location model; Permutation tests; Monte Carlo

Abstract: The critical values for various tests for changes in location model are obtained through the use of permutation tests principle. Theoretical results show that in the limit these new "permutation tests" behave in the same way as the "classical tests" stemming from both maximum likelihood and Bayes principles. However, the results of the simulation study show that the permutation tests behave considerably better than the corresponding classical tests if measured by the critical values attained

Asgharian, Masoud and Wolfson, David B.(2001-3)
Covariates in Multipath Change-Point Problems: Modelling and Consistency of the MLE

Abstract: Although the single-path change-point problem has been extensively treated in the statistical literature, its multipath counterpart has largely been ignored. In the multipath change-point setting, it is often of interest to assess the impact of covariates on the change point itself as well as on the parameters before and after the change point. This paper is concerned only with the inclusion of covariates in the change-point distribution. This is achieved through the hazard of change. Maximum likelihood estimation is discussed and consistency of the maximum likelihood estimators established.

Avery, Peter J.(2001-4)
The effect of dependence in a binary sequence on tests for a changepoint or a changed segment

Abstract: The effect of partial dependence in a binary sequence on tests for the presence of a changepoint or changed segment are investigated and exemplified in the context of modelling non-coding deoxyribonucleic acid (DNA). For the levels of dependence that are commonly seen in such DNA, the null distributions of the test statistics are approximately correct and so conclusions based on them are still valid. A strong dependence would, however, invalidate the use of such procedures

Boudjellaba, H.; MacGibbon, B. and Sawyer, P.(2001-5)
On exact inference for change in a poisson sequence.

Abstract: Since Hinkley's original work on exact inference for a change in a sequence of random variables, many authors have proposed different methods based either on exact distributions or asymptotic approximations to test for and estimate a change point in such a sequence. Here we concentrate on the Poisson case. We propose estimators, tests and confidence intervals obtained by adapting and modifying Hinkley's and Worsley's work and we do an extensive study of the small sample properties of these proposed methods of inference, also comparing them to methods based on asymptotic approximations. The methods are applied to a neural train data set.

Chang, Yi-Ping (2001-6)
Estimation of parameters for nonhomogeneous Poisson process: Software reliability with change-point model

55

Abstract: Nonhomogeneous Poisson process (NHPP) models, frequently employed in reliability engineering, are used to estimate the number of software errors remaining in a software system. Traditionally, the error detection rate of NHPP models is usually assumed to be a continuous and monotonic function. The error detection rate may, however, not be smooth and can change if the testing environment, strategy or resource allocation is changed. This paper describes NHPP with change-point software reliability models. Due to the irregularity imbedded in the model, the classical maximum likelihood method and the conditional maximum likelihood method cannot be used with interfailure data. The parameters of the NHPP with change-point model can however be estimated by the least squares method. According to the results of a simulation study and analysis of real data, the least squares estimates shown to perform well.

Chen, Jie and Gupta, A.(2001-7)
On change point detection and estimation.
Communications in Statistics: Simulation and Computation; 30 665-697
Keywords: Change-point; Bayes solution; Likelihood-ratio procedure; Information
ccriterian; Model selection; CUSUM; Wavelet transformation.

Abstract: Provides a survey of the change point detection and estimation. Origin of the change point problem; Determination of the presence of a statistically significant change point in a sequence of chronologically ordered data; Likelihood-ratio procedure; Informational approach; Bayes solution.

Chong, Terence Tai-Leung(2001-8)
Estimating the locations and number of change points by the sample-splitting method .
Statistical Papers, 42, p53-79
Keywords: Multiple structural change; Change-point estimator; Brownian motion; Bessel
process.

Abstract: Provides a sample-splitting procedure to deal with the problem of multiple changes without an enormous computational cost. Definition of the sample-splitting method; Analysis of the multiple structural-change model; Assessment of situations in which the sample splitting method may fail

Davydov, Yu.; Paulauskas, V. and A. Rakauskas, A.(2001-9)
More on P-Stable Convex Sets in Banach Spaces

Abstract: We study the asymptotic behavior and limit distributions for sums $S_n = b_n^{-1} \sum_{i=1}^n \xi_i$, where $\xi_i, i \geq 1$, are *i.i.d.* random convex compact (cc) sets in a given separable Banach space B and summation is defined in a sense of Minkowski. The following results are obtained: (i) Series (LePage type) and Poisson integral representations of random stable cc sets in B are established; (ii) The invariance principle for processes $S_n(t) = b_n^{-1} \sum_{i=1}^{[nt]} \xi_i$, $t \in [0,1]$, and the existence of p-stable cc Levy motion are proved; (iii) In the case, where ξ_i are segments, the limit of S_n is proved to be countable zonotope. Furthermore, if $B = R^d$, the singularity of distributions of two countable zonotopes Y_{p1}, σ_1, Y_{p2}, σ_2, corresponding to values of exponents p_1, p_2 and spectral measures σ_1, σ_2, is proved if either $p_1 \neq p_2$ or $\sigma_1 \neq \sigma_2$; (iv) Some new simple estimates of parameters of stable laws in R^d, based on these results are suggested.

Dietmar Ferger(2001-10)
Analysis of change-point estimators under the null hypothesis

Abstract: We consider estimators for the change-point in a sequence of independent observations. These are defined as the maximizing points of weighted U-statistic type processes. Our investigations focus on the behaviour of the estimators in the case of independent and identically distributed random variables (null hypothesis of no change), but contiguous alternatives in the sense of Oosterhoff and van Zwet are also taken into account. If the weight functions belong to the Chibisov-O'Reilly class we derive convergence in distribution, including a special Berry-Esseen result. The limit variable is the almost sure unique maximizing point of a weighted (standard or reflected) Brownian bridge with drift. For general weight functions the limiting null distribution is analytically not known. However, in the special case where no weight functions are involved it is known that the maximizer of a standard Brownian bridge is uniformly distributed on the unit interval. A corresponding result for the reflected Brownian bridge seems to be unknown in the literature. In this paper we fill this gap and actually compute the common density of the maximum and its location for a reflected Brownian bridge. From this one can find the density of the maximizer, which analytically can be expressed in terms of a series. In a special case even the finite sample size distribution of our estimator is established. Besides distributional results, we also determine the almost sure set of cluster points.

Ebrahimi, Nader and Ghosh, Sujit K(2001-11)

Bayesian and frequentist methods in change-point problems

Abstract: The change-point problem is one of the important problems of statistical inference in which one tries to detect abrupt change in a given sequence of random variables. This problem, which originally started with statistical control theory (see Page, 1955), has now been applied to different fields, including but not restricted to survival analysis and reliability studies. The literature about change-point problem, by now is quite extensive. In this paper, our goal is to review recent developments in this area. In particular, statistical procedures to estimate discrete change-point as well as continuous change-point are reviewed.

Exponential and polynomial tailbounds for change-point estimators

Abstract: Let $X_{1n}, \cdots, X_{nn}$ be independent random elements with an unknown change point $\theta \in (0,1)$, that is X_{in} has a distribution υ_1 or υ_2, respectively, according to $i \leq [n\theta]$ or $i > [n\theta]$. We propose an estimator θ_n of θ, which is defined as the maximizer of a weighted empirical process on (0,1). Finding upper bounds of polynomial and exponential type for the tails of $n\theta_n - [n\theta]$, we are able to derive rates of almost sure convergence, of distributional convergence, of L_p-convergence and of convergence in the Ky-Fan- and in the Prokhorov-metric

A note on change point estimation in doseresponse trials

Abstract: An often used model for the shape of a drug's doseresponse relationship is the following: the curve increases to a certain dose until a plateau is reached. In this situation, the minimum dose with maximum effect can be seen as a change point in a regression model. We present four estimators for the change point and compare their performance with respect to selection rates and loss function based criteria by extensive simulations. The choice of the loss function depending on the practical situation is discussed. Application of the estimators is illustrated by an example.

Giraitis, Liudas; Kokoszka, Piotr and Leipus Remigijus(2001-14)
Testing for Long Memory in the Presence of a General Trend
Journal of Applied Probability, 38, 1033-1054.
Keywords: Long memory; Trend; Change point.

Abstract: The paper studies the impact of a broadly understood trend, which includes a change point in mean and monotonic trends studied by Bhattacharya et al. (1983, J. of applied probability,20,649-662), on the asymptotic behaviour of a class of tests designed to detect long memory in a stationary sequence. Our results pertain to a family of tests which are similar to Lo's (1991, Econometrica,59,1279-1313) modified R/S test. We show that both long memory and nonstationarity (presence of trend or change points) can lead to rejection of the null hypothesis of short memory, so that further testing is needed to discriminate between long memory and some forms of nonstationarity. We provide quantitative description of trends which do or do not fool the R/S-type long memory tests. We show, in particular, that a shift in mean of a magnitude larger than $N^{-1/2}$, where N is the sample size, affects the asymptotic size of the tests, whereas smaller shifts do not do so.

Gombay, Edit(2001-15)
U-Statistics for Change under Alternatives
Journal of Multivariate Analysis, 78, 139-158
Keywords: U-statistic; Degenerate and nondegenerate kernels; Change-point estimator; Projection; Law of iterated logarithm

Abstract:Asymptotic distributions of U-statistics to test for possible changes in the distribution will be derived when the change occurred. We will show that for all possible types of kernels, symmetric, antisymmetric, degenerate, non-degenerate, the test statistics are asymptotically normally distributed. We also study the distribution of the estimator of the time of change. Its large sample behavior is approximately that of the maximum of a two-sided random walk. The terms in these random walks explain the exact nature of bias in the change-point estimator. Several examples will be given as illustration.

Gössl, Christoff and Küchenhoff, Helmut(2001-16)
Bayesian analysis of logistic regression with an unknown change point and covariate measurement error
Statistics in Medicine, 20, 3109-3121

Abstract: We discuss Bayesian estimation of a logistic regression model with an unknown threshold limiting value (TLV). In these models it is assumed that there is no effect of a covariate on the response under a certain unknown TLV. The estimation of these models in a Bayesian context by Markov chain Monte Carlo (MCMC) methods is considered with focus on the TLV. We extend the model by accounting for measurement error in the covariate.

The Bayesian solution is compared with the likelihood solution proposed by Küchenhoff and Carroll using a data set concerning the relationship between dust concentration in the working place and the occurrence of chronic bronchitis.

Grossi, L.; Zurlini, G. and Rossi, O. (2001-17)
Statistical detection of multiscale landscape patterns
Environmental and Ecological Statistics, 8, 253-267
Keywords: Boundary fractal dimension; CORINE biotopes; Piecewise regression

Abstract: Detection of discontinuities in landscape patterns is a crucial problem both in ecology and in environmental sciences since they may indicate substantial scale changes in generating and maintaining processes of landscape patches. This paper presents a statistical procedure for detecting distinct scales of pattern for irregular patch mosaics using fractal analysis. The method suggested is based on a piecewise regression model given by fitting different regression lines to different ranges of patches ordered according to patch size (area). Proper shift-points, where discontinuities occur, are then identified by means of an iterative procedure. Further statistical tests are applied in order to verify the statistical significance of the best models selected. Compared to the method proposed by Krummel et al. (1987, Oikos,48,321-324), the procedure described here is not influenced by subjective choices of initial parameters. The procedure was applied to landscape pattern analysis of irregular patch mosaics (CORINE biotopes) of a watershed within the Map of the Italian Nature Project. Results for three different CORINE patch types are herein presented revealing different scaling properties with special pattern organizations linked to ecological traits of vegetation communities and human disturbance.

Gupta, Arjun K. and Ramanayake, Asoka(2001-18)
Change points with linear trend for the exponential distribution
Journal of Statistical Planning and Inference, 93, 181-195
Keywords: Change points; Likelihood ratio statistics; Estimation; Consistency; Asymptotic distribution

Abstract: A sequence of independent exponential random variables is hypothesized to have equal means, and we would like to test whether the means have been subjected to a linear trend change at an unknown point, for an unknown duration in the sequence. Two likelihood ratio-type statistics and Rao's efficient score statistics are derived. The distribution theory and related properties of the test statistics are discussed. Numerical critical points and powers of the test statistics are tabulated for certain selected values of the parameters. The tests are then applied to find linear trend changes in the set of Stanford heart transplant data and air traffic arrival data.

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Hawkins, Douglas M. (2001-19)
Fitting multiple change-point models to data
Computational Statistics and Data Analysis, 37, 323-341
Keywords: Segmented regressions; Quality improvement; Regression trees; Time series

Abstract Change-point problems arise when different subsequences of a data series follow different statistical distributions commonly of the same functional form but having different parameters. This paper develops an exact approach for finding maximum likelihood estimates of the change points and within-segment parameters when the functional form is within the general exponential family. The algorithm, a dynamic program, has execution time only linear in the number of segments and quadratic in the number of potential change points. The details are worked out for the normal, gamma, Poisson and binomial distributions.

Holmes, C. C. and Mallick, B. K. (2001-20)
Bayesian regression with multivariate linear splines
Journal of the Royal Statistical Society, Series B: Statistical Methodology, 63, 3-17
Keywords: Bayesian model averaging; Bayesian piecewise linear regression; Local linear regression; Multivariate splines; Non-linear regression

Abstract: We present a Bayesian analysis of a piecewise linear model constructed by using basis functions which generalizes the univariate linear spline to higher dimensions. Prior distributions are adopted on both the number and the locations of the splines, which leads to a model averaging approach to prediction with predictive distributions that take into account model uncertainty. Conditioning on the data produces a Bayes local linear model with distributions on both predictions and local linear parameters. The method is spatially adaptive and covariate selection is achieved by using splines of lower dimension than the data.

Hsu, Chih-Chiang(2001-21)
Change point estimation in regressions with I(d) variables
Economics Letters, 70, 147-155
Keywords: Change point estimation; Fractional Brownian motion; Spurious change

Abstract: In this paper we study the least-squares change-point estimator in regressions with stationary and invertible I(d) regressors and disturbances. We find that the least-squares estimator remains consistent when there is a one-time break, but it may identify a spurious change when there is none

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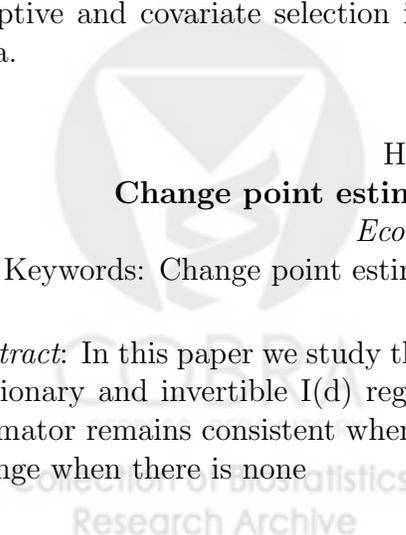
Hsu, Chih-Chiang and Kuan, Chung-Ming(2001-22)
Distinguishing between trend-break models: method and empirical evidence.

Abstract: We demonstrate that in time trend models, the likelihood-based tests of partial parameter stability have size distortions and cannot be applied to detect the changing parameter. A two-step procedure is then proposed to distinguish between different trend-break models. This procedure involves consistent estimation of break dates and properly-sized tests for changing coefficient. In the empirical study of the Nelson-Plosser data set, we find that the estimated change points and trend-break specifications resulting from the proposed procedure are quite different from those of Perron ((1989, Econometrica,57,1361-1401) (1997, J. of Econometrics,80,355-385)), Chu and White (1992, J. of Business and Econometric Statist. 10,289-299), and Zivot and Andrews (1992, J. of Business and Econometric Statist., 10,251-270). In another application, our procedure provides formal support for the conclusion of Ben-David and Papell (1995, Econometrics Letters,49,351-357) that real per capita GDPs of most OECD countries exhibit a slope change in trend.

Hušková, M.(2001-23)
A note on estimators of gradual changes

Abstract: This paper concerns the location model with a change after an unknown time point $m_n < n$, defined as follows (1) $Y_i = \ + \delta_n((i - m_n)/n)_+^{\alpha_0} + e_i$, $i = 1, \ldots, n$, where $a_+ = \max(0, a)$, , $\delta_n \neq 0$, m_n and $\alpha_0 \in [0, 1]$ are unknown parameters; it is assumed that (2) $e_1, \ldots, e_n$ are i.i.d. random variables with $Ee_1 = 0$, $0 < \sigma^2 = Ee_1^2 < \infty$, $E|e_1|^{2+\Delta} < \infty$ for some $\Delta > 0$, and $m_n = [\gamma n]$ for some $\gamma \in (0, 1)$, where $[]$ is the integer part. Concerning the slope parameter δ_n, it is assumed that, as $n \to \infty$

$$\delta_n \to 0, \sqrt{n}|\delta_n|\sqrt{\log \log n} \to \infty, \tag{3}$$

which covers local alternatives ($\delta_n \to \infty$), and, if $\alpha_0 \neq 0$, fixed alternatives as well ($\delta_n = \delta \neq 0$). Least squares type estimators $\hat{\mu}_n$, $\hat{\delta}_n$, $\hat{\alpha}_n$, $\hat{m}_n$ are defined and studied for the unknown parameters , δ_n, α_0, m_n. It is shown that the asymptotic behaviour (both the rate of consistency and the limit distribution) of the estimators of the change-point m_n in these models, as summarized by (1)–(3) above, depends on the parameter α_0, which characterizes the type of change (abrupt if $\alpha_0 = 0$, or gradual if $\alpha_0 > 0$). In particular, the limiting behaviour of the estimators $\hat{m}_n$ and $\hat{\alpha}_n$ is studied and compared to the asymptotic behaviour of the corresponding estimators when only one of the parameters m_n and α_0 is assumed to be unknown. (Reviewed by M. Csrg)

Hušková, M. and Antoch, J.(2001-24)
M-estimators of structural changes in regression models
Tatra Mountains Mathematical Publications, 22, 197-208
Keywords: Linear regression; Structural changes; Change point problem; $L_1 - L_2$
-estimators; M-estimators; Huber estimator; Parametric inference–point estimation;
Functional limit theorems; Asymptotic distribution theory; Parametric inference;
Asymptotic properties of estimators

Abstract: A class of M-estimators of change point(s) in a linear regression model is introduced and their properties are studied. Selected conclusions of a simulation study are presented.

Irina, Grabovsky and Lajos, Horváth(2001-25)
Change-Point Detection in Angular Data
Annals of the Institute of Statistical Mathematics 53, 552 - 566
Keywords: Angular data; Change-point; Pulsar; Brownian bridge; Von Mises distribution

Abstract: We suggest a modification of the CUSUM procedure to detect changes in angular data. We obtain limit theorems for the test statistics under the no change null hypothesis. We discuss the estimation of the times of changes and show that the binary segmentation provides the times of all changes. Our method is applied to a data set on the activity of a pulsar.

Jandhyala, V. K. and Fotopoulos, S. B. (2001-26)
Rate of convergence of the maximum likelihood estimate of a change-point
SankhyÄ, Series A, 63, 277-285
Keywords: Maximum of a random walk; Negative drift; Maximum likelihood estimate;
Parameter change

Abstract: Maximum likelihood method is applied to estimate the change-point of a parameter associated with a sequence of independent random variables. Exact expressions are derived for the limiting distribution of the maximum likelihood estimator of a change-point and computable bounds are suggested. Exponential rate of convergence is obtained from finite sample to the case of infinite sample.

Jarušková, D.(2001-27)
Change-point estimator in continuous quadratic regression
Commentationes Mathematicae Universitatis Carolinae, 42, 741-752
Keywords: Nonlinear regression; Limit distribution

Abstract: The paper deals with the asymptotic distribution of the least squares estimator of a change point in a regression model where the regression function has two phases — the first linear and the second quadratic. In the case when the linear coefficient after change is non-zero the limit distribution of the change point estimator is normal whereas it is non-normal if the linear coefficient is zero.

Jarušková, D.(2001-28)
Change-point estimator in continuous quadratic regression
Commentationes Mathematicae Universitatis Carolinae, 42, 741-752
Keywords: Change point estimator; Nonlinear regression; Limit distribution

Abstract: The paper deals with the asymptotic distribution of the least squares estimator of a change point in a regression model where the regression function has two phases, the first linear and the second quadratic. In the case when the linear coefficient after change is non-zero the limit distribution of the change point estimator is normal whereas it is non-normal if the linear coefficient is zero.

Jose, C. T. and Ismail, B. (2001-29)
Nonparametric inference on jump regression surface
Journal of Nonparametric Statistics, 13, 791-813
Keywords: Change point; Discontinuity; Kernel estimator; Local polynomial regression; Nonparametric regression

Abstract: Estimators for jump location curve and jump size function of a two-dimensional jump regression function (jump regression surface) are proposed. The estimators are obtained by fitting kernel weighted least squares regression based on the observations in the four quadrants of a neighborhood of a given point. The proposed procedure can be used in the case of jump in the regression surface and/or in its slope (jump in the partial derivatives). The limiting distributions and the asymptotic properties of the estimators are investigated. The procedure is illustrated through a simulation study.

Julious, Steven A. (2001-30)
Inference and estimation in a changepoint regression problem
Journal of the Royal Statistical Society, Series D: The Statistician, 50, 51-61
Keywords: Bootstrapping; Least squares estimates; Piecewise regression; Split lines; Two-phase regression

Abstract: The two-line model when the location of the changepoint is known is introduced, with an F-test to detect a change in the regression coefficient. The situation when the change-point is unknown is then introduced and an algorithm proposed for parameter estimation.

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It is demonstrated that when the location of the changepoint is not known the F-test does not conform to its expected parametric distribution. Nonparametric bootstrap methods are proposed as a way of overcoming the problems encountered. Finally, a physiology example is introduced where the regression change represents the change from aerobic to anaerobic energy production.

Kruglov, V.M. (2001-31)
The asymptotic behavior of the Pearson statistic.
Theory of Probability and its Applications, 45, 69-92
Keywords: Chi-square statistic; Random broken lines; Polynomial distribution; Functional limit theorems; Asymptotic distribution theory; Inference from stochastic processes; Stochastic processes

Abstract: Some limit theorems are proved for some functionals of the Pearson statistic constructed from the polynomial distribution with parameters n and p_k, $k = 1, 2, \ldots, s = s(n)$, under the assumption that $\inf_n\{n \min_{1 \le k \le s} p_k\} > 0$, $s \to \infty$, $n \min\{p_k : k \in W_n\} \longrightarrow \infty$, $N_n/s \to 1$ as $n \to \infty$, where N_n is the number of elements in the set $W_n \subset \{1, 2, \ldots, s\}$. In particular, multivariate and functional limit theorems are proved for this statistic. As a whole, the statements proved in this paper demonstrate that the Pearson statistic in many respects behaves as an asymptotically normal sum of independent random variables.

Kühn, Christoph(2001-32)
An estimator of the number of change points based on a weak invariance principle
Statistics and Probability Letters, 51, 189-196
Keywords: Change points; Change in the mean; Schwarz criterion; Invariance principle; Partial sum; Renewal process; Linear process

Abstract: We study an estimator of the number of change points in the drift of a stochastic process based on the Schwarz criterion. In a general statistical model where the additive measurement noise satisfies a certain weak invariance principle (examples included are partial sums, renewal processes, and linear processes in time series analysis) consistency can be shown under the condition that the number of jumps is not greater than a given upper bound.

Pastor, Roberto and Guallar, Eliseo (2001-33)
RE: "Use of two-segmented logistic regression to estimate change-points in epidemiologic studies"
American Journal of Epidemiology, 153, 615-615
Keywords: Case-control studies; Epidemiologic methods; Logistic models; Risk assessment

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Abstract : In many epidemiologic data, the dose-response relation between a continuous exposure and the risk of disease abruptly changes when the exposure variable reaches an unknown threshold level, the so-called change-point. Although several methods are available for dose-response assessment with dichotomous outcomes, none of them provide inferential procedures to estimate change-points. In this paper, we describe a two-segmented logistic regression model, in which the linear term associated with a continuous exposure in standard logistic regression is replaced by a two-segmented polynomial function with unknown change-point, which is also estimated. A modified, iteratively reweighted least squares algorithm is presented to obtain parameter estimates and confidence intervals, and the performance of this model is explored through simulation. Finally, a two-segmented logistic regression model is applied to a case-control study of the association of alcohol intake with the risk of myocardial infarction and compared with alternative analyses. The ability of two-segmented logistic regression to estimate and provide inferences for the location of change-points and for the magnitude of other parameters of effect will make this model a useful complement to other methods of dose-response analysis in epidemiologic studies.

Screening Based on the Risk of Cancer Calculation From Bayesian Hierarchical Changepoint and Mixture Models of Longitudinal Markers

Abstract: The standard approach to early detection of disease with a quantitative marker is to set a population-based fixed reference level for making further individual screening or referral decisions. For many types of disease, including prostate and ovarian cancer, additional information is contained in the subject-specific temporal behavior of the marker, which exhibits a characteristic alteration early in the course of the disease. In this article we derive a Bayesian approach to screening based on calculation of the posterior probability of disease given longitudinal marker levels. The method is motivated by a randomized ovarian cancer screening trial in the United Kingdom comprising 22,000 women screened over 4 years with an additional 5 years of follow-up on average. Levels of the antigen CA125 were recorded annually in the screened arm. CA125 profiles of cases and controls from the U.K. trial are modeled using hierarchical changepoint and mixture models, posterior distributions are calculated using Markov chain Monte Carlo methods, and the model is used to calculate the Bayesian posterior risk of having ovarian cancer given a new subject's single or multiple longitudinal CA125 levels. A screening strategy based on the risk calculation is then evaluated using data from an independent screening trial of 5,550 women performed in Sweden. A longitudinal CA125 screening strategy based on calculation of the risk of ovarian cancer is proposed. Simulations of a prospective trial using a strategy based on the risk calculated from longitudinal CA125 values indicate potentially large increases in sensitivity

for a given specificity compared to the standard approach based on a fixed CA125 reference level for all subjects.

Stergios Fotopoulos, and Venkata Jandhyala(2001-35)
Maximum likelihood estimation of a change-point for exponentially distributed random variables

Abstract: We consider the problem of estimating the unknown change-point in the parameter of a sequence of independent and exponentially distributed random variables. An exact expression for the asymptotic distribution of the maximum likelihood estimate of the change-point is derived. The analysis is based on the application of WeinerHopf factorization identity involving the distribution of ascending and descending ladder heights, and the renewal measure in random walks.

Tae Young Yang and Lynn Kuo(2001-36)
Bayesian binary segmentation procedure for a Poisson process with multiple changepoints

Abstract: We observe n events occurring in $(0, T]$ taken from a Poisson process. The intensity function of the process is assumed to be a step function with multiple changepoints. This article proposes a Bayesian binary segmentation procedure for locating the changepoints and the associated heights of the intensity function. We conduct a sequence of nested hypothesis tests using the Bayes factor or the BIC approximation to the Bayes factor. At each comparison in the binary segmentation steps, we need only to compare a single-changepoint model to a no-changepoint model. Therefore, this method circumvents the computational complexity we would normally face in problems with an unknown (large) number of dimensions. A simulation study and an analysis on a real dataset are given to illustrate our methods

Thum, Yeow Meng and Bhattacharya, Suman K. (2001-37)
Detecting a change in school performance: A Bayesian analysis for a multilevel join point problem

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Abstract: A substantial literature on switches in linear regression functions considers situations in which the regression function is discontinuous at an unknown value of the regressor, X_k, where k is the so-called unknown "change point." The regression model is thus a two-phase composite of $y_i \sim N(\beta_{01} + \beta_{11}x_i, \sigma_1^2)$, i = 1, 2, ..., k and $y_i \sim N(\beta_{02} + \beta_{12}x_i, \sigma_2^2), i = k+1, k+2, ..., n$. Solutions to this single series problem are considerably more complex when we consider a wrinkle frequently encountered in evaluation studies of system interventions, in that a system typically comprises multiple members (j = 1, 2, ..., m) and that members of the system cannot all be expected to change synchronously. For example, schools differ not only in whether a program, implemented system-wide, improves their students' test scores, but depending on the resources already in place, schools may also differ in when they start to show effects of the program. If ignored, heterogeneity among schools in when the program takes initial effect undermines any program evaluation that assumes that change points are known and that they are the same for all schools. To describe individual behavior within a system better, and using a sample of longitudinal test scores from a large urban school system, we consider hierarchical Bayes estimation of a multilevel linear regression model in which each individual regression slope of test score on time switches at some unknown point in time, k_j. We further explore additional results employing models that accommodate case weights and shorter time series.

Truong, Young K. and Patil, Prakash N. (2001-38)
Asymptotics for wavelet based estimates of piecewise smooth regression for stationary time series
Annals of the Institute of Statistical Mathematics, 53, 159-178
Keywords: Convergence rate; Density estimation; Nonparametric regression; Piecewise-smoothness; Wavelet

Abstract: Wavelet methods are used to estimate density and (auto-) regression functions that are possibly discontinuous. For stationary time series that satisfy appropriate mixing conditions, we derive mean integrated squared errors (MISEs) of wavelet-based estimators. In contrast to the case for kernel methods, the MISEs of wavelet-based estimators are not affected by the presence of discontinuities in the curves. Applications of this approach to problems of identification of nonlinear time series models are discussed.

Wu, Wei Biao, Woodroofe, Michael and Mentz, Graciela (2001-39)
Isotonic regression: Another look at the changepoint problem
Biometrika, 88, 793-804
Keywords: changepoint problem; Isotonic regression; Penalised likelihood function; Short range dependence

Abstract: A test based on isotonic regression is developed for monotonic trends in short range dependent sequences and is applied to Argentina rainfall data and global warming

data. This test provides another perspective for changepoint problems. The isotonic test is shown to be more powerful than some existing tests for trend.

Zhenmin Chen ; Dietmar Ferger and Jie Mi(2001-40)
Estimation of the change point of a distribution based on the number of failed test items
Metrika, 53, 31-38

Abstract: This paper discusses the family of life distributions with failure rate functions which decrease initially until a change point and remain constant thereafter. The paper focuses on the estimation for the change point of the failure rate function. While point estimation of the change point of the failure rate function has been discussed by some authors, one can hardly find any existing work on the interval estimation of the change point. In this paper, a method for constructing approximate confidence intervals for the change point is proposed. The proposed approximate confidence intervals are based on the number of failed test items at or before a fixed inspection time

Abd-Rabou, Abd-Elnaser S and Aly, Emad-Eldin A. A.(2000-1)
Weighted tests for a change in the regression slope
Statistics and Decisions, 18, 79-97

Abstract: We consider the problem of testing for a change in the slope of a simple linear regression model. We generalize the least squares (LS) approach of P. K. Sen[Z. Wahrsch. Verw. Gebiete 52 (1980), no. 2, 203-218] by proving weighted approximations of certain LS change point processes. These results are then used to develop Anderson-Darling and Erdös-Darling type test statistics. The limiting distributions of these two statistics are derived."

Abd-Rabou, Abd-Elnaser S. and Aly, Emad-Eldin A. A.(2000-2)
A comparison of weighted tests for a slope change in simple regression
Biometrical Journal, 42, 1033-1038

Abstract: Testing for a change in the slope of the simple linear regression model has many applications in bio-sciences, quality control and survival analysis. This paper compares

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Anderson-Darling and Erdös-Darling type test statistics which are based on the least squares change point process of Sen (1980) with the corresponding Kolmogorov-Smirnov and Crámer-von Mises type test statistics. We estimated the limiting critical values of these test statistics and conducted Monte Carlo simulation studies to compare their powers.

Abramovich, Felix, Bailey, Trevor C. and Sapatinas, Theofanis (2000-3)
Wavelet analysis and its statistical applications
Journal of the Royal Statistical Society, Series D: The Statistician, 49, 1-29
Keywords: Changepoint analysis; Density estimation; Fourier analysis; Inverse problems; Nonparametric regression; Signal processing; Spectral density estimation; Time series analysis; Bibliography

Abstract: In recent years there has been a considerable development in the use of wavelet methods in statistics. As a result, we are now at the stage where it is reasonable to consider such methods to be another standard tool of the applied statistician rather than a research novelty. With that in mind, this paper gives a relatively accessible introduction to standard wavelet analysis and provides a review of some common uses of wavelet methods in statistical applications. It is primarily orientated towards the general statistical audience who may be involved in analysing data where the use of wavelets might be effective, rather than to researchers who are already familiar with the field. Given that objective, we do not emphasize mathematical generality or rigour in our exposition of wavelets and we restrict our discussion to the more frequently employed wavelet methods in statistics. We provide extensive references where the ideas and concepts discussed can be followed up in greater detail and generality if required. The paper first establishes some necessary basic mathematical background and terminology relating to wavelets. It then reviews the more well-established applications of wavelets in statistics including their use in nonparametric regression, density estimation, inverse problems, changepoint problems and in some specialized aspects of time series analysis. Possible extensions to the uses of wavelets in statistics are then considered. The paper concludes with a brief reference to readily available software packages for wavelet analysis.

Antoch, J. and Hušková, M.(2000-4)
Bayesian-type estimators of change points
Journal of Statistical Planning and Inference, 91, 195-208
Keywords: Estimators of change point(s); Fixed and local changes; Location model; Bootstrap

Abstract: The purpose of this paper is to introduce Bayesian-type estimators of change point(s). These estimators have smaller variance than related argmax-type estimators and can be also viewed as a one-step estimators, where the argmax-type estimators are used as the preliminary ones. The Bayesian least-squares (LS)-type estimators are introduced and

studied. Confidence intervals based on bootstrap approximation for the change point are also constructed. Finite sample performance is checked in a simulation study. We concentrate on the shift in location model; however, appropriately modified procedure can be applied to the other models and other types of preliminary estimators as well.

Antoniadis, Anestis; Gijbels, Irene and MacGibbon, Brenda(2000-5)
Non-parametric estimation for the location of a change-point in an otherwise smooth hazard function under random censoring

Abstract: A non-parametric wavelet based estimator is proposed for the location of a change-point in an otherwise smooth hazard function under non-informative random right censoring. The proposed estimator is based on wavelet coefficients differences via an appropriate parametrization of the time-frequency plane. The study of the estimator is facilitated by the strong representation theorem for the Kaplan-Meier estimator established by Lo and Singh (1986). The performance of the estimator is checked via simulations and two real examples conclude the paper

Antonio Pievatolo1 and Renata Rotondi(2000-6)
Analysing the interevent time distribution to identify seismicity phases: a Bayesian nonparametric approach to the multiple-changepoint problem

Abstract: In the study of earthquakes, several aspects of the underlying physical process, such as the time non-stationary of the process, are not yet well understood, because we lack clear indications about its evolution in time. Taking as our point of departure the theory that the seismic process evolves in phases with different activity patterns, we have attempted to identify these phases through the variations in the interevent time probability distribution within the framework of the multiple-changepoint problem. In a nonparametric Bayesian setting, the distribution under examination has been considered a random realization from a mixture of Dirichlet processes, the parameter of which is proportional to a generalized gamma distribution. In this way we could avoid making precise assumptions about the functional form of the distribution. The number and location in time of the phases are unknown and are estimated at the same time as the interevent time distributions. We have analysed the sequence of main shocks that occurred in Irpinia, a particularly active area in southern Italy: the method consistently identifies changepoints at times when strong stress releases were recorded. The estimation problem can be solved by stochastic simulation methods

based on Markov chains, the implementation of which is improved, in this case, by the good analytical properties of the Dirichlet process.

Bhatti, M. Ishaq and Wang, Jinglong(2000-7)
On Testing for a Change-Point in Variance of Normal Distribution
Biometrical Journal, 42, 1021-1032
Keywords: Normal random sequence; Change-point; Bayesian; Cusum of square; Lagrange multiplier; Likelihood-methods

Abstract: This paper addresses the problem of testing for a change about the variance of sequence of normal random variables with unknown means. It compares the power performance of five tests, namely: L-test based on Lehmann's (1951, Annals of Mathematical Statistics 22, 165-179) U-statistic, B-test based on bayesian method, R-test derived from likelihood method, C-test based on Brown et al.'s (1975,Journal of the Royal Statistical Society, Series B 37, 149-163.) CUSUM of squares method and finally, the LM-test based on Nyblom's (1989, Journal of the American Statistical Association 84, 223-230.) and Hansen's (1991, University of Rochester Working Paper) Lagrange multiplier method. An empirical power comparison of the five tests suggests that none of the tests power lead to another. However, it is observed that when change-point is between the beginning and the mid portion, the L and R test are better and when the change-point is between the mid portion and the end, CUSUM of square and the B test are better as compared to the other tests

Bischoff, Wolfgang and Miller, Frank(2000-8)
Asymptotically optimal tests and optimal designs for testing the mean in regression models with applications to change-point problems
Annals of the Institute of Statistical Mathematics, 52, 658-679
Keywords: Linear regression; F-test; Likelihood ratio test; Gaussian processes; Quality control

Abstract: Let a linear regression model be given with an experimental region [a, b] R and regression functions $f_1,...,f_{d+1}$: [a, b] R. In practice it is an important question whether a certain regression function f_{d+1}, say, does or does not belong to the model. Therefore, we investigate the test problem H_0 : f_{d+1} does not belong to the model against K : f_{d+1} belong to the model based on the least-squares residuals of the observations made at design points of the experimental region [a, b]. By a new functional central limit theorem given in Bischoff (1998, Ann. Statist. 26, 1398-1410), we are able to determine optimal tests in an asymptotic way. Moreover, we introduce the problem of experimental design for the optimal test statistics. Further, we compare the asymptotically optimal test with the likelihood ratio test (F-test) under the assumption that the error is normally distributed. Finally, we consider real change-point problems as examples and investigate by simulations the behavior of the asymptotic test for finite sample sizes. We determine optimal designs for these examples.

Braun, J. V., Braun, R. K. and H. G. Müller, H. G.(2000-9)
Multiple Changepoint Fitting via Quasilikelihood, with Application to DNA Sequence Segmentation
Biometrika, 87, 301-314.
Keywords: Bacteriophage lambda; Deviance; Generalized linear model; Model selection; Schwarz criterion; Step function.

Abstract: We consider situations where a step function with a variable number of steps provides an adequate model for a regression relationship, while the variance of the observations depends on their mean. This model provides for discontinuous jumps at changepoints and for constant means and error variances in between changepoints. The basic statistical problem consists of identification of the number of changepoints, their locations and the levels the function assumes in between. We embed this problem into a quasilikelihood formulation and utilise the minimum deviance criterion to fit the model; for the choice of the number of changepoints, we discuss a modified Schwarz criterion. A dynamic programming algorithm makes the segmentation feasible for sequences of moderate length. The performance of the segmentation method is demonstrated in an application to the segmentation of the Bacteriophage λ sequence.

Brodsky, B. E. and Darkhovsky, B. S.(2000-10)
Non-parametric statistical diagnosis. Problems and methods.
Mathematics and its Applications, 509. Kluwer Academic Publishers, Dordrecht, 2000.

Abstract: This book deals with the statistical analysis of data from complex objects with more than one statistical mechanism of data generation. Almost all classical statistical inference is based on the assumption that there exists a fixed statistical mechanism of data generation. When the hypothesis of statistical homogeneity holds true, i.e., there exists only one mechanism of data generation, all statistical inferences are based on the fundamental laws of large numbers. The situation is completely different when the distribution law of data generation can change in time or in the phase space. In this case the obtained data must be "sorted" in subsamples generated by different mechanisms. Another type of problem is to detect possible (but unpredictable) changes of data generation process on-line with data collection. These problems form a comparatively new field of research in mathematical statistics—statistical diagnosis. The main goal of statistical diagnosis is to correctly answer the question of whether data obtained are generated by one or by many statistical mechanisms. Problems of statistical diagnosis become very actual due to implementation of complex systems in all spheres of human life. Some important examples are the following. Economics, econometrics, and analysis of financial systems. Many types of exogenous and endogenous "shocks" in economic systems, as well as financial crises, can be described by dynamic models with unpredictable changes in mechanisms of data generation. The problem of detection of structural changes in economic models, systems, and data is really an actual problem.

The research of functioning of the human brain on the basis of measurements of its electric activity. It was discovered recently that the electro-encephalographic signal cannot be described by a unique mathematical model but only by a rich enough set of such models. Therefore the problem of structural analysis of electro-encephalographic signals is very important for the analysis of the functioning of the human brain and the development of modern diagnostics and therapy.

The authors point out two large classes of problems of statistical diagnosis: (1) retrospective problems (detection of disorders and non-stationarities a posteriori, i.e., in the whole of the information received); (ii) sequential problems (decisions on homogeneity or non-homogeneity of processes observed are made on-line with data collection). The book deals with disorder problems in retrospective and sequential settings, as well as contamination problems for random processes and fields.

The present book is not only a corrected edition of the previous one ??B. Brodskiĭand B. Darkhovskiĭ, Nonparametric methods in change-point problems, Kluwer Acad. Publ., Dordrecht, 1993; MR1228205 (95d:62068)]. It proposes generalizations of many ideas and new theoretical and practical results obtained during the 1993–1999 period of time. As in the first book the nonparametric approach to problems of statistical diagnosis is exploited. It means that the proposed methods do not require a priori knowledge of probabilistic distributions of data observed. Alexander Kaplan and Sergei Shishkin wrote a special chapter for this book devoted to applications of the proposed methods to structural analysis of electro-encephalographic signals. This chapter will be informative for professional biologists and neurophysiologists.

The authors' description of the chapters will give some impression of the book.

"The book consists of two parts. The first part (Chapters 1–6) deals with the theory; the second part (Chapters 7, 8 and the Appendix) is devoted to applications.

"In the first chapter results from the theory of probability, the theory of random processes, the theory of random fields, and optimisation theory, as well as some auxiliary results which will be used in the following chapters, are presented. Here we formulate the main ideas of our approach to the problems of statistical diagnosis and the main assumptions which are used in the sequel.

"The second chapter contains a short review of works in the field of statistical diagnosis up to the end of 1998.

"The third and the fourth chapters deal with retrospective problems of statistical diagnosis.

"In the third chapter change-point problems for random sequences are considered. Here we analyse problems with single and multiple change-points in mean values of random sequences, in coefficients of the linear functional model, in derivatives, as well as gradual disorders. Methods for solving all these problems are proposed and investigated. Here we give asymptotic analysis of change-point estimates and compare asymptotic properties of our estimates with the maximum likelihood estimates. A special paragraph is devoted to a priori estimates in change-point problems: the Rao-Cramr type of inequality and the minimax boundaries for the estimation error probabilities in different problems of statistical diagnosis.

"The fourth chapter deals with contamination problems for random sequences. Methods of solving these problems are proposed and analysed.

"The fifth chapter deals with sequential problems of statistical diagnosis. Here we analyse nonparametric analogs of the cumulative sums (CUSUM) method, the quasi-Bayesian method of Girshick-Rubin and Shiryaev, the exponential smoothing method, and the moving sample methods. For each of these methods the characteristics of the normed delay time in change-point detection, the normed false alarm probability, and the rate of convergence of the normed delay time are investigated. The a priori informational boundaries for the normed delay time in detection and the rate of convergence of the normed delay time to its limit are proved. The asymptotic comparative analysis of the nonparametric change-point detection methods based upon the a priori informational boundaries is carried out. Then we formulate and analyse the problem of early detection of non-stationarities. In the last section of this chapter the robust properties of nonparametric sequential change-point detection methods are investigated and robust modifications of these methods are proposed.

"The sixth chapter deals with problems and methods of statistical diagnosis for random fields. Retrospective problems for random fields, i.e., multidimensional analogs of disorder and contamination problems, are considered, as well as sequential problems for random fields. The multi-dimensional analog of disorder problems consists in the search for a boundary which divides the domain of the field into two areas with different probabilistic characteristics of observations. The a priori low boundaries for the quality characteristics of detection in multi-dimensional disorder and contamination problems are established. An approach to solving sequential problems of statistical diagnosis for random fields is proposed.

"Chapter 7 has been written by our colleagues A. Kaplan and S. Shishkin. It deals with applications of statistical diagnosis methods to the analysis of electro-encephalographic signals. Here our readers can find the professional description of the modern state of the art of this field and actual problems of statistical diagnosis of electro-encephalographic signals. Results of applications of our nonparametric methods to real electro-encephalographic signals are presented.

"Chapter 8 deals with problems of statistical diagnosis in economic models and systems. Here we analyse the problem of the structural breaks in non-stationary econometric models, the problem of the statistical analysis of contaminated econometric models, and the problem of the early detection of crises in financial and economic systems.

The Appendix is devoted to the description of nonparametric algorithms and programs of statistical diagnosis and some results of their experimental testing. (Reviewed by M. P. Moklyachuk)

Chen, Jie; Gupta, A.K.(2000-11)
Detecting change point for a sequence of random vectors under nonnormality.
Random Operators and Stochastic Equations, 8, 127-142

Abstract: Presents information on a study which tested and estimated the change point in mean vectors and covariance matrices. Derivation of the asymptotic null distribution of

nonnormality.

Chen, Jie and Gupta, A. K.(2000-12)
Parametric statistical change point analysis.
Birkhäuser Boston, Inc., Boston, MA, 2000.

Abstract: The book summarizes recent developments in parametric change-point analysis. The emphases are on the discussion of a variety of models and formation of test statistics based on three basic methods, namely, the generalized likelihood ratio test (GLRT), Bayesian and information criterion approaches. The main results focus on deriving asymptotically null distributions for the corresponding tests. A major contribution made by the authors is the use of an information criterion to form a test statistic. Another attractive feature is the application of different models to a variety of different data sets. The main body of the book is presented in six chapters. In Chapter 2, the univariate normal model is discussed. First, the mean change model with known or unknown variance is considered. A numerical method is provided to calculate the null distribution based on the GLRT and the asymptotic null distribution is derived by using the extreme value theory. Then the variance change model is considered and the information criterion approach is used to form a test statistic. The Bayesian approach is introduced as a model selection tool to determine the number of change points. The model with both mean and variance changes is then considered by using the GLRT and information criterion approaches.

In Chapter 3, the corresponding problems in the multivariate normal model are considered by using the GLRT and information criterion approaches.

Chapter 4 further generalizes the results to the linear regression model where both the intercept and coefficients are subject to change. The information criterion and Bayesian approaches are used as the main tools.

Chapter 5 and Chapter 6 discuss the change-point problems in the gamma model and the exponential model by using all three approaches, respectively.

In Chapter 7, discrete distribution models including the binomial model and the Poisson model are discussed, mainly by using the GLRT and information criterion approach.

A bibliography containing the references used in the book and other related articles is given at the end along with two indices.

Overall, the book gives a clear and systematic presentation of the models and methods. It will be an excellent source for theoretical and applied statisticians who are interested in research on change-point analysis and its applications to many areas.

As the research on the change-point problem is too extensive to be covered in a single monograph, certain aspects are not discussed in this book, which is to be expected. Two main topics which are missing are the confidence sets or intervals for the change point and power studies, which consequently involve comparisons between the several proposed tests. Some literature on these two topics is thus not listed in the bibliography.

Technically, the book gives many details on the derivation of asymptotically null distributions. A point of confusion about the consistency of the change-point estimator could be

avoided, e.g., in Theorem 2.21, Theorem 3.10 and Theorem 3.21, by showing $\hat{\lambda} = \hat{k}/n \to \lambda \in (0,1)$ with probability 1 by defining $k = [\lambda n]$ as the change point.

Of course, as in most books, some typographical errors are unavoidable. A noticeable one is the definition of V_k on p. 6, which should be $V_k = (k(n-k)/n)(\overline{x}_k - \overline{x}_{n-k})$. Also, throughout the whole book, the notations $\overline{x}_k$ and $\overline{x}_{n-k}$ are a little confusing and it would be much better to replace them with $\overline{x}_{1,k}$ and $\overline{x}_{k+1,n}$. (Reviewed by Yanhong Wu)

Abstract: The field of archaeology provides a rich source of complex, non-standard problems ideally suited to Bayesian inference. We discuss the application of Bayesian methodology to prehistoric corbelled tomb data collected from a variety of sites around Europe. We show how the corresponding analyses may be carried out with the aid of reversible jump Markov chain Monte Carlo simulation techniques and, by calculating posterior model probabilities, we show how to distinguish between competing models. In particular, we discuss how earlier analyses of tomb data by Cavanagh and Laxton and by Buck and co-workers, where structural changes are anticipated in the shape of the tomb at different depths, can be extended and improved by considering a wider range of models. We also discuss the extent to which these analyses may be useful in addressing questions concerning the origin of tomb building technologies, particularly in distinguishing between corbelled domes built by different civilizations, as well as the processes involved in their construction.

Abstract: Knowledge of the direction, magnitude, and timing of changes in bird population abundance is essential to enable species of priority conservation concern to be identified, and reasons for the population changes to be understood. We give a brief review of previous techniques for the analysis of large-scale survey data and present a new approach based on generalized additive models (GAMs). GAMs are used to model trend as a smooth, nonlinear

function of time, and they provide a framework for testing the statistical significance of changes in abundance. In addition, the second derivatives of the modeled trend curve may be used to identify key years in which the direction of the population trajectory was seen to change significantly. The inclusion of covariates into models for population abundance is also discussed and illustrated, and tests for the significance of covariate terms are given. We apply the methods to data from the Common Birds Census of the British Trust for Ornithology for 13 species of farmland birds. Seven of the species are shown to have experienced statistically significant declines since the mid-1960s. Two species exhibited a significant increase. The population trajectories of all but three species turned downward in the 1970s, although in most cases the 1980s brought either some recovery or a decrease in the rate of decline. The majority of populations have remained relatively stable in the 1990s. The results are comparable with those from other analysis techniques, although the new approach is shown to have advantages in generality and precision. We suggest extensions of the methods and make recommendations for the design of future surveys.

Ghosal, Subhashis; Ghosh, Jayanta K. and Samanta, Tapas(2000-15)
Approximation of the Posterior Distribution in a Change-Point Problem
Annals of the Institute of Statistical Mathematics, 51, 479-497
Keywords: Change-point; Gibbs sampling; Hazard rate; Posterior distribution

Abstract: We consider a family of models that arise in connection with sharp change in hazard rate corresponding to high initial hazard rate dropping to a more stable or slowly changing rate at an unknown change-point θ . Although the Bayes estimates are well behaved and are asymptotically efficient, it is difficult to compute them as the posterior distributions are generally very complicated. We obtain a simple first order asymptotic approximation to the posterior distribution of θ . The accuracy of the approximation is judged through simulation. The approximation performs quite well. Our method is also applied to analyze a real data set.

Gombay, Edit and Shuangquan Liu(2000-16)
A Nonparametric Test for Change in Randomly Censored Data
The Canadian Journal of Statistics / La Revue Canadienne de Statistique, Vol. 28, No. 1., 113-121.

Abstract:A simple method for checking the assumption of identical distribution in a sequence of censored data is presented. It is based on a generalization of Wilcoxon's rank statistic. The authors demonstrate the usefulness of this technique on the Stanford Heart Transplant and on the Radiation Therapy Oncology Group data. Clear and simple plots make it easy to visualize and interpret the type of potential change

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Grabovskya, Irina; Horváth, Lajos and Hušková, Marie(2000-17)
Limit theorems for kernel-type estimators for the time of change
Journal of Statistical Planning and Inference, 89, 25-56
Keywords: Change-point; Kernel; Brownian motion; Strong approximation; Limit
distribution

Abstract: We use kernel-type estimators to estimate the time of change in the mean in a sequence of independent observations. Assuming that the size of the change is small two types of limit distributions are derived. The forms of the limit distributions depend on the behavior of the kernel at the end points. The argmax of a two-sided Brownian motion with polynomial drift is a possible limit, while the normal distribution is the limit when the kernel is zero at both boundaries.

Hennig, Christian (2000-18)
Identifiability of models for clusterwise linear regression
Journal of Classification, 17, 273-296
Keywords: partial identifiability; Switching regression; Mixture model; Fixed partition
model; Change point problem; Gaussian mixtures with covariates

Abstract: Identifiability of the parameters is a necessary condition for the existence of consistent estimators. In this paper the identifiability of the parameters of models for data generated by different linear regression distributions with Gaussian errors is investigated. It turns out that such models cause other identifiability problems than do simple Gaussian mixtures. This problem was heretofore ignored; thus there are no satisfying consistency proofs in this area. Three different models are treated: Finite mixture models with random and fixed covariates and a fixed partition model. Counterexamples and sufficient conditions for identifiability are given, including an example for nonidentifiable parameters with an invertible information matrix. The model choice and the interpretation of the parameters are discussed as well as the use of the identifiability concept for fixed partition models.The concept is generalized to partial identifiability

Hušková, M. and Steinebach, J.(2000-19)
Limit theorems for a class of tests of gradual changes
Journal of Statistical Planning and Inference., 89, 57-77
Keywords: Change-point test; Linear and nonlinear regression; Gradual change; Limiting
extreme value distribution; Weighted embedding

Abstract: We study a class of asymptotic tests for detecting various types of gradual changes in a location model. The results of Jaruková (1998. J. Statist. Plann. Inference 70, 263-276) and Hušková (1998. In: Szyszkowicz, B. (Ed.), Asymptotic Methods in Probability and

Statistics. Elsevier, Amsterdam, pp. 577-584.) are extended to other families of change alternatives. Under the null hypothesis of "no change", limiting extreme value distributions of Gumbel type are derived for some asymptotic log-likelihood ratio test statistics. Consistency under certain alternatives is also discussed.

Kang, Kee-Hoon; Koo, Ja-Yong and Park, Cheol-Woo (2000-20)
Kernel estimation of discontinuous regression functions
Statistics and Probability Letters, 47, 277-285
Keywords: Boundary kernel; Change-points; Jump location; Jump size; L_2 convergence; Rate of convergence; Weak convergence

Abstract: A kernel regression estimator is proposed wherein the regression function is smooth, except possibly for a finite number of points of discontinuity. The proposed estimator uses preliminary estimators for the location and size of discontinuities or change-points in an otherwise smooth regression model and then uses an ordinary kernel regression estimator based on suitably adjusted data. Global L2 rates of convergence of curve estimates are derived. It is shown that these rates of convergence are the same as those for ordinary kernel regression estimators of smooth curves. Moreover, pointwise asymptotic normality is also obtained. The finite-sample performance of the proposed method is illustrated by simulated examples.

Koul, Hira L. (2000-21)
Fitting a two phase linear regression model
Journal of the Indian Statistical Association, 38, 331-353
Keywords: Asymptotically distribution free; Fixed jump size; Martingale type transform

Abstract: This paper discusses asymptotically distribution free tests for fitting a two phase linear regression model in the case jump size at the change-point is either fixed or tends to zero at a rate slower than the square root of the sample size. These tests are based on an appropriate partial sum process of the underlying residuals. The paper also discusses the asymptotic behavior of the empirical and a certain weighted empirical processes of these residuals. These latter results are useful in fitting an error distribution in these models.

Leybourne, Stephen J. and Newbold, Paul (2000-22)
Behavior of Dickey-Fuller t-tests when there is a break under the alternative hypothesis
Econometric Theory, 16, 779-789
Keywords: Unit root; Asymptotic analysis; Broken linear trend

Abstract: This paper analyzes the limiting behavior of Dickey-Fuller t-tests when the true generating model is stationary around a broken linear trend. The cases of a break in level and a break in slope are considered separately and found to generate qualitatively different outcomes. In the asymptotic analysis, appropriate normalizations are applied to the break sizes. This leads to theoretical results that generate interesting predictions for sample sizes and break amounts of practical interest. Simulation evidence confirms the value of this approach to an asymptotic theory.

Evolutionary Monte Carlo: Applications to C_P model sampling and change point problem

Abstract: Motivated by the success of genetic algorithms and simulated annealing in hard optimization problems, the authors propose a new Markov chain Monte Carlo (MCMC) algorithm called an evolutionary Monte Carlo algorithm. This algorithm has incorporated several attractive features of genetic algorithms and simulated annealing into the framework of MCMC. It works by simulating a population of Markov chains in parallel, where a different temperature is attached to each chain. The population is updated by mutation (Metropolis update), crossover (partial state swapping) and exchange operators (full state swapping). The algorithm is illustrated through examples of -based model selection and change-point identification. The numerical results and the extensive comparisons show that evolutionary Monte Carlo is a promising approach for simulation and optimization.

Comparison of parameter estimation methods for detecting climate-induced changes in productivity of Pacific salmon

Abstract: Pacific salmon (Oncorhynchus spp.) populations can experience persistent changes in productivity, possibly due to climatic shifts. Management agencies need to rapidly and reliably detect such changes to avoid costly suboptimal harvests or depletion of stocks. However, given the inherent variability of salmon populations, it is difficult to detect changes quickly, let alone forecast them. We therefore compared three methods of annually updating estimates of stock-recruitment parameters: standard linear regression, Walters' bias-corrected regression, and a Kalman filter. We used Monte Carlo simulations that hypothesized a wide range of future climate-induced changes in the Ricker a parameter of a salmon

stock. We then used each parameter estimation method on the simulated stock and recruitment data and set escapement targets and harvest goals accordingly. In these situations with a time-varying true Ricker a parameter, Kalman filter estimation resulted in greater mean cumulative catch than was produced by the standard linear regression approach, Walters' bias correction method, or a fixed harvest rate policy. This benefit of the Kalman filter resulted from its better ability to track changing parameter values, thereby producing escapements closer to the optimal escapement each year. However, errors in implementing desired management actions can significantly reduce benefits from all parameter estimation techniques.

Pauler; Donns K. and Laird, Nan M.(2000-25)

A Mixture Model for Longitudinal Data with Application to Assessment of Noncompliance

Abstract: In clinical trials of a self-administered drug, repeated measures of a laboratory marker, which is affected by study medication and collected in all treatment arms, can provide valuable information on population and individual summaries of compliance. In this paper, we introduce a general finite mixture of nonlinear hierarchical models that allows estimates of component membership probabilities and random effect distributions for longitudinal data arising from multiple subpopulations, such as from noncomplying and complying subgroups in clinical trials. We outline a sampling strategy for fitting these models, which consists of a sequence of Gibbs, Metropolis-Hastings, and reversible jump steps, where the latter is required for switching between component models of different dimensions. Our model is applied to identify noncomplying subjects in the placebo arm of a clinical trial assessing the effectiveness of zidovudine (AZT) in the treatment of patients with HIV, where noncompliance was defined as initiation of AZT during the trial without the investigators' knowledge. We fit a hierarchical nonlinear change-point model for increases in the marker MCV (mean corpuscular volume of erythrocytes) for subjects who noncomply and a constant mean random effects model for those who comply. As part of our fully Bayesian analysis, we assess the sensitivity of conclusions to prior and modeling assumptions and demonstrate how external information and covariates can be incorporated to distinguish subgroups.

William J. Reed(2000-26)

Reconstructing the History of Forest Fire Frequency: Identifying Hazard Rate Change Points Using the Bayes Information Criterion

Abstract: A problem in model selection, namely the identification of multiple change points for a piece-wise constant hazard rate, is discussed. A methodology using the Bayes Information Criterion is developed in an overdispersed survival model (with corresponding quasi-likelihood function). The technique is used to identify changes in the historical frequency of forest fire. It is applied to two data sets derived from time-since-fire maps in the Canadian Rockies.

Ulm, Kurt and Küchenhoff, Helmut (2000-27)

RE: "Use of two-segmented logistic regression to estimate change-points in epidemiologic studies"

American Journal of Epidemiology, 152, 289-289

Abstract: Pastor and Guallar (1998, Am J Epidemoil,148,631-642) have described an interesting problem in epidemiology, the use of two-segmented logistic regression. In the introduction to their paper, they stated that none of the usual methods provides inference procedures for estimating the location of the change-point. However, a method for estimating change-points was described several years ago Ulm K. A(1991, Stat Med,10,341-349). The situation considered there described a threshold with no effect below a certain level. There has been much discussion about appropriate test statistics Cox C. (1987, Biometrics,43,511-523) Silvapulle, JM (1989, Biometrics,47,1629). More recently, an exact algorithm for estimating breakpoints in segmented generalized linear models was described Küchenhoff H(1997, Commut Stat,12,235-247). Finally, the results were compared by using different statistical models Küchenhoff H, Ulm K(1997) that showed how the estimation can depend on the model used for the analysis. Careful modeling and interpretation seems to be very important. When consequences are important, such as in the assessment of threshold values in occupational medicine, it is obvious that only one value is required. The work of Pastor and Guallar (1998) is a first step, but a lot more must be done to enable this method to be used in practice. Regarding the example used in the paper (Pastor R, Guallar E(1998)), several questions remain unanswered. How can a threshold be established? Is a formal test available? No value of any likelihood function (with and without a threshold) was given. What is the interpretation of a threshold or change-point if the corresponding parameter (β_2) is nonsignificant? In the example considered, it is unclear whether the parameters β_1 and β_2 are indeed significantly different from zero (table 2 (Pastor R, Guallar E(1998))). In the paper, the authors compared five models that led to different estimates of the change-point. How can we discriminate between these models?

Walkowiak, Ryszard and Kala, Radoslaw(2000-28)
Two-phase nonlinear regression with smooth transition
Communications in Statistics: Simulation and Computation, 29, 385-397
Keywords: Two-phase regression; Transition point; Transition function; Least square method

Abstract: A method of estimating parameter of two-phase nonlinear regression with smooth transition between phases is described. It consists of two stages, both utilizing the least square fit. In the first one, each phase is fitted separately and, simultaneously, a transition point is determined. In the second stage, the two phases are joined smoothly by a proper transition function, which depends on the transition point chosen by the grid search. The practical aspects of the method proposed are demonstrated on the data concerning the soil bulk density in dependence on the soil water content

Abstract: This article studies the problem of testing and locating changepoints in stochastic ordering. We propose a sequential process to detect the changepoints from two multinomial distributions. We also obtain the maximum likelihood estimators of two multinomial probability vectors under the assumption that the cumulative distributions have a changepoint. Asymptotically unbiased Akaike's information criterion is used to estimate the changepoints of two discrete probability distributions. Finally, we demonstrate our procedure by studying a data set pertaining to average daily insulin dose from the Boston Colloative Drug Surveillance Program and locate the changepoints in stochastic ordering.

Abstract: This paper considers the use of a local linear kernel regression method to test whether the mean function of a sequence of long-range dependent processes has discontinuities or change-points. It proposes a non-parametric estimation procedure and then establishes an asymptotic theory for the estimation procedure. Examples, simulated and real, illustrate the estimation procedure.

Abstract:In the classical setting of the change-point problem, the maximum-likelihood estimator and the traditional confidence region for the change-point parameter are considered. It is shown that the probability of the correct decision, the coverage probability and the expected size of the confidence set converge exponentially fast as the sample size increases to infinity. For this purpose, the tail probabilities of the first passage times are studied. General inequalities are established, and exact asymptotics are obtained for the case of Bernoulli distributions. A closed asymptotic form for the expected size of the confidence set is derived for this case via the conditional distribution of the first passage times.

Borovkov, A. A(1999-2)
Asymptotically optimal solutions in the change-point problem.
Theory of Probability and Its Applications; 43, 539-561

Abstract: Asymptotically optimal procedures in the classical change-point problem are established under the assumption that the change point, being an unknown parameter, increases unlimitedly

Csörgö, M. and Horvath, L.(1999-3)
Limit Theorems in Change-Point Analysis
Csörgö, M. and Horvath, L., 1997 Chichester, Wiley
Review author[s]: R. H. Glendinning The Statistician, Vol. 48, No. 1. (1999), 138

Abstract: This monograph brings together research on changepoint problems. Techniques which address this issue are central to fault detection and analysis of a wide range of phenomena subject to change over time. This monograph gives mathematically rigorous treatment of this problem under a variety of assumptions about the data. These include regression structures and weak or long-range dependence. The particular focus of this work is the application of the of the asymptotic approximations developed by the authors; see Csörgö and Horvath(1990). In chapters one and two, the authors consider a wide variety of changepoint problems. Increasing complexity these are testing for a change in the mean, in location, in variance and finally in distributional shape. Results for the last are extended to vector-valued random variables. Problems where a change may occurs at a single time point are given most attention, although other alternative hypotheses based on smooth changes, change-back and multiple changes are briefly covered. The general approach is to introduce techniques based

on specific distributional assumption(Gaussianity) and to develop asymptotic approximations under less restrictive assumptions. Chapters One and Two end with a brief discussion of results not covered in the body of the text. In Chapter Three, the authors consider the important problem of detecting change in structure of linear regressions. The authors introduce concepts through a progression of problems of increasing complexity. Initial discussions center on the changepoint problem where at least one parameter value changes at unknown design point (the variance of the errors remaining constant). This likelihood ratio approach is extended to changepoint problems where the variance of the errors may change. Techniques based on various definitions of regression residuals are introduced, with the importance of the 'union intersection principle' emphasized. In all cases asymptotic results of varying scope are developed under weak assumptions. The asymptotic properties of the maximum likelihood estimate of the design point where a change occurs are described in the penultimate section of this chapter. The final section gives a brief description of related results. In the final chapter, the authors consider the challenging problem of detecting changes in sequence of the dependent observations. The authors focus on two types of problem. In the first, changes in the mean level of weakly or strongly dependent sequences are considered. Asymptotic results are determined for both cases. The second problem is considered with changes in parameter values of autoregressive moving average models. Particular attention is payed explosive regressions. This chapter finished with a short description of additional results. This monograph requires a high level of mathematical sophistication, although some numerical examples are included. Its main audience appears to be researchers in this area, and therefore it includes an extensive bibliography, subject index and useful author index. This book is an important addition to the research literature and should be a valuable source for researchers in this area.

Abstract: Two-step methods are suggested for obtaining optimal performance in the problem of estimating jump points in smooth curves. The first step is based on a kernel-type diagnostic, and the second on local least-squares. In the case of a sample of size n the exact convergence rate is n^-1, rather than $n^-1 + [\delta]$ (for some $[\delta] > 0$) in the context of recent one-step methods based purely on kernels, or $n^-1(\log n)^1 + [\delta]$ for recent techniques based on wavelets. Relatively mild assumptions are required of the error distribution. Under more stringent conditions the kernel-based step in our algorithm may be used by itself to produce an estimator with exact convergence rate $n^-1(\log n)^1/2$. Our techniques also enjoy good numerical performance, even in complex settings, and so offer a viable practical alternative to existing techniques, as well as providing theoretical optimality.

Hibey, J.L. and Charalambos, C.D.(1999-5)
Performance analysis for a changepoint problem
Automatic Control, IEEE Transactions on Publication Date, 44, 1628-1632

Abstract: Nonlinear stochastic differential equations are used to model a version of the changepoint problem. State estimates of the minimum mean-square-error-type are used in likelihood-ratio tests to detect the time of change. System performance is evaluated by deriving theoretical expressions for the false alarm probability and a special case of the miss error probability. The approach uses a Fokker-Planck equation

Hušková, M.(1999-6)
Gradual changes versus abrupt changes
Journal of Statistical Planning and Inference, 76, 109-125
Keywords: Gradual changes in location model; Estimators

Abstract: A number of papers have been published on the estimation problem in location models with abrupt changes. In the present paper we focus on estimators in location models with various gradual changes. Least type squares estimators of the parameters are proposed and studied. It appears that the limit behavior (both the rate of consistency and limit distribution) of the estimators of the change point in location models depends on how the type of gradual changes differ.

Jandhyala, V.K. and Fotopoulos, S.B.(1999-7)
Capturing the distributional behaviour of the maximum likelihood estimator of a changepoint
Biometrika, 86, 129-140
Keywords:Maximum likelihood estimator; Maximum of a random walk; Negative drift;
Parameter change.

Abstract: We consider the problem of estimating the unknown changepoint in a sequence of time-ordered observations. Upper and lower bounds are derived for the asymptotic distribution of the maximum likelihood estimator and methods of approximation are suggested. A computationally efficient algorithm is presented for deriving the bounds and approximations for the asymptotic probabilities of the maximum likelihood estimator when the parameters before and after the changepoint are unknown. We also show an essentially exponential rate of convergence of the probability distribution of the maximum likelihood estimator from finite samples to the case of infinite samples. We apply the algorithm to the cases of normal and exponential distributions. For the exponential distribution the lower and upper bounds for the right tail probabilities of the maximum likelihood estimator, and the two approximations, are identical. This is not the case for the normal distribution. Finally, we apply

our changepoint analysis for the case of the exponential distribution to data on explosions in British coal mines.

Jandhyala, Venkata K. and Al-Saleh, Jamal A. (1999-8)
Parameter changes at unknown times in non-linear regression
EnvironMetrics, 10, 711-724
Keywords: Brownian motion; Changepoint; Bayesian testing

Abstract: Introducing the change-point problem for the non-linear regression model, we derive Bayes-type statistics for tests of parameter changes at unknown times in the non-linear parameters. We then establish asymptotic distribution theory, mainly, for the two-sided statistics. Throughout the paper, we emphasize the specific case of an exponential type non-linear regression model. The methodology is illustrated through data on pre-school boys' weight/height ratio.

Jandhyala, V. K.; S. Zacks and A. H. El-Shaarawi (1999-9)
Change-point methods and their applications: contributions of Ian MacNeill
Environmetrics, 10, 657-676
Keywords: Change-point problem; Partial sum process; Serial correlation; Brownian motion; Brownian bridge; Weak convergence

Abstract: The present paper reviews the important contributions of Ian MacNeill to the theory and methodology of change-point analysis and environmental statistics. The review concentrates on four areas of change-point analysis: sequences of independent random variables; linear regression models with independent as well as serially correlated random errors; regression models with continuity constraints and spatial models of change-points.

Jose, C. T. and Ismail, B. (1999-10)
Change points in nonparametric regression functions
Communications in Statistics: Theory and Methods, 28, 1883-1902
Keywords: Kernel estimator; Local polynomial regression

Abstract: Estimators of location and size of jumps or discontinuities in a regression function and/or its derivatives are proposed. The estimators are based on the analysis of residuals obtained from the locally weighted least squares regression. The proposed estimators adapt to both fixed and random designs. The asymptotic properties of the estimators are investigated. The method is illustrated through simulation studies
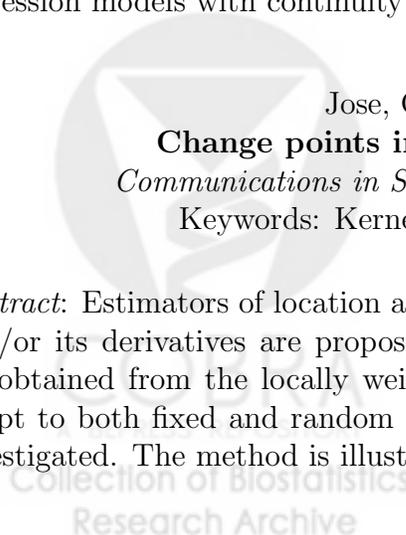
Kazuo Anraku (1999-11)
An Information Criterion for Parameters under a Simple Order Restriction
Biometrika, 86, 141-152
Keywords: AIC; Changepoint; Information criterion; Non-observed-adverse-effect level;
Simple order restriction

Abstract: Suppose we have independent random samples from each of k populations speci-
fied by scalar-valued, unknown parameters $\theta_1, \cdots, \theta_k$ satisfying the simple order restriction
$\theta_1 \leq 1 \cdots \leq \theta_k$. Our concern is to seek distinct parameters among $\theta_1, \cdots, \theta_k$ based on
the data. To find a configuration of distinct parameters among the ?'s, one may consider
employing Akaike's information criterion (Akaike, 1973, 2nd International Symposium on
Information Theory, Ed. B.N. Petrov and F. Csaki, pp 267-281.). However, the criterion
is not appropriate for the order-restricted maximum likelihood estimator of $\theta = (\theta_1, \cdots \theta_k)$,
since the normality or the asymptotic normality of the estimator is not valid. In this paper
an information criterion is proposed for detecting the configuration of the true parameters
with the simple order restriction. This method may also be applied for detecting a change-
point in a sequence of parameters with a monotone trend. A Monte Carlo study indicates
that our new criterion is effective, compared to Akaike's information criterion, for detecting
the configuration of normal means satisfying the simple order restriction.

Kohler, Michael (1999-12)
Nonparametric estimation of piecewise smooth regression functions
Statistics and Probability Letters, 43, 49-55
Keywords: Least squares; Convergence rate

Abstract: Estimation of univariate regression functions from bounded i.i.d. data is consid-
ered. Estimates are defined by minimizing a complexity penalized residual sum of squares
over all piecewise polynomials. The integrated squared error of these estimates achieves for
piecewise p-smooth regression functions the rate $(\ln^2(n)/n)^{2p/(2p+1)}$.

Marc Lavielle(1999-13)
Detection of multiple changes in a sequence of dependent variables
Stochastic Processes and their Applications, 83, 79-102
Keywords: Detection of change-points; Minimum contrast estimator; Penalized minimum
contrast estimator; Strongly mixing processes; Strongly dependent

Abstract: We present some results of convergence for a minimum contrast estimator in a
problem of change-points estimation. Here, we consider that the changes affect the marginal
distribution of a sequence of random variables. We only consider parametric models, but the
results are obtained under very general conditions. We show that the estimated configuration
of changes converges to the true configuration, and we show that the rate of convergence

does not depend on the dependance structure of the process: we obtain the same rate for strongly mixing and strongly dependent processes. When the number of changes is unknown, it is estimated by minimizing a penalized contrast function. Some examples of application to real data are given.

Müller, Hans-Georg and Stadtmüller, Ulrich(1999-14)
Discontinuous Versus Smooth Regression
The Annals of Statistics, 27, 299-337.

Abstract: Given measurements (x_i, y_i), i = 1,..., n, we discuss methods to assess whether an underlying regression function is smooth (continuous or differentiable) or whether it has discontinuities. The variance of the measurements is assumed to be unknown, and is estimated simultaneously. By regressing squared differences of the data formed with various span sizes on the span size itself, we obtain an asymptotic linear model with dependent errors. The parameters of this asymptotic linear model include the sum of the squared jump sizes as well as the variance of the measurements. Both parameters can be consistently estimated, with mean squared error rates of convergence of $n^{-2/3}$ for the sum of squared jump sizes and n^{-1} for the error variance. We derive the asymptotic constants of the mean squared error (MSE) and discuss the dependence of MSE on the maximum span size L. The test for the existence of jumps is formulated for the null hypothesis that the sum of squared jump sizes is 0. The asymptotic distribution of the test statistic is obtained essentially via a central limit theorem for U-statistics. We motivate and illustrate the methods with data surrounded by a scientific controversy concerning the question whether the growth of children occurs smoothly or rather in jumps.

Rao, Ajit V., Miller, David J., Rose, Kenneth and Gersho, Allen (1999-15)
A deterministic annealing approach for parsimonious design of piecewise regression models
IEEE Transactions on Pattern Analysis and Machine Intelligence, 21, 159-173
Keywords: CART

Abstract: A new learning algorithm is proposed for piecewise regression modeling. It employs the technique of deterministic annealing to design space partition regression functions. While the performance of traditional space partition regression functions such as CART and MARS is limited by a simple tree-structured partition and by a hierarchical approach for design, the deterministic annealing algorithm enables the joint optimization of a more powerful piecewise structure based on a Voronoi partition. The new method is demonstrated to achieve consistent performance improvements over regular CART as well as over its extension to allow arbitrary hyperplane boundaries. Comparison tests, on several benchmark data sets from the regression literature, are provided.

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Raymond J. Carroll; Kathryn Roeder; Larry Wasserman (1999-16)
Flexible Parametric Measurement Error Models
Biometrics, Vol. 55, No. 1., 44-54.

Abstract:Inferences in measurement error models can be sensitive to modeling assumptions. Specifically, if the model is incorrect, the estimates can be inconsistent. To reduce sensitivity to modeling assumptions and yet still retain the efficiency of parametric inference, we propose using flexible parametric models that can accommodate departures from standard parametric models. We use mixtures of normals for this purpose. We study two cases in detail: a linear errors-in-variables model and a change-point Berkson model.

Sidoravicius, V., Vares, M. E. and Surgailis, D. (1999-17)
Poisson broken lines process and its application to Bernoulli first passage percolation
Acta Applicandae Mathematicae, 58, 311-325

Abstract: In this note we introduce a process, which we call "the Poisson broken lines process", and we compute the intensity of a point process which is obtained by intersecting the Poisson broken lines process with an abscissa axis. In the second part we apply this result to compute an explicit lower bound for the time constant of a planar Bernoulli first passage percolation model with the parameter $p < p_c$.

Sugiura, Nariaki and Takaoka, Naoko (1999-18)
Homogeneity of variances in normal linear regression with a change point
Communications in Statistics: Theory and Methods, 28, 2785-2801

Abstract: Two methods for testing the equality of variances in straight lines regression with a change point are considered. One is likelihood ratio test and the other is Bayesian confidence interval, based on the highest posterior density for the ratio of variances, using non-informative priors. Methods are applied to the renal transplant data analyzed by Smith and Cook(1980) and Stephens(1994).
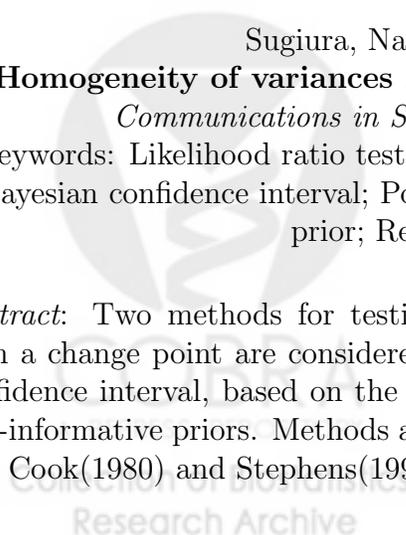
Yakir, Benjamin, Krieger, Abba M. and Pollak, Moshe (1999-19)
Detecting a change in regression: first-order optimality
The Annals of Statistics, 27, 1896-1913
Keywords: Change point detection; Regression; Information bound; Stopping rule

Abstract: Observations are generated according to a regression with normal error as a function of time, when the process is in control. The process potentially changes at some unknown point of time and then the ensuing observations are normal with the same mean function plus an arbitrary function under suitable regularity conditions. The problem is to obtain a stopping rule that is optimal in the sense that the rule minimizes the expected delay in detecting a change subject to a constraint on the average run length to a false alarm. A bound on the expected delay is first obtained. It is then shown that the CUSUM and Shiryayev-Roberts procedures achieve this bound to first order.

Zhou, Shanggang and Wolfe, Douglas A. (1999-20)
Local piecewise linear regression
Journal of Nonparametric Statistics, 12, 109-131
Keywords: Polynomial regression; Kernel estimator; Regression spline.

Abstract: In this paper, we propose a new nonparametric estimator called the local piecewise linear regression estimator. The proposed estimator has the advantages of the regression spline and the local linear regression estimator but overcomes the drawback of both. Here we study the asymptotic behavior of the proposed estimator. Under suitable conditions, we drive the leading bias and variance terms of the local piecewise linear regression estimator at the interior and boundary points for both the fixed design and the random design. As a result, we are able to see clearly many optimal properties of the local piecewise linear regression estimator. For example, the proposed estimator is efficient, design adaptive and computationally inexpensive, and it suffers no boundary effects.

Bai, J. and Perron, P.(1998-1)
Estimating and testing linear models with multiple structural changes
Econometrica., 66, Iss. 1; 47-78
Keywords: Asymptotic distribution; Change point; Rate of convergence; Model selection.

Abstract: This paper considers issues related to multiple structural changes, occurring at unknown dates, in the linear regression model estimated by least squares. The main aspects are the properties of the estimators, including the estimates of the break dates, and the construction of tests that allow inference to be made about the presence of structural change and the number of breaks. The general case of a partial structural change model is considered ware not all parameters are subject to shifts. Both fixed and shrinking magnitudes of shifts are studied, and the rates of convergence for the estimated break fractions are obtained.

A procedure is also proposed that allows one to test the null hypothesis of, say, l changes, versus the alternative hypothesis of l + 1 changes.

Bai, Jushan (1998-2)
Estimation of multiple-regime regressions with least absolute deviations
Journal of Statistical Planning and Inference, 74, 103-134
Keywords:Multiple change points; Multiple-regime regressions; Least absolute deviation; Asymptotic distribution

Abstract: This paper considers least absolute deviations estimation of a regression model with multiple change points occurring at unknown times. Some asymptotic results, including rates of convergence and asymptotic distributions, for the estimated change points and the estimated regression coefficient are derived. Results are obtained without assuming that each regime spans a positive fraction of the sample size. In addition, the number of change points is allowed to grow as the sample size increases. Estimation of the number of change points is also considered. A feasible computational algorithm is developed. An application is also given, along with some Monte Carlo simulations.

Belisle, Patrick; Joseph, Lawrence; MacGibbon, Brenda; Wolfson, David B; Berger, Roxane du(1998-3)
Change-Point Analysis of Neuron Spike Train Data
Biometrics, Vol. 54, No. 1., 113-123

Abstract: In many medical experiments, data are collected across time, over a number of similar trials, or over a number of experimental units. As is the case of neuron spike train studies, these data may be in the form of counts of events per unit of time. These counts may be correlated within each trial. It is often of interest to know if the introduction of an intervention, such as the application of a stimulus, affects the distribution of the counts over the course of the experiment. In such investigations, each trial generates a sequence of data that may or may not contain a change in distribution at some point in time. Each sequence of integer counts can be viewed as arising from a Poisson process and are therefore independently distributed or as an integer-valued time series that allows for correlations between these counts. The main aim of this paper is to show how the ensemble of sample paths may be used to make inference about the distribution of the instantaneous times of change in a given population. This will be accomplished using a Bayesian hierarchical model for these change-points in time. A bonus of these models is they also allow for inference about the probability of a change in each unit and the magnitude of the effects, if any. The use of such change-point models on integer-valued time series is illustrated on neuron spike train data, although the methods can be applied to other situations where integer-valued processes arise.

Bischoff, Wolfgang (1998-4)
A functional central limit theorem for regression models
The Annals of Statistics, 26, 1398-1410
Keywords: Linear regression; regression residuals; partial sums process; Functional central limit theorem; functions of Brownian motion; Change-point problems; Tests; Brownian motion; Change point

Abstract: Let a linear regression be given. For detecting change-points, it is usual to consider the sequence of partial sums of least squares residuals whence a partial sums process is defined. Given a sequence of exact experimental designs, we consider for each design the corresponding partial sums process. If the sequence of designs converges to a continuous design, we derive the explicit form of the limit process of the corresponding sequence of partial sums processes. This is a complicated function of the Brownian motion. These results are useful for the problem of testing for change of regression at known or unknown times.

Braun, Jerome and Müller, H. G.(1998-5)
Statistical Methods for DNA Sequence Segmentation
Statistical Science, Vol. 13, No. 2., 142-162
Keywords: Statistical genetics; Change-point; Hidden Markov chain; Patchiness; Quasideviance; Split local polynomial; Chromosome banding; Bacteriophage λ.

Abstract: This article examines methods, issues and controversies that have arisen over the last decade in the effort to organize sequences of DNA base information into homogeneous segments. An array of different models and techniques have been considered and applied. We demonstrate that most approaches can be embedded into a suitable version of the multiple change-point problem, and we review the various methods in this light. We also propose and discuss a promising local segmentation method, namely, the application of split local polynomial fitting. The genome of bacteriophage λ serves as an example sequence throughout the paper.

Chen, Jie (1998-6)
Testing for a change point in linear regression models
Communications in Statistics: Theory and Methods, 27, 2481-2493

Abstract: In this paper, the Schwarz Informayion Criterion (SIC) is proposed to locate a change point in the simple linear regression model, as well as in the multiple linear regression model. The method is then applied to a financial data set, and a change point is successfully detected.

Correa, José Andrés(1998-7)
Weighted approximations of parameters-estimated empirical processes and changepoint analysis
Asymptotic Methods in Probability and Statistics. A volume in Honor of Miklós Csörgő 515-549 Szyszkowicz, Barbara (ed.)
Elsevier/North-Holland [Elsevier Science Publishing Co., New York; North-Holland Publishing Co., Amsterdam] (New York; Amsterdam)

Abstract: We study the weighted asymptotic behaviour of empirical processes based on independent observations from parametric families and defined after the unknown parameter(s) have been estimated by suitably chosen estimator(s). We derive the limiting Gaussian processes under the null hypothesis of the observations being identically distributed. An application to testing for a change in the distribution at an unknown point of a random sequence is considered.

Cooper, Suzanne J. (1998-8)
Multiple regimes in U.S. output fluctuations
Journal of Business and Economic Statistics, 16, 92-100
Keywords: Monte Carlo; Nonlinear time series; Regression tree; Change point

Abstract: This article investigates the existence of multiple regimes in the U.S. economy during the 1923-1991 period. A technique known as regression tree analysis is applied to search for splits in the data, if any exist, rather than choosing a splitting point a priori as has been done in previous work. Using this technique, strong evidence for the existence of nonlinear behavior of U.S. output is found over this period. Monte Carlo results are presented to assess the significance of the regime changes that are found.

Darkhovskiĭ, B. S.(1998-9)
A method for the retrospective estimation of the change points of linear regression coefficients
Avtomat. i Telemekh. 1998, , no. 8, 185–189; translation in Automat. Remote Control 59 (1998), no. 8, part 2, 1201–1204 (1999)

Abstract: The author considers the problem of multiple change points in linear regression models with possibly dependent error terms. The procedure worked out in the paper by B. E. Brodskiĭand Darkhovskiĭ [Avtomat. i Telemekh. 1993, no. 1, 62-67] is adapted for this case. The consistency of the proposed procedure is shown. (Reviewed by M. Huškov)

Freidlin, Boris and Gastwirth, Joseph L.(1998-10)
The application of change point tests to data occuring in fair hiring cases
Asymptotic Methods in Probability and Statistics B. Szyszkowicz(Editor), 1998, Elsevier Science B. V.

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Abstract: When a complaint of discrimination is made an employer may respond to it by hiring more minorities. From a legal viewpoint the practices in effect during the time period prior to the complaint are more relevant for determining liability than those of the post charge period. In **Gay v. Waiters**, Judge Schwarzer observed that the data suggested a change occurred after the charge was filed. His decision did not utilize this observation, however, because the data had not been subject to a formal statistical analysis. This article presents two adaptations of tests based on the Cusums, originally developed to detect in quality control, to analyze several data sets that were submitted to the courts in the U.S.

Hušková, M.(1998-11)
Estimators in the location model with gradual changes.
Comment. Math. Univ. Carolin. 39, 147–157.

Abstract: The author considers a location model with gradual changes after an unknown time point m. Namely, $Y_i = + \delta_n((i - m)/n)^+ + e_i$, $1 \le i \le n$, where $a^+ = \max(a, 0)$, and , δ_n and m are unknown parameters. It is assumed that $e_1, e_2, \cdots, e_n$ are independent identically distributed random variables with $e_1 = 0$, $Ee_1^2 = \sigma^2 > 0$ and $E|e_i|^{2+\Delta} < \infty$ with some $\Delta > 0$. The model corresponds to the situation when the observations are identically distributed until the unknown time m when the model changes to a simple regression model with slope δ_n. The parameter m is the change point. The least squares estimators $\hat{m}, \hat{}$ and $\hat{\delta}_n$ and m, and δ_n are the solutions of

$$\min_{*, \delta^*, m^*} \sum_{1 \le i \le n} \left(Y_i - {}^* - \delta^* \left(\frac{i - m^*}{n} \right)^+ \right)^2.$$

If $m = [n\theta]$ with some $0 < \theta < 1$, $\delta_n = O(1)$ and $\delta_n^2 n / (\log \log n)^2 \to \infty$, then

$$\frac{\delta_n}{\sigma} \frac{\hat{m} - m}{n^{1/2}} \left(\frac{\theta(1 - \theta)}{1 + 3\theta} \right)^{1/2} \xrightarrow{D} N(0, 1),$$

where $N(0, 1)$ stands for a standard normal random variable. Similar central limit theorems are established for $\hat{}$ and $\hat{\delta}_n$. (Reviewed by Lajos Horvth)

Hušková, M.(1998-12)
Remarks on test procedures for gradual changes.
Asymptotic methods in probability and statistics (Ottawa, ON, 1997), 577–583, North-Holland, Amsterdam, 1998.

Abstract: Jaruškov, D.(1998), developed a test procedure for detecting gradual changes in simple regression models and studied its limit behavior under the null hypothesis 'no change'. The purpose of the present paper is to propose a new class of test procedures for this problem

and to study their limit behavior both under the null hypothesis (no change) and under some alternatives.

Istas, Jacques and Stryhn, Henrik(1998-13)
Discretely Observing a White Noise Change-Point Model in the Presence of Blur
Bernoulli, Vol. 4, No. 2., 185-201
Keywords: Asymptotic distribution; Blur; Maximum likelihood; Orthogonalization

Abstract:In discretely observed diffusion models, inference about unknown parameters in a smooth drift function has attracted much interest of late. This paper deals with a diffusion-type change-point model where the drift has a discontinuity across the point of change, analyzed in detail in continuous time by Ibragimov and Hasminskii. We consider discrete versions of this model with integrated or blurred observations at a regular lattice. Asymptotic convergence rates and limiting distributions are given for the maximum likelihood change-point estimator when the observation noise and the lattice spacing simultaneously decrease. In particular, it is shown that the continuous and discrete model convergence rates are generally equal only up to a constant; under specific conditions on the blurring function this constant equals unity, and the normalized difference between the estimators tends to zero in the limit.

Jaruskova, D.(1998-14)
Testing appearance of linear trend
Journal of Statistical Planning and Inference, 70, 263-276
Keywords: Change-point problem; Linear regression; Gradual type of change; Limit theorem

Abstract: A test for the detection of a gradual change in simple linear regression is studied. Asymptotic distribution of the "maximum type" statistic is derived. Asymptotic critical values are compared with critical values obtained by simulations. The problem was motivated by the effort of meteorologists to discover a change in meteorological measurements.

Jaruškov, Daniela(1998-15)
Change-point estimator in gradually changing sequences.
Comment. Math. Univ. Carolin. 39 (1998), no. 3, 551–561.

Abstract: In this paper, the author considers the problem of testing for gradual changes in a polynomial regression model where a change may occur on a high-order term at an unknown time point. This extends the work by M. Hušková [Comment. Math. Univ. Carolin. 39 (1998), no. 1, 147-157] in which a linear trend may appear in the mean at an unknown time

point. By assuming that all other related parameters are known and using the expression in terms of the discrete orthogonal polynomials, the limiting distribution for the least square estimation of the change point is shown to be normal as the sample size goes to infinity and the magnitude of the change goes to zero under certain conditions. (Reviewed by Yanhong Wu)

Kim, Jaehee H. and Hart, Jeffrey D. (1998-16)
Tests for change in a mean function when the data are dependent
Journal of Time Series Analysis, 19, 399-424
Keywords: Change-point models; Nonparametric regression; Fourier series; local linear smoothing; Change point; Nonparametric regression; Local smoothing

Abstract: Detecting changes in the mean of a stochastic process is important in many areas, including quality control. We develop powerful omnibus tests for the null hypothesis that the underlying mean is constant. The proposed tests can be applied to test for any kind of change, whether it be abrupt, smooth or cyclical. Nonparametric function estimation techniques are used in deriving these tests. The test statistics are derived from a Fourier series smoother that minimizes an estimate of mean integrated squared error. An important example of correlated data is that arising from a stationary time series. To obtain a valid test of mean constancy, it is necessary to estimate the spectrum of the error process, and we consider various methods of doing this. We have found that, in the case of an $AR(1)$ model, the spectrum is well estimated if local linear smoothing is used in conjunction with generalized least squares. A power study of the proposed tests is done by simulation when the errors follow an $AR(1)$ model, and the tests are applied to a set of astronomy data.

Lamprecht, E. A. and Zacks, S.(1998-17)
Two armed bandits with change point in one arm
Journal of Statistical Planning and Inference, 73, 47-60
Keywords: Bernoulli trials; Two armed bandits; Change point; Surveillance; Dynamic programming; Optimal strategy; Wald SPRT

Abstract: The problem of allocating N Bernoulli trials to two alternative experiments (arms) is considered. The success probability on Arm1 is fixed at a known value λ. The success probability on Arm2 is ϕ before a change point and after the change,$\theta < \lambda < \phi$Both θ and ϕ are known. The change point t is unknown. Bayesian strategies are considered. The objective is to maximize the total expected number of successes. The procedure considered consists of multiple surveillance cycles combined with a final cycle. The efficiency of the procedure is evaluated relative to the maximal possible expected number of successes.

Lee, Chung-Bow(1998-18)

Bayesian estimation of the number of change points.
Statist. Sinica 8 , 923-939.

Abstract: The problem of estimating the number of change points in a sequence of independent random variables is considered in a Bayesian framework. We find that, under mild assumptions and with respect to a suitable prior distribution, the posterior mode of the number of change points converges to the true number of change points in the frequentist sense. Furthermore, the posterior mode of the locations of the change points is shown to be within $O_p(\log n)$ of the true locations of the change points where n is the sample size. The prior distribution on the locations of the change points may be taken to be uniform. Finally, some simulated results are given, showing that the method works well in estimating the number of change points

Lombard, F.(1998-19)
Tests for Constancy of a Mean
Asymptotic Methods in Probability and Statistics, B. Szyszkowicz (Editor), 1998, Elsevier Science B. V.

Abstract: Consider a regression model of the form $y_t = g((t-1/2)/n) + \tau_t$, $t = 1, \cdots, n$. Where g is a function on $(0,1)$ and the noise components τ_t are iid random variables with zero mean and variance $\sigma^2 > 0$. We want to test the hypothesis that g is constant against the alternative that it is non-constant. No specified alternative is postulated. The test should have good power against a wide range of alternatives. In particular we wish to avoid making excessive smoothness assumptions on g. In this paper we propose a class of test statistics which are simple weighted averages of periodogram ordinates. The asymptotic distributions of the test statistics are non-normal. The associated tests are shown to have good power against a wide range of alternatives. We also exhibit the application of the tests to a number of statistical hypothesis testing problems, some of which seemingly bear no relationship to tests for constancy of a mean. In particular, a white noise test and a test of goodness of the fit of a specified distribution to a set of independent observations are proposed.

Müller, Hans-Georg(1998-20)
Non-parametric models for non-smooth functions.
Asymptotic methods in probability and statistics (Ottawa, ON, 1997), 595–609, North-Holland, Amsterdam, 1998

Abstract: This paper provides a review of earlier and recent developments on the statistical analysis of functions which combine discontinuous and smooth features. Such situations have been the focus of much recent research and this area involves many theoretical and applied questions of interest. The issues range from estimation of change-points and boundary curves to testing for the presence of change-points and determination of the number of

change-points. It is shown that some popular change-point detection methods such as one-sided kernel differences, wavelets, semiparametric modelling and local polynomial fitting are simply different versions of the same underlying idea. This entire class of one-sided methods is best illustrated and understood relative to the implementation with one-sided kernel estimators. We review some basic results and discuss some pertinent issues regarding the modelling of discontinuous phenomena

Use of two-segmented logistic regression to estimate change-points in epidemiologic studies

Abstract : Many epidemiologic data, the dose-response relation between a continuous exposure and the risk of disease abruptly changes when the exposure variable reaches an unknown threshold level, the so-called change-point. Although several methods are available for dose-response assessment with dichotomous outcomes, none of them provide inferential procedures to estimate change-points. In this paper, we describe a two-segmented logistic regression model, in which the linear term associated with a continuous exposure in standard logistic regression is replaced by a two-segmented polynomial function with unknown change-point, which is also estimated. A modified, iteratively reweighted least squares algorithm is presented to obtain parameter estimates and confidence intervals, and the performance of this model is explored through simulation. Finally, a two-segmented logistic regression model is applied to a case-control study of the association of alcohol intake with the risk of myocardial infarction and compared with alternative analysis. The ability of two-segmented logistic regression to estimate and provide inferences for the location of change-points and for the magnitude of other parameters of effect will make this model a useful complement to other methods of dose-response analysis in epidemiologic studies. Am J Epidemiol 1998; 148: 631-642.

Minimax estimation of sharp change points

Abstract: We define the sharp change point problem as an extension of earlier problems in change point analysis related to nonparametric regression. As particular cases, these include estimation of jump points in smooth curves. More generally, we give a systematic treatment of the correct rate of convergence for estimating the position of a "cusp" of an arbitrary order. We propose a test function for the local regularity of a signal that characterizes such a point

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as a global maximum. In the sample implementation of our method, from observations of the signal at discrete time positions i/n, $i = 1, \cdots, n$, we use a wavelet transformation to approximate the position of the change point in the no-noise case. We study the noise effect, in the worst case scenario over a wide class of functions having a unique irregularity of "order α" and propose a sequence of estimators which converge at the rate $n^{-1/(1+2\alpha)}$, as n tends to infinity. Finally we analyze the likelihood ratio of the problem and show that this is actually the minimax rate of convergence. Examples of thresholding empirical wavelet coefficients to estimate the position of sharp change points are also presented.

Estimation and comparison of multiple change-point models

Abstract: This paper provides a new Bayesian approach for models with multiple change points. The centerpiece of the approach is a formulation of the change-point model in terms of a latent discrete state variable that indicates the regime from which a particular observation has been drawn. This state variable is specified to evolve according to a discrete-time discrete-state Markov process with the transition probabilities constrained so that the state variable can either stay at the current value or jump to the next higher value. This parameterization exactly reproduces the change point model. The model is estimated by Markov chain Monte Carlo methods using an approach that is based on Chib (1996, Econometric Theory 12, pp. 409431). This methodology is quite valuable since it allows for the fitting of more complex change point models than was possible before. Methods for the computation of Bayes factors are also developed. All the techniques are illustrated using simulated and real data sets.

Estimation of a function with discontinuities via local polynomial fit with an adaptive window choice

Abstract:We propose a method of adaptive estimation of a regression function which is near optimal in the classical sense of the mean integrated error. At the same time, the estimator is shown to be very sensitive to discontinuities or change-points of the underlying function f or its derivatives. For instance, in the case of a jump of a regression function, beyond the intervals of length (in order) $n^{-1} \log n$ around change-points the quality of estimation is essentially the same as if locations of jumps were known. The method is fully adaptive

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and no assumptions are imposed on the design, number and size of jumps. The results are formulated in a nonasymptotic way and can therefore be applied for an arbitrary sample size.

Aly,Emad-Eldin A. A. Kochar,Subhash C.(1997-1)
Change Point Tests Based on U-Statistics with Applications in Reliability.
Metrika, 45 , 259-269

Abstract: We consider the problem of testing the null hypothesis of no change against alternative of exactly one change point. The proposal tests are based on generalized two-sample U-statistics processes. We drive the limiting null distributions of the proposal tests. Some applications in Statistical Reliability are given.

Antoch, J.; Hušková , M. and Práková, Z.(1997-2)
Effect of dependence on statistics for determination of change
Journal of Statistical Planning and Inference Volume 60, 291-310
Keywords: Change-point problem; Linear process; Location model; AR(1)

Abstract: Quite a number of test statistics and estimators for detection of a change in the mean of a series of independent observations were proposed and studied. The purpose of this paper is to examine the behavior of these statistics if the observations are dependent, particularly, if they form a linear process.

Bai, Jushan(1997-3)
Estimation of a Change Point in Multiple Regression Models
The Review of Economics and Statistics, Vol. 79, No. 4. (Nov., 1997), pp. 551-563.

Abstract: This paper studies the least squares estimation of a change point in multiple regressions. Consistency, rate of convergence, and asymptotic distributions are obtained. The model allows for lagged dependent variables and trending regressors. The error process can be dependent and heteroskedastic. For nonstationary regressors or disturbances, the asymptotic distribution is shown to be skewed. The analytical density function and the cumulative distribution function for the general skewed distribution are derived. The analysis applies to both pure and partial changes. The method is used to analyze the response of market interest rates to discount rate changes.

Balakumar, Sivanandan(1997-4)
Changepoint detection using nonparametric procedures
Missouri J. Math. Sci. 9 (1997), no. 3, 178-183

Abstract: This study investigates the detection of a changepoint due to location or scale or both location and scale changes in a sequence of univariate random variables. Two changepoints models, namely, the single change and the continuous change are considered and the appropriate test statistics are derived. The proposed test statistics are functions of two linear rank statistics, of which one is odd-translation invariant, sensitive to location shift and the other one is even-translation invariant, sensitive to scale shift. The asymptotic distribution of the proposed statistics are obtained under the null hypothesis. Jie

Baron, Michael and Rukhin, Andrew L.(1997-5)
Asymptotic behavior of confidence regions in the change-point problem
Journal of Statistical Planning and Inference, 58, 263-282
Keywords: Change-point problem; Confidence estimation; Credible region; Exponential family; Information numbers; Random walk

Abstract: The confidence estimation of the change-point is considered. The asymptotic behavior of the coverage probability and the expected width of a traditional confidence region is derived as the threshold constant increases. The nature of this bound is related to information numbers and first passage probabilities for random walks. In the situation when pre- and after-change distributions belong to a one-parameter exponential family but are otherwise unknown, similar results are obtained for the confidence region based on the maximum likelihood procedure.

Brostrom, Goran(1997-6)
A Martingale Approach to the Changepoint Problem
Journal of the American Statistical Association, Vol. 92, No. 439. 1177-1183
Keywords: Binary data; Bootstrap; Brownian motion; Conditional inference; Recursive residuals; Reliability.

Abstract:The changepoint problem for a binary sequence is considered. A test statistic based on recursive residuals is compared to the test statistic suggested by Pettitt. The new test statistic has more interesting properties for use in sequential testing. However, neither of the two test statistics dominates the other. Sequential versions of the martingale-based test, forward and reverse, are given and compared to other tests by means of a simulation study. The reverse martingale tests detect a shift earlier, if it is detected. The price to be paid is a slightly higher probability of not detecting a shift

Chang, Yi-Ping and Huang, Wen-Tao(1997-7)
Inferences for the Linear Errors-in-Variables With Changepoint Mode
Journal of the American Statistical Association, Vol. 92, No. 437, 171-178.
Keywords: Asymptotic distribution; Asymptotic efficiency; Likelihood ratio test.

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Abstract: A new linear structural errors-in-variables regression with changepoint model is considered. In this model we consider the likelihood ratio test based on the maximum Hotelling T^2 for the test of no change against the alternative of exactly one change. If there is a change, either known a priori or by testing, then we estimate the unknown changepoint parameter and some other related parameters by maximum likelihood. The limiting distribution of the changepoint estimator is investigated and it is shown that the asymptotic efficiency increases as the absolute regression slope coefficient increases. A Monte Carlo study shows that the proposed estimator performs satisfactorily

Carlstein, Edward; Müller, Hans-Georg and Siegmund, David (Eds) Review author[s]: RVL (1997-8)

Change-Point Problems (Lecture Notes Monograph Series Vol. 23)

Journal of the American Statistical Association. Alexandria: 92, Iss. 437, 389

Abstract: This volume contains 28 selected invited papers from an ASAIMS-SIAM-sponsored conference on changepoint problems held at Mt. Holyoke College in 1992. A review paper by P. K. Bhattacharya provides some orientation to the subject area, and the Preface does a good job of summarizing the contents in broad topical areas. Several papers concern the general problem of detecting a change in the distribution of a sequence of random variables. Articles on sequential changepoint methods are particularly useful in quality-control settings. Other topics include changepoints in regression functions, splines, and kernel smoothers; hazard functions and censored data; multivariate methods, such as edge detection; and dependent data.

Caussinus, Henri and Lyazrhi, Faouzi(1997-9)

Choosing a Linear Model with a Random Number of Change-Points and Outliers

Annals of the Institute of Statistical Mathematics, 49, 761 - 775

Keywords: Akaike information criterion; Bayesian information criterion; Hypothesis testing.

Abstract: The problem of determining a normal linear model with possible perturbations, viz. change-points and outliers, is formulated as a problem of testing multiple hypotheses, and a Bayes invariant optimal multi-decision procedure is provided for detecting at most k $(k > 1)$ such perturbations. The asymptotic form of the procedure is a penalized log-likelihood procedure which does not depend on the loss function nor on the prior distribution of the shifts under fairly mild assumptions. The term which penalizes too large a number of changes (or outliers) arises mainly from realistic assumptions about their occurrence. It is different from the term which appears in Akaikes or Schwarz criteria, although it is of the same order as the latter. Some concrete numerical examples are analyzed.

Chang, Yi-Ping and Huang, Wen-Tao(1997-10)
Inferences for the linear errors-in-variables with changepoint models.
Journal of the American Statistical Association, 92, 171-178
Keywords: Asymptotic distribution; Asymptotic efficiency; Consistency; Likelihood ratio test

Abstract: A new linear structural errors-in-variables regression with changepoint model is proposed. In the model, the likelihood ratio test based on the maximum Hoteling T2 for the test of no change counter to the alternative of one change is examined. Asymptotic efficiency is observed to go up as the family goes down.

Chen and A.K. Gupta(1997-11)
Testing and locating variance changepoints with application to stock prices
Journal of the American Statistical Association, 92, 739-747.
Keywords: Asymptotic distribution; Consistency; Cummulative sum; Hypothesis testing; Information criterion; Return series;; Unbiased estimator.

Abstract: A binary procedure combined with Swarz information criterion (SIC) was utilized for testing and locating variance changepoints in a sequence of independent Gaussian random variables. The binary segmentation method involved the testing and estimation of the single changepoint hypothesis which was then repeated for subsequent test until the null hypothesis was accepted. Furthermore, the SIC and binary segmentation method reduced long computations incurred in changepoint analysis

Dunnigan, Gerri M.; Hammenb, John L. and Harris, T. Robert(1997-12)
A SAS-IML program for implementing two-phase regression analysis of geophysical time series data
Computers and Geosciences Volume 23, Issue 7, 763-770
Keywords: Linear regression; Two-phase regression; Change point; SAS-IML

Abstract: Two-phase regression analysis has been shown to have utility in geophysical time series analysis. Based on linear regression, the technique operates by locating a change point, if one exists, where a significant change in slope occurs. The timing of the break can then be associated with natural and anthropogenic variables which are thought to impact the behavior of the dependent variable. The technique is not widely available in commercial statistical packages. A SAS Interactive Matrix Language program is presented here to implement the technique.

Durazo-Arvizu; McGee, Daniel; Li, Zhaohai and Cooper, Richard (1997-13)

Establishing the nadir of the body mass index-mortality relationship: A case study

Journal of the American Statistical Association, 92, 1312-1319

Keywords: Changepoint; Logistic regression; Mortality; Obesity; Quetelet index

Abstract: Many studies have demonstrated a nonmonotonic relationship between mortality and body mass index (BMI), with excess mortality occurring at both low and high levels. Although much discussion and many different analyses have appeared, to our knowledge no attempt has been made to quantitatively establish the BMI at which minimum mortality (BMI_{min}) occurs or to establish confidence intervals for this BMI, accounting for the asymmetry of the relationship. We model the nonmonotonic relationship between BMI and mortality in 13,242 black and white participants of the NHANES-I Epidemiologic Follow-up Study to estimate the BMI at which minimum mortality occurs. In our analyses we consider the joint relationship of age, smoking status, and BMI to mortality. We present two methodologies for estimating BMI_{min}: a logistic regression model with a transformation of BMI to accommodate asymmetry and a changepoint model as suggested by Goetghebeur and Pocock. We establish confidence intervals for BMI_{min} using the delta method and bootstrap sampling for the logistic and the profile likelihood and bootstrap sampling for the changepoint model. We also present formal tests for the heterogeneity of BMI_{min} by smoking status, sex, race, and age. Only the interaction between race and BMI is significant; the BMI_{min} is somewhat higher for blacks than whites. Finally, we discuss the problem of goodness-of-fit statistics when the relationship between the characteristic and the outcome is nonmonotonic.

Gerard, Patrick D. and Schucany, William R. (1997-14)

Locating exotherms in differential thermal analysis with nonparametric regression

Journal of Agricultural, Biological, and Environmental Statistics, 2, 255-268

Keywords: Derivative estimation; Bandwidth; Change point; Local polynomial regression

Abstract: Differential Thermal Analysis (DTA) is a technique used to quantify cold tolerance in plants. Plant tissue is cooled, and the ambient temperature and the temperature differential between tissue and ambient are measured. Freezing episodes, called exotherms, can be identified as changepoints, local minima, or selected inflection points in a plot of differential temperature against ambient temperature. The primary exotherm typically manifests itself as a discontinuity and can be identified using techniques similar to those described by Müller (1992, The Annals of Statistics,20,737-761) and Loader (1996, The Annals of Statistics,24,1667-1678). Modifications for locally weighted polynomial regression are proposed here to locate additional exotherms through estimation of first and second derivatives. These techniques involve an innovative combination of local and global bandwidth selection. Finally, the estimators are applied to crepe myrtle (Lagerstremia indica) and pecan (Carya illinoensis) DTA data.

Hirotsu, Chihiro(1997-15)
Two-way change-point model and its application.
Australian Journal of Statistics. 39, 205-218.

Abstract: Exact null and alternative distributions of the two-way maximally selected χ^2 for interaction between the ordered rows and columns are derived for each of the normal and Poisson models, respectively. The method is one of the multiple comparison procedures for ordered parameters and is useful for defining a block interaction or a two-way change-point model as a simple alternative to the two-way additive model. The construction of a confidence region for the two-way change-point is then described. An important application is found in a dose-response clinical trial with ordered categorical responses, where detecting the dose level which gives significantly higher responses than the lower doses can be formulated as a problem of detecting a change in the interaction effects.

Honda, Toshio (1997-16)
The CUSUM tests with nonparametric regression residuals
Journal of the Japan Statistical Society, 27, 45-63
Keywords: Structural stability; Polynomial regression; Change point

Abstract: Let $\{(X_t', \epsilon_t')'\}, t = 1, 2, \cdots$, be a $(p+1)$-dimensional stationary stochastic process. We observe $\{(Y_t, X_t')'\}$ satisfying $Y_t = g_t(X_t) + \epsilon_t, t = 1, 2, \cdots, T$. The author wants to test the null hypothesis H_0 $g_t(x) = g(x)$. The test is based on the cumulative sum $S_\tau = (1/T^{1/2}) \sum_{t=1}^{T\tau} Q_t$, where

$$Q_t = \hat{f}(X_t)Y_t - \widehat{fg}(X_t), \hat{f}(X_t) = \frac{1}{Th^p} \sum_{s=1}^{T} K\left(\frac{X_s - X_t}{h}\right), \widehat{fg}(X_t) = \frac{1}{Th^p} \sum_{s=1}^{T} K\left(\frac{X_s - X_t}{h}\right)Y_s.$$

K is the kernel in the estimators and h is the bandwidth. It is shown that under some regularity conditions S_τ converges weakly to a constant multiple of a one-dimensional Brownian bridge in $D[0, 1]$. The author proves a similar result when instead of the Nadaraya-Watson and Parzen-Rosenblatt estimates local polynomial smoothing is used. (Reviewed by Lajos Horváth)

Horváth, Lajos(1997-17)
Detection of Changes in Linear Sequences
Annals of the Institute of Statistical Mathematics 49, 271-283
Keywords:Change-point; Linear sequence; Dependent observations

Abstract: We discuss the asymptotic properties of some tests to detect possible changes in the mean of linear processes.

Horváth, Lajos and Kokoszka, piotr(1997-18)
The effect of long-range dependence on change-point estimators
Journal of Statistical Planning and Inference, 64, 57-81
Keywords: Change-point estimation; Long-range dependence; Fractional Brownian motion

Abstract: We study the asymptotic behaviour of a class of estimators of the time of change in the mean of Gaussian observations having long-range dependence. We prove that after a suitable normalization the estimators converge in distribution to functionals of fractional Brownian motion.

Horváth, Lajos; Hušková, Marie; Serbinowska, Monika(1997-19)
Estimators for the time of change in linear models.
Statistics 29, 109-130

Abstract: We derive an estimator for the time of change in a linear model when the regression coefficients and the variance may change. The asymptotic distribution of the estimator is computed. The method is used to construct confidence intervals for the time of change in gross domestic product in the United States between 1929 and 1967.

Jandhyala, V. K. and MacNeill, I. B. (1997-20)
Iterated partial sum sequences of regression residuals and tests for changepoints with continuity constraints
Journal of the Royal Statistical Society, Series B: Methodological, 59, 147-156
Keywords: Polynomial regression; Bayesian testing; Change point

Abstract: Iterated partial sum sequences of regression least squares residuals are defined and large sample properties of sequences of stochastic processes defined by these iterated partial sums are discussed. Also, finite sample properties of the iterated partial sum sequences are obtained. These include a property of least squares residuals of polynomial fits to equi-spaced data, namely the iterated partial sums sum to 0 provided that the order of iteration is not greater than the order of the polynomial, thus extending the well-known result that residuals sum to 0. Iterated partial sums are shown to play an important role in testing regression parameters for changes at unknown times under the constraint of continuity.

Janardan, K. G.; Mohan, N. R.; Vasudeva, R. and Hebbar, H. V.(1997-21)
A stochastic model with random change point for the study of oviposition tactics of weevils on beans
Journal of Statistical Planning and Inference, 61, 193-201
Keywords: Stochastic models; Probability generating function; Mean and frequency estimation

Abstract: Mitchell (1975, Callosobruchus maculatus (F)Ecology 56, pp. 696702.) experimentally investigated the oviposition behavior of bruchid beetles on mung beans. Janardan (1980, Math. BioSci. 50, pp. 231238) developed a stochastic model, assuming a rate λ until one egg is laid and a rate $\mu(< \lambda)$ on a bean having already one egg. From Mitchell's data it appears that rate of laying eggs (on a bean) is influenced by a random phenomenon by which the bettle changes the rate either after one egg is laid or after two eggs are laid on the same bean. Thus here we have developed a new stochastic model incorporating this random phenomenon. Estimation of the parameters of the model and measure of fit of the model to the experimental data are also provided

Joshi, S. N. And Maceachern, Steven N.(1997-22)

Isotonic maximum likelihood estimation for the change point of a hazard rate

Sankhya: The Indian Journal of Statistics 1997, Volume 59, Series A, Pt. 3, 392-407

Keywords: Isotonic model; Failure rate; Change point; Asymptotic distribution; Hypothesis testing, Confidence interval

Abstract: A hazard rate $\lambda(t)$ is assumed to be of the shape of the "first" part of a "bathtub" model, i.e., $\lambda(t)$ is non-increasing for $t < \tau$ and is constant for $t \geq \tau$. The isotonic maximum likelihood estimator of the hazard rate is obtained and its asymptotic distribution is investigated. This leads to the maximum likelihood estimator and a confidence interval for a new version of the change point parameter. Their asymptotic properties are investigated. Some simulations are reported.

Jose, C. T. and Ismail, B. (1997-23)

Estimation of jump points in nonparametric regression through residual analysis

Communications in Statistics: Theory and Methods, 26, 2583-2607

Keywords: Change point; Diagnostic; Kernel estimation

Abstract: This paper deals with the estimation of location and size of jumps in the equally spaced design nonparametric regression function itself and/or in its first derivative. The estimators are based on the analysis of residuals from the nonparametric kernel regression estimate. The proposed estimators are used to accommodate discontinuities in the nonparametric regression function estimate. The method is illustrated through simulation study.

Koo, Ja-Yong (1997-24)

Spline estimation of discontinuous regression functions

Journal of Computational and Graphical Statistics, 6, 266-284

Abstract: This article deals with regression function estimation when the regression function is smooth at all but a finite number of points. An important question is: How can one produce discontinuous output without knowledge of the location of discontinuity points? Unlike most commonly used smoothers that tend to blur discontinuity in the data, we need to find a smoother that can detect such discontinuity. In this article, linear splines are used to estimate discontinuous regression functions. A procedure of knot-merging is introduced for the estimation of regression functions near discontinuous points. The basic idea is to use multiple knots for spline estimates. We use an automatic procedure involving the least squares method, stepwise knot addition, stepwise basis deletion, knot-merging, and the Bayes information criterion to select the final model. The proposed method can produce discontinuous outputs. Numerical examples using both simulated and real data are given to illustrate the performance of the proposed method.

Koop, Gary and Poirier, Dale J. (1997-25)
Learning about the across-regime correlation in switching regression models
Journal of Econometrics, 78, 217-227
Keywords:Bayesian; Gibbs; identification; Roy model

Abstract: Vijverberg (1993, Journal of Econometrics 57, pp. 6989) considers the extended Roy/switching regression model of selectivity, focusing attention on the nonidentified correlation between the regime disturbances and describing how the positive definiteness of the covariance matrix implies that it is possible to learn about this covariance. In this paper, we show that this learning derives from prior dependence between identified and nonidentified parameters. Even though beliefs about the nonidentified covariance are updated, we show under reasonable a priori independence assumptions that beliefs about the partial correlation between disturbances, control of the switching index, are not updated. Empirical illustrations show how an exact Bayesian analysis can be carried out using Gibbs sampling and related techniques.

Korostelev, Alexander(1997-26)
Minimax large deviations risk in change-point problems.
Math. Methods Statist. 6 (1997), no. 3, 365-374

Abstract: The author considers the minimax large deviation risk for the change-point estimation in both continuous and discrete time models. In the continuous model, it is assumed that the observation process follows the It equation $\dot{X}^{(n)}(t) = AI_{(t>\theta)} + (\sigma/\sqrt{n})\dot{W}^{(n)}(t)$ for $-\infty < t < \infty$, where A and σ are given positive constants, $\dot{W}^{(n)}$ is a standard Gaussian

white noise, $I_{()}$ denotes the indicator function, and $\theta \in (-T, T)$ for a given positive constant T. For a fixed constant $c > 0$, define the minimax large deviation probability risk for an estimator $\hat{\theta}_n$ for the change point θ as

$$\beta_n(c) = \inf_{\hat{\theta}_n} \sup_{|\theta| < T} \tfrac{1}{n} \ln P_\theta^{(n)}(|\hat{\theta}_n - \theta| \geq c),$$

where $P_\theta^{(n)}$ denote the probability of $X^{(n)}$ corresponding to the true value of θ. It is shown that, for any $c < T, \lim_{n \to \infty} \beta_n(c) = A^2 c/(4\sigma^2)$.

In the discrete-time model, we assume that $X_i = X_i^{(n)}$ are observed at $t_i = t_i^{(n)} = (i/n)T$ for $|i| = 0, 1, 2, \cdots, n$. Assume that X_i has a probability density $p(x)$ for $t_i < \theta$ and a density $q(x)$ for $t_i \geq \theta$ with $|\theta| < T$. For $\alpha \in (0, 1)$, define $S(\alpha) = \ln \int p^\alpha(x) q^{1-\alpha}(x) dx$, and let $S^* = \min_\alpha S(\alpha) < 0$. Then it is shown that $\lim_{n \to \infty} \beta_n(c) = 2cS^*$. (Reviewed by Yanhong Wu)

Abstract : This paper concerns segmented multivariate regression models, models which have different linear forms in different subdomains of the domain of an independent variable. Without knowing that number and their boundaries, we first estimate the number of these subdomains using a modified Schwarz criterion. The estimated number of regions proves to be weakly consistent under fairly general conditions. We then estimate the subdomain boundaries ('thresholds') and the regression coefficients within subdomains by minimizing the sum of squares of the residuals. We show that the threshold estimates converge (at rates $1/n$ and $n^{-1/2}$, respectively, at the model's threshold points of discontinuity and continuity) and that the regression coefficients as well as the residual variances are asymptotically normal. The basic condition on the error distribution required for the veracity of our asymptotic results is satisfied by any distribution with zero mean and a moment generating function (having bounded second derivative around zero). As an illustration, a segmented bivariate regression model is fitted to real data and the relevance of the asymptotic results is examined via simulations.

Abstract: The change-point problem for normal regression models is considered here as the problem of choosing the hypothesis H_0 of no change or one of the hypotheses H_i that one or more parameters change after the ith observation. The observations are often associated with a known increasing sequence τ_i(for example, τ_i is the date of the ith observation). It then seems natural to introduce a quadratic loss function involving $(\tau_i - \tau_j)^2$ for selecting H_i instead of the true hypothesis H_j. A Bayes optimal invariant procedure is derived within such a framework and compared to previous proposals. When H_0 is rejected, large errors may arise in the estimation of the change point. To get around this difficulty another procedure is introduced whose main feature is to select one of the H_i's when H_0 is rejected only if there is sufficient evidence in favor of this choice.

Lingyun Ma(1997-29)
The asymptotic distributions of maximum likelihood ratio test and maximally selected χ^2-test in binomial observations
Journal of Statistical Planning and Inference, 65, 17-43

Abstract: We considered binomial distributed random variables whose parameters are unknown and some of those parameters need to be estimated. We studied the maximum likelihood ratio test and the maximally selected χ^2-test to detect if there is a change in the distributions among the random variables. Their limit distributions under the null hypothesis and their asymptotic distributions under the alternative hypothesis were obtained when the number of the observations is fixed. We discussed the properties of the limit distribution and found an efficient way to calculate the probability of multivariate normal random variables. Finally, those results for both tests have been applied to examples of Lindisfarne's data, the Talipes Data. Our conclusions are consistent with other researchers' findings.

Müller, Hans-Georg and Songb, Kai-Sheng(1997-30)
Two-stage change-point estimators in smooth regression models
Statistics and Probability Letters, 34, 323-335

Abstract: We consider a fixed design regression model where the regression function is assumed to be smooth, i.e., Lipschitz continuous, except for a point where it has only one-sided limits and a local discontinuity occurs. We propose a two-step estimator for the location of this change point and study its asymptotic convergence properties. In a first step, initial pilot estimates of the change point and associated asymptotically shrinking intervals which contain the true change point with probability converging to 1 are obtained. In the second step, a weighted mean difference depending on the assumed location of the change point is

maximized within these intervals and the maximizing argument is then the final change point estimator. It is shown that this estimator attains the rate $O_p(n^{-1})$ in the fixed jump case. In the contiguous case, the estimator attains the rate $O_p(n^{-1}\triangle_n^{-2})$, where $\triangle_n$ is the sequence of jump sizes which in this case is assumed to converge to 0. For the contiguous case an invariance principle is established. A sequence of appropriately scaled deviation processes is shown to converge to a two-sided Brownian motion with triangular drift.

Optimal Change-point Estimation in Inverse Problems

Abstract: We develop a method of estimating a change-point of an otherwise smooth function in the case of indirect noisy observations. As two paradigms we consider deconvolution and non-parametric errors-in-variables regression. In a similar manner to well-established methods for estimating change-points in non-parametric regression, we look essentially at the difference of one-sided kernel estimators. Because of the indirect nature of the observations we employ deconvoluting kernels. We obtain an estimate of the change-point by the extremal point of the differences between these two-sided kernel estimators. We derive rates of convergence for this estimator. They depend on the degree of ill-posedness of the problem, which derives from the smoothness of the error density. Analysing the Hellinger modulus of continuity of the problem we show that these rates are minimax

Piecewise convex function estimation: pilot estimators.

Abstract: This paper assumes that a regression function to be estimated consists of a small number of regions, where the function is either convex or concave. The nonparametric function estimator should obey this geometric constraint. The number and the location of convexity change points are unknown and have to be estimated. The author proposes a two-stage estimation procedure. In the first stage, the data are strongly smoothed to estimate the number of convex/concave regions (smoothing is based on kernel estimators or splines without any constraints). Then a bound can be obtained for a wrong change point and an asymptotic expression for the expected number of change points is given.

In the second stage, a nonparametric function estimator is applied which is based on an MISE-optimal data-adaptive smoothing parameter. In addition, the convexity/concavity constraints are imposed.

This paper builds on earlier work by H.-G. Müller[Scand. J. Statist. 12 (1985), no. 3, 221-232], E. Mammen [Ann. Statist. 19 (1991), no. 2, 741-759], Mammen, J. S. Marron and

N. J. Fisher[Probab. Theory Related Fields 91 (1992), no. 1, 115-132] and M. B. Mächler [Ann. Statist. 23 (1995), no. 5, 1496-1517]. (Reviewed by Theo Gasser)

Abstract: A special class of change-point models, where the change is defined as a shift of observation means, is considered. We show that such models can be transformed into a nonlinear regression problem. It is proven that M-estimators can localize the change point, and at the same time, consistently estimate the unknown parameters characterizing the change behavior. For a special class of continuous models we prove the asymptotic normality of M-estimators simultaneously estimating the change-point and the related parameters.

Abstract: In the asymptotic setting of the change-point estimation problem the limiting behavior of Bayes procedures for a general loss function is studied. It is demonstrated that for prior distributions approaching a geometric probability law, the difference between the Bayes estimator and the parameter converges in distribution to a nondegenerate random variable, whose explicit form is given. The sequence of minimum Bayes risks is shown to converge to its supremum, and the exact formula for this limit is obtained for the geometric prior. Special loss functions, in particular, the linex loss, are considered in detail. (Reviewed by Barbara H. Szyszkowicz)

Abstract: Medical researchers often desire to categorize patients into monotonic response groups based on the relationship between continuous variables. Isotonic regression fits consist of level sets of increasing value, for which the estimated response is constant. However, the number of level sets obtained is often large, preventing simple description. This article introduces two new nonparametric methods called reduced isotonic regression and reduced monotonic regression, the latter being a two-sided extension of the former for use when

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the direction of the trend is unknown. Using a backward elimination algorithm, the new procedures reduce the number of level sets by combining those whose values do not differ greatly. For the statistical relations examined here, the reduced monotonic method averaged at most 30% of the number of level sets obtained for isotonic regression. The method is illustrated with an example that examines the relationship between risk factors for survival among children with leukemia. In simulation studies, the reduced monotonic method fits the data as closely as alternative methods that combine isotonicity and smoothing, while improving greatly on isotonic regression. The method is also related to changepoint models of normally distributed sequences.

Smith, Philip J.(1997-36)
Power and Sample Size Considerations for Detecting Deviations from Secular Trends in Surveillance Surveys

Abstract: On-going public health surveillance is essential to the detection and monitoring of epidemics. We present statistical methods for determining the sample size that is required to detect unacceptable deviations from existing secular trends in prevalence with specified power. Between 1958 and 1969, a large study was conducted for providing surveillance of the prevalence of infection for a well-known disease. Data from this study indicated that the prevalence of infection in the mid-1960s increased significantly over previous secular trends among important demographic groups tested. These data are used to illustrate the statistical methods that we propose for detecting departures from existing temporal trends, estimating the year in which the changepoint occurred and specifying sample sizes for on-going active surveillance studies.

Sugakova, O. V.(1997-37)
A search for change points in a flow of independent observations.

Abstract: Let $\xi_1, \cdots, \xi_N$ be a sequence of independent observations such that the first $[\theta_1 N]$ observations have distribution of type 1, the further $[\theta_2 N]$ observations have distribution of type 2, etc. for an unknown (but finite) total number R of different types. The corresponding estimator is chosen to minimise $\sum_{i=1}^{N}(\pi_N(h_i, h_{i-1}) + \phi(\xi_i, h_i))$ over all $h = (h_1, \cdots, h_N)$ with components $h_i \in \{1, \cdots, R\}$ representing different options for the distribution of ξ_i. This estimator of h naturally yields an estimator for the change point vector $\theta = (\theta_1, \cdots, \theta_R)$. The author specifies conditions on the function ϕ and on the penalising term π_N that guarantee consistency of the corresponding estimator for θ and provides bounds for the speed of convergence. (Reviewed by Ilya S. Molchanov)

Andrews, Donald W. K., Lee, Inpyo and Ploberger, Werner (1996-1)
Optimal changepoint tests for normal linear regression
Journal of Econometrics, 70, 9-38
Keywords: Changepoint test; Linear regression; Multiple changepoints; Optimal test;
Structural change test

Abstract: This paper determines a class of finite-sample optimal tests for the existence of a changepoint at an unknown time in a normal linear multiple regression model with known variance. Optimal tests for multiple changepoints are also derived. It is shown that the results cover some models of cointegration. Power comparisons of several tests are provided based on simulations.

Csörgö, Miklos and Horvath, Lajos(1996-2)
A note on the change-point problem for angular data
Statistics and Probability Letters, 27, 61-65
Keywords: Angular data; Rank tests; Standardized statistics

Abstract: Limit theorems are obtained for test statistics that detect changes in the distribution of angular data.

Emad-Eldin A. A. Aly and Sana S. BuHamr(1996-3)
Rank tests for two change points
Computational Statistics and Data Analysis, 22, 363-372
Keywords: Brownian bridge; Limit theorems; Rank tests; Monte Carlo powers

Abstract: We consider the problem of testing the null hypothesis of no change against two-change points alternatives in a series of independent observations. We propose some Kruskal-Walis-type tests and give their asymptotic null distributions. We also give approximations of their limiting critical values and tables of small sample Monte Carlo critical values. We conducted Monte Carlo simulation studies to compare the powers of the proposed tests with their competitors.

Farrell,A. P.and Thorarensen, H.(1996-4)
Inference on Segmented Polynomial Models
Biometrics, Vol. 52, No. 1., 321-327.
Keywords: Change-point; Confidence curve; Curvature; Likelihood ratio; Wald test.

Abstract: Segmented polynomial regression models with unknown change-points are used in a wide variety of biological settings. The application that stimulated this work uses a segmented polynomial model to examine the optimal hematocrit hypothesis. We discuss

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problems in the fitting of these models and compare, by simulation, two methods of inference in these models: that based on the chi-squared approximation to the distribution of the likelihood ratio statistic and that based on the asymptotic normality of the least-squares estimates. The results show that, of the two, only the likelihood ratio statistic produces reliable inference concerning the change-point.

Ghosh, Jayanta K.; Joshi, Shrikant N. and Mukhopadhyay, Chiranjit(1996-5)
Asymptotics of a Bayesian approach to estimating change-point in a hazard rate.
Communication in Statistics. Theory and Methods 25, 3147-3166

Abstract: Let $X_1, \cdots, X_n$ be a random sample on a lifetime random variable with the hazard rate function $h(t) = a$ if $t \le \tau$, $h(t) = b$ if $t > \tau$, where $a > b > 0$, $\tau > 0$, and the parameters τ and (a, b) are assumed to be independent a priori, with τ having a uniform prior on $0 < t_1 < t_2 < \infty$, while the prior of (a, b) is assumed to be smooth, as spelled out in the paper. It is shown that (i) the values of the marginal posterior density of τ, outside an n^{-1} neighborhood of a sample value τ_0 of τ from this model, are negligible compared to τ_0, (ii) the integral of the marginal posterior density of τ outside an n^{-1} neighborhood of τ_0 can be made arbitrarily small, (iii) the posterior distribution of (a, b) can be approximated by a normal distribution, (iv) the random variables a, b and τ are a posteriori independent, and (v) one can approximate the posterior mean and variance of (a, b) by easily computable quantities. The accuracies of the mentioned approximation results are studied via simulation. (Reviewed by Barbara H. Szyszkowicz)

Ghorbanzadeh, Dariush(1996-6)
Detection of random change point in one-parameter exponential families
Applied Mathematics and Computation, 77, 167-177

Abstract: We consider a sequence $X_1, X_2, \cdots, X_n$ of independent random variables which are susceptible to changing their distribution after the $[nT]$ first observations where T is a random variable of a distribution known with support in $]0, 1[$. The object of this work is to detect the eventual change of distributionfor thatwe study the performance of a test based on the statistic of Log-likelihood. It shows that when the number of observations gets larger, the distribution of the statistic Log-likelihood behaves like that of an affine function of the Brownian motion; this allows, by using the concept of contiguity in the sense of LeCam, the evaluation of the asymptotic power function of the test.

Giraitisa, Liudas; Leipusb, Remigijus and Surgailisc, Donatas(1996-7)
The change-point problem for dependent observations
Journal of Statistical Planning and Inference, 53, 297-310

Keywords: Change-point problem; Empirical processes; Long-memory; Kolmogorov-Smirnov test

Abstract: We consider the change-point problem for the marginal distribution function of a strictly stationary time series. Asymptotic behavior of Kolmogorov-Smirnov type tests and estimators of the change point is studied under the null hypothesis and converging alternatives. The discussion is based on a general empirical process' approach which enables a unified treatment of both short-memory (weakly dependent) and long-memory time series. In particular, the case of long-memory moving-average process $X_j = \Sigma_{s \leq j} b_{j-s} \xi_s$ studied, using the recent results of Giraitis and Surgailis (1994, Lithuanian Math. J. 32, pp. 2038).

Gombaya, Edit and Horváth, Lajos(1996-8)
Approximations for the time of change and the power function in change-point models

Keywords: Change-point model; Exponential family; Maximum likelihood estimator and test; Weak convergence; Partial sums

Abstract: Assuming that the observations are from an exponential family we obtain the asymptotic distribution of the maximum likelihood estimator of the time of change. We also prove that the maximum likelihood ratio test is asymptotically normal, if there is a change in the parameters at an unknown time.

Gombaya, Edit and Horváth, Lajos(1996-9)
Approximations for Maximum Likelihood Tests in Change-Point Models

Keywords: Likelihood ratio processes; Maximum likelihood estimators; Weighted approximations; Extreme value; Brownian bridge;

Abstract: We study the asymptotics of maximum-likelihood ratio-type statistics for testing a sequence of observations for no change in parameters against a possible change while some nuisance parameters remain constant over time. We obtain extreme value as well as Gaussian-type approximations for the likelihood ratio. We get necessary and sufficient conditions for the weak convergence of supremum and L_p-functionals of the likelihood ration process. We also approximate the maximum likelihood ratio with Ornstein-Uhlenbeck processes and obtain bounds for the rate of approximation. We show that the Ornstein-Uhlenbeck approach is superior to the extreme value limit in case of moderate sample sizes.

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Tests and estimators for the change point problem based on M-statistics.

Abstract: Let X_i be independent observations with distribution function $F(x, \theta_i)$ for $i = 1, 2, \cdots, n$. The author considers the following change-point problem: Test $H_0: \theta_1 = \cdots = \theta_m (= \theta_0)$ against $H_1: (\theta_0 =) \theta_1 = \cdots = \theta_m \neq \theta_{m+1} = \cdots = \theta_n (= \theta_0 + \delta_n)$, for some unknown θ_0, δ_n and the change point $1 \leq m < n$. Three test statistics based on the M-estimator θ_n satisfying $\sum_1^n \psi(x_j; \theta_n) = 0$ are considered:

$$T_{n1}(\psi) = \max_{1 \leq k \leq n} \left\{ n^{-1/2} \left| \sum_{i=1}^k S_{n,k}(\psi) \right| \right\},$$

$$T_{n2}(\psi) = \max_{1 \leq k \leq n} \left\{ \left(\frac{n}{k(n-k)} \right)^{1/2} |S_{n,k}(\psi)| \right\}, \text{and}$$

$$T_{n3}(G, \psi) = \max_{G \leq k \leq n} \{ G^{-1/2} |S_{n,k}(\psi) - S_{n,k-G}(\psi)| \},$$

where $S_{n,k}(\psi) = \sum_1^k \psi(x_j; \theta_n)$. The corresponding estimators based on the three statistics are also defined. Under suitable conditions, the limiting null distributions for the three test statistics are obtained, and the limiting distributions for the change-point estimators are also derived. (Reviewed by Yanhong Wu)

Estimation of a change in linear models

Abstract: An estimator of the change point in the linear model is proposed and its asymptotic properties (the rate of consistency and the limit distribution) are derived. This estimator is then used to construct estimators for the magnitude of the change of the regression parameters and of the scale. Their asymptotic properties are studied too.

Joseph, Lawrence; Vandal, A. C. and Wolfson, D. B.(1996-12)
Estimation in the Multipath Change Point Problem for Correlated Data

Abstract: In many experiments, several measurements on the same variable are taken over time, a geographic region, or some other index set. It is often of interest to know if there

has been a change over the index set in the parameters of the distribution of the variable. Frequently, the data consist of a sequence of correlated random variables, and there may also be several experimental units under observation, each providing a sequence of data. A problem in ascertaining the boundaries between the layers in geological sedimentary beds is used to introduce the model and then to illustrate the proposed methodology. It is assumed that, conditional on the change point, the data from each sequence arise from an autoregressive process that undergoes a change in one or more of its parameters. Unconditionally, the model then becomes a mixture of nonstationary autoregressive processes. Maximum-likelihood methods are used, and results of simulations to evaluate the performance of these estimators under practical conditions are given.

Koul, Hira L. (1996-13)
Asymptotics of some estimators and sequential residual empirical in nonlinear time series
The Annals of Statistics, 24, 380-404
Keywords: Asymptotic uniform linearity; minimum distance estimators; threshold autoregression models; asymptotically distribution free; Change point; Goodness-of-fit

Abstract: This paper establishes the asymptotic uniform linearity of M- and R-scores in a family of nonlinear time series and regression models. It also gives an asymptotic expansion of the standardized sequential residual empirical process in these models. These results are, in turn, used to obtain the asymptotic normality of certain classes of M-, R- and minimum distance estimators of the underlying parameters. The classes of estimators considered include analogs of Hodges-Lehmann, Huber and LAD (least absolute deviation) estimators. Some applications to the change point and testing of the goodness-of-fit problems in threshold and amplitude-dependent exponential autoregression models are also given. The paper thus offers a unified functional approach to some aspects of robust inference for a large class of nonlinear time series models.

Koziol, James A. and Wu, Shu-Chen(1996-14)
A review of nonparametric tests for changepoint problems, with application to a recombinant drug therapy clinical trial
Journal of Biopharmaceutical Statistics,6(4), 425-441
Keywords: Erythropoietin; Weak convergence of empirical processes; Bootstrap; Confidence intervals.

Abstract: We review general classes of nonparametric tests for the changepoint problem. We extend these tests to analyze the results of a clinical trial examining whether pretreatment of surgical patient with erythropoietin reduces their subsequent need for blood transfusions. We find a changepoint in baseline hemoglobin levels, below which pretreatment with erythropoietin appears partially beneficial relative to placebo.

Lee, Chung-Bow(1996-15)

Nonparametric multiple change-point estimators

Statistics Probability Letters, 27, 295-304

Keywords: Change points; Nonparametric estimation; Weighted empirical distribution

Abstract: A simple method is proposed to detect the number of change points in a sequence of independent random variables with no distributional assumption. The method is based on the weighted empirical measures over a window of observations and then runs the window over the full extent of the data. We find that the class of estimators based on our method will be consistent a.s. (almost surely) to the true number of change points and the difference between the true location of change points and the estimated location will be of order $O(\log n)$ a.s. Three examples are investigated by the proposed method.

Loader, Clive R. (1996-16)

Change point estimation using nonparametric regression

The Annals of Statistics, 24, 1667-1678

Keywords: 62G07 (MSC1991); Change point; Nonparametric regression; Boundary crossing

Abstract: We consider a regression model in which the mean function may have a discontinuity at an unknown point. We propose an estimate of the location of the discontinuity based on one-side nonparametric regression estimates of the mean function. The change point estimate is shown to converge in probability at rate $O(n^{-1})$ and to have the same asymptotic distribution as maximum likelihood estimates considered by other authors under parametric regression models. Confidence regions for the location and size of the change are also discussed.

Mira, A. and Petrone, S.(1996-17)

Bayesian hierarchical nonparametric inference for change-point problems.

Bayesian statistics, 5 (Alicante, 1994), 693–703, Oxford Sci. Publ., Oxford Univ. Press, New York, 1996.

Abstract: Bayesian nonparametric inference for a nonsequential change-point problem is studied. We use a mixture of products of Dirichlet processes as our prior distribution. This allows the data before and after the change-point to be dependent, even when the change point is known. A Gibbs sampler algorithm is also proposed in order to overcome analytic difficulties in computing the posterior distributions of interest, some of which have support on the space of all distribution functions

Mitra, Murari and Basu, Sujit K.(1996-18)

Shock models leading to non-monotonic ageing classes of life distributions.

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Abstract: Consider the survival function $\overline{H}(t)$, $t \geq 0$, of a single device subject to shocks occurring randomly in time according to a homogeneous Poisson process with rate λ. Suppose $N(t)$ denotes the number of shocks the device experiences in the time interval $(0, t]$ and let $\overline{P}_k$ be the probability that the device survives the first k shocks, $k = 0, 1, 2, \cdots$. It is further assumed that the $\overline{P}_k$'s satisfy the condition $1 = \overline{P}_0 \geq \overline{P}_1 \geq \cdots$. Under the above setup, the probability $\overline{H}(t)$ of the device surviving beyond time $t \geq 0$ is given by $\overline{H}(t) = \sum_{k=0}^{\infty} P[N(t) = k]\overline{P}_k = \sum_{k=0}^{\infty} \{\exp(-\lambda t)(\lambda t)^k / k!\}\overline{P}_k$. It has been known from earlier works which focused only on monotonic aging phenomena, that (a) if the sequence $\{\overline{P}_k\}$ possesses what is called the discrete IFR property, then $\overline{H}(t)$ is also an IFR survival function; and (b) when $\{\overline{P}_k\}$ has a discrete NBUE [NWUE] property, then $\overline{H}(t)$ is also continuous NBUE [NWUE]. Analogous results for the HNBUE [HNWUE] class are also available.

The present paper deals with non-monotonic aging classes. It is proved that if the sequence $\{\overline{P}_k\}$ possesses a discrete NWBUE (new worse then better than used in expectation) then the continuous NWBUE property is inherited by $\overline{H}(t)$. Similar results are shown for BFR (bathtub failure rate) distributions under suitable conditions. These results then enable one to apply the change-point estimation techniques for NWBUE and BFR distributions [see M. Mitra and S. K. Basu, Ann. Inst. Statist. Math. 47 (1995), no. 3, 483-491] in estimating the change-point from lifetime data for components subject to the above-mentioned shock models. (Reviewed by Nabendu Pal)

Ogden, Todd and Parzen, Emanuel (1996-19)
Change-point approach to data analytic wavelet thresholding

Abstract: Previous proposals in data dependent wavelet threshold selection have used only the magnitudes of the wavelet coefficients in choosing a threshold for each level. Since a jump (or other unusual feature) in the underlying function results in several non-zero coefficients which are adjacent to each other, it is possible to use change-point approaches to take advantage of the information contained in the relative position of the coefficients as well as their magnitudes. The method introduced here represents an initial step in wavelet thresholding when coefficients are kept in the original order.

Ogden, Todd and Parzen, Emanuel (1996-20)
Data dependent wavelet thresholding in nonparametric regression with change-point applications

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*Keywords:*Wavelet thresholding; Change-point problems; Nonparametric regression; Likelihood ratio

Abstract: In one version of the change-point problem one has independent observationsY_1,..., Y_n which have the same mean under the null hypothesis. This problem is transformed into a nonparametric regression problem by considering each Y_i to have mean $f(i/n)$, and then estimating the function f on [0,1] from the data. Wavelets provide a useful tool for estimating such a function, which may have multiple abrupt jumps. A data dependent technique for selecting a threshold with which to shrink empirical wavelet coefficients is introduced. The technique, based on standard statistical tests of hypotheses, is shown to give good results both when the underlying function is constant, and when it undergoes multiple abrupt changes. By adjusting the level of the tests of significance, it is possible to control the smoothness of the resulting estimator, allowing one to give preference to good expected MSE performance or to favor good visual appearance of the estimator in exploratory data analysis settings.

Razzaghi, Mehdi and Govindarajulu, Z.(1996-21)
A locally optimal test for the change-point.
Journal of Applied Statistical Sciences, 4 , 159-173.

Abstract: In this paper, the authors consider the change-point detection problem where $x_1, \cdots, x_\theta$ follow the density $f_0(x)$ and $x_{\theta+1}, \cdots, x_n$ follow the mixture density $(1-\alpha)f_0(x) + \alpha f_1(x)$ with the change point θ. This introduces an extra parameter α. By assuming that θ has a truncated geometric prior distribution function, a locally optimal test is derived by letting $\alpha \to 0$ in the mixture likelihood. Normal approximation and simulation studies are considered for studying the power behaviour by using the normal and exponential distributions. (Reviewed by Yanhong Wu)

Rukhin, A. L.(1996-22)
Change-point estimation: linear statistics and asymptotic Bayes risk.
Math. Methods Statist. 5 (1996), no. 4, 424-442.

Abstract: In the retrospective setting of the change-point estimation problem, the author introduces a class of fairly simple linear estimators. The Bayes estimator within this class is determined and its mean squared error is compared with that of the unbiased estimator. The author also obtains the explicit formulas for the Bayes estimator and its quadratic risk for the uniform distribution. In addition, he establishes the asymptotic admissibility and the minimaxity of the unbiased linear estimator. Related work can be found in papers by D. V. Hinkley [Biometrika 57 (1970), 1-17] and G. W. Cobb [Biometrika 65 (1978), no. 2, 243-251]. (Reviewed by Jian-Lun Xu)

Rukhin, A. L.(1996-23)
Linear Statistics in Change-Point Estimation and Their Asymptotic Behaviour
The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 24, 503-515.
Keywords: Bayes risk; Change-point problem; Integral operator; Linear estimator; Prior distribution; Quadratic loss.

Abstract: The limiting behaviour of Bayes procedures in the asymptotic setting of the change-point estimation problem is studied. It is shown that the distribution of the difference between the Bayes estimator and the parameter converges to the distribution of a fairly complicated random variable. A class of linear statistics is introduced, and the form of the Bayes estimator within this class is deduced. The asymptotic properties of this linear estimator are investigated in two different settings for the prior distribution.

Rukhin, A. L.(1996-24)
Change-point analysis as a multiple decision problem.
Statistical Decisions 14, 103-114.

Abstract: The author shows that, in the classical setting of the change-point estimation problem, a positive limit of the minimum Bayes risk for the uniform prior exists for any loss. Its explicit form and some inequalities are derived for the zero-one loss function. The nature of the minimum Bayes risk is shown to be related to the multiple decision problem and to information-type characteristics. (Reviewed by M. Hušková)

Rukhin, A. L.(1996-25)
The rates of convergence of Bayes estimators in change-point analysis
Statistics and probability letters, 27, 319 -329
Keywords: Bayes risk; Change-point problem; Convergence rate; Geometric distribution; Maximum likelihood estimator; Spitzer's formula; Zero-one loss function

Abstract: In the asymptotic setting of the change-point estimation problem the limiting behavior of Bayes procedures for the zero-one loss function is studied. The limiting distribution of the difference between the Bayes estimator and the parameter is derived. An explicit formula for the limit of the minimum Bayes risk for the geometric prior distribution is obtained from Spitzer's formula, and the rates of convergence in these limiting relations are determined.

Stute, Winfried(1996-26)
Changepoint problems under random censorship.
Statistics 27, 255-266.

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Abstract: Let $X_1, \cdots, X_n$ be independent random variables. We assume that $X_1, \cdots, X_{[n\theta]}$ have distribution function F_1 and $X_{[n\theta]+1}, \cdots, X_n$ have distribution function F_2, $F_1 \neq F_2$. The index θ is the change point of the distribution. The case $\theta = 0$ corresponds to "no change". We observe $(Z_1, \delta_1), \cdots, (Z_n, \delta_n)$, with $Z_i = \min\{X_i, Y_i\}$ and $\delta_i = I\{X_i \leq Y_i\}$, where $Y_1, \cdots, Y_n$ is a sequence of independent, identically distributed random variables. Let ${}^t_s\tilde{H}_n(x) = n^{-1} \sum_{[ns]+1 \leq i \leq [nt]} I\{Z_i \leq x, \delta_i = 1\}$ and $H_n(x) = n^{-1} \sum_{1 \leq i \leq n} I\{Z_i \leq x\}$. We estimate θ by $\hat{\theta}_n$, the point where $|r_n(t)|$ reaches its maximum. The process $r_n(t)$ is given by

$$r_n(t) = \int_{-\infty}^{T} \int_{-\infty}^{T} \frac{K(x,y) \, {}^t_0\tilde{H}(dx) \, {}^1_t\tilde{H}(dy)}{(1 - H_n(x-))(1 - H_n(y-))},$$

where K denotes a bounded antisymmetric kernel. Let $1 - F_0 = \theta(1 - F_1) + (1 - \theta)(1 - F_2)$. If

$$\int_{-\infty}^{T} \int_{-\infty}^{T} \frac{K(x,y) F_1(dx) F_2(dy)}{(1 - F_0(x-))(1 - F_0(y-))} \neq 0,$$

then $|\hat{\theta}_n - \theta| = O(\log n / n)$ with probability one. If $\theta = 0$ (no change), then $n^{1/2} r_n(t) \overset{\mathbf{D}}{[0,1]} \to \to \sigma B^0(t)$, where $B^0(t)$ is a Brownian bridge and σ is a constant. (Reviewed by Lajos Horvth)

Ventera, J. H. and Steelb, S. J.(1996-27)
Finding multiple abrupt change points
Computational Statistics and Data Analysis, 22, 481-504
Keywords: Change points; Hypothesis testing; Non-parametric tests; Computer intensive methods

Abstract: The problem of identifying multiple abrupt change points in a sequence of observations is approached via hypothesis testing. The null hypothesis of no change points is tested and if it is rejected the number of change points present are indicated by the procedures introduced here. These procedures allow an arbitrary number of change points and patterns of changes. Apart from being computationally intensive, they are easy to apply and interpret and do not require a great deal of user input.

Darkhovskiĭ, B. S.(1995-1)
Retrospective detection of a "change point" in some regression-type models.
Teor. Veroyatnost. i Primenen. 40 (1995), no. 4, 898–903; translation in Theory Probab. Appl. 40 (1995), no. 4, 748–753 (1996)
Keywords: Regression model; The change-point of a random sequence

Abstract: Two problem for detecting the change-point of the mean of a random sequence are considered. The first problem examines the points of discontinuity of the kth derivative of

the mean, and the second problem analyze the moment of instant change of the coefficients by means of a linear combination of known function.

Ferger, D.(1995-2)
Nonparametric tests for nonstandard change-point problems
The Annals of Statistics, 23, 1848-1861.

Abstract: We consider independent random elements $X_1, \ldots, X_n, n \in \mathbb{N}$, with values in a measurable space (X, B) so that $X_1, \ldots, X_{[n\theta]}$ have a common distribution ν_1 and the remaining $X_{[n\theta]+1}, \ldots, X_n$ have a common distribution $\nu_2 \neq \nu_1$, for some $\theta \in (0, 1)$. The change point θ as well as the distributions are unknown. A family of tests is introduced for the nonstandard change-point problem $H_0 : \theta \in \Theta_0$ versus $H_1 : \theta \notin \Theta_0$, where Θ_0 is an arbitrary subset of $(0, 1)$. The tests are shown to be asymptotic level-α tests and to be consistent on a large class of alternatives. The same holds for the corresponding bootstrap versions of the tests. Moreover, we present a detailed investigation of the local power.

Kiuchi, Amy S.;Hartigan, J. A.;Holford, Theodore R.; Rubinstein, Pablo and Stevens, Cladd E.(1995-3)
Change Points in the Series of T4 Counts Prior to AIDS
Biometrics, Vol. 51, No. 1., 236-248

Abstract:The absolute number of $T4$ cells has been established as an important clinical marker of disease progression to acquired immunodeficiency syndrome (AIDS) in persons infected with human immunodeficiency virus (HIV). Series of $T4$ counts are analyzed from the 131 homosexual men who entered the New York Blood Center Study in 1984, mostly seropositive for HIV, and who developed AIDS as participants by 1990. These series exhibit a gradual decline of the $\log(T4)$ count followed by a more rapid decline close to the time of the development of AIDS. Empirical Bayes and hierarchical Bayes change point models are proposed to estimate the distribution of the time before AIDS when this rapid decline begins. Results using the EM Algorithm and Markov chain Monte Carlo indicate that the mean change point occurs approximately 1 year before diagnosis with a standard deviation of 9 months. Detection of a change point may indicate that an AIDS diagnosis is increasingly likely for an individual HIV-positive but AIDS-free.

Lee, Chung-Bow(1995-4)
Estimating the number of change points in a sequence of independent normal random variables
Statistics and Probability Letters, 25, 241-248
Keywords: Change points; Schwarz's criterion

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Abstract: This work concerns the detection of the number of change points in a sequence of independent normal random variables. An estimator is proposed through some criterion, $SC(k)$, of maximizing the log likelihood function with some penality term. The criterion is similar to that given by Yao (1988, Statist. Probab. Lett. 6, pp. 181187.) only with a different penality term. An interesting result is that, under mild assumptions, the criterion $SC(k)$ will be monotonically increasing in $k \leq k_0$ but decreasing in $k \geq k_0$ with probability approaching 1 as $n \rightarrow \infty$. Thus, weak consistency of the estimator based on the criterion can easily be obtained.

Parzen, Emanuel(1995-5)
Comparison change analysis approach to changepoint estimation.
Applied change point problems in statistics (Baltimore, MD, 1993), 57-79, Nova Sci. Publ., Commack, NY, 1995.

Abstract: The aim of this paper is to show how to apply our theory of change analysis first introduced in our 1992 paper [in Nonparametric statistics and related topics (Ottawa, ON, 1991), 3–15, North-Holland, Amsterdam, 1992]. It proposes a strategy for change analysis of indexed data $Y_1, \cdots, Y_n$. A non-parametric analysis forms for each $m = 1, \cdots, n - 1$ a function on $0 < u < 1$, called a change PP plot, which non-parametrically compares the distribution of the data up to time m with the distribution of all the data. Diagnostics of the change PP plot are computed by linear functionals (which are non-parametric statistics to test for equality of distributions) using data score functions which are orthonormal Legendre polynomials of degrees $k = 1, \cdots, 8$. For each value one obtains a function of m, called a score change process, which is a CUSUM which can be used to test and estimate change over time m (in a non-parametric statistic for equality of the distributions of the data before and after m). A parametric analysis is outlined using Fisher-score change processes. We call our approach to change analysis a comparison change approach, because it uses concepts of comparison distribution and comparison density functions. In addition to providing methods of change modeling it provides a unified framework for understanding the standard theory of parametric and non-parametric inference. Elsewhere [J. Amer. Statist. Assoc. 74 (1979), no. 365, 105–131; in Statistical data analysis and inference (Neuchâtel, 1989), 71–84, North-Holland, Amsterdam, 1989; in Proceedings of Computer Science–Statistics INTERFACE '90, 235–242, Springer, New York, 1991; per bibl.] we discussed various aspects of a unified framework relating diverse statistical methods. "To begin with we assume that the sequence of random variables $Y_1, \cdots, Y_n$ consists of independent random variables (but we expect to be able to extend the methods to a time series). A change analysis approach to modeling considers the possibility that the observations are heterogeneous (not identically distributed) rather than homogeneous (identically distributed). The diverse applications of methods for detecting and modeling change can be found elsewhere [see Detection of abrupt changes in signals and dynamical systems, edited by M. Basseville and A. Benveniste, Springer, Berlin, 1986; A. N. Kolmogorov, Yu. V. Prokhorov and A. N. Shiryaev, Proc. Steklov Inst. Math. 182 (1990), 1–21; Zbl 706.62076

Sinha, Bimal; Rukhin, Andrew and Ahsanullah, Mohammed(1995-6)
Applied change point problems in statistics.

Contents: Daniel Barry and J. A. Hartigan, Change points in 0-1 sequences, with an application to predicting divisional winners in major league baseball (1–14); Marie Huškov, Nonparametric procedures for detecting a change in simple linear regression models (15–36); I. B. MacNeill and Y. Mao, Change-point analysis for mortality and morbidity rate (37–55); Emanuel Parzen, Comparison change analysis approach to changepoint estimation (57–79); Adrian E. Raftery, Change point and change curve modeling in stochastic processes and spatial statistics (81–102); Pranab K. Sen [Pranab Kumar Sen1], Some change-point problems in survival analysis: relevance of nonparametrics in applications (103–122); M. S. Srivastava, Comparison of CUSUM and EWMA procedures for detecting a shift in the mean or an increase in the variance (123–146); S. Zacks, Sequential testing of reliability system with change points (147–161); Lyle Broemeling, Bayesian analysis of threshold autoregressions (163); Xiaolong Luo, Bruce W. Turnbull, Haiyan Cai and Larry C. Clark, Regression for censored survival data with lag effects (163–164); R. Douglas Martin, Tree based robust Bayesian estimation of time series structural models (164); R. H. Riffenburgh, Detecting a point of any change in a time series (165–166); Jianming Ye, Deconvolution and jump detection by the method of local approximation (166–167).

Yashchin, Emmanuel(1995-7)
Estimating the Current Mean of a Process Subject to Abrupt Changes

Abstract: This article discusses estimation of the current process mean in situations in which this parameter is subject to abrupt changes of unpredictable magnitude at some unknown points in time. It introduces performance criteria for this estimation problem and discusses in detail the relative merits of several estimation procedures. I show that an estimate based on exponentially weighted moving average of past observations has optimality properties within the class of linear estimators, and I propose alternative estimating procedures to overcome its limitations. I consider two primary types of estimation procedures, Markovian estimators, in which the current estimate is obtained as a function of the previous estimate and the most current data point, and adaptive estimators, based on identification of the most recent changepoint. We give several examples that illustrate the use of the proposed techniques

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Zhan, Z.; C. B. Dean; R. Routledge; P. Gallaugher; Hušková, M. (1995-8)

Nonparametric procedures for detecting a change in simple linear regression models

Applied Change Point Problems in Statistics, Nova Science Publisher, Inc. Edited by Bimal Sinha, Andrew Rukhin and Mohammed Ahsanullah

Abstract: The paper studies procedures for detecting changes in simple linear models based on ranks (R-procedures) or on M-estimators and related statistics (M-procedures). Both test procedures for detecting a change and estimators of the change point(s) are described, their limit properties are formulated and some recommendations for practical applications, based on theoretical and simulation results are made.

Antoch, J. and Hušková, M.(1994-1)

Procedures for the detection of multiple changes in series of independent observations.

Asymptotic statistics (Prague, 1993), 3–20, Contrib. Statist., Physica, Heidelberg, 1994.

Abstract: In this paper, the problem of detecting change points in a series of n independent observations from an L_2-generated regression model is considered. Emphasis is on the problem with more than one change, distinguishing between continuous and abrupt changes. The authors investigate test procedures based on M-estimators, whose asymptotic properties are studied. An extensive simulation study illustrates their performance in some standard situations. Reviewed by Hartmut Milbrodt

Azari, A. S. and Tsai, Chih-Ling (1994-2)

Non-parametric regression approach for model checking on the two-phase regression problem

Journal of Applied Statistics, 21, 597-606
Keywords: Goodness-of-fit

Abstract: A non-parametric approach is derived for assessing the goodness of fit for the two-phase regression problem. It is shown that the validity of the constant error variance assumption and the constraint of the continuity of change in two-phase regression can be examined. This method is also used to test nested two-phase regression models. Three numerical examples are presented.

Bhattacharya, P. K. (1994-3)

Some aspects of change-point analysis
Change-point problems, 28-56

Carlstein, Edward G. (ed.), Müller, Hans-Georg (ed.) and Siegmund, David (ed.) Institute of Mathematical Statistics (Hayward)
Keywords: Two phase regression; Quality control; Two-sided random walk

Abstract: Change-points divide statistical models into homogenous segments. Inference about change-points is discussed here in the context of testing the hypothesis of "no change", point and interval estimation of a change-point, changes in nonparametric models, changes in regression, and detection of change in distribution of sequentially observed data.

Chang, I-Shou; Chen, Chen-Hsin and Hsiung, Chao A.(1994-4)

Estimation in change-point hazard rate models with random censorship.

Change-point problems (South Hadley, MA, 1992), 78-92, IMS Lecture Notes Monogr. Ser., 23, Inst. Math. Statist., Hayward, CA, 1994.

Abstract: Hazard rate models with a change-point allowing for random censorship are considered. An estimator of the change-point is proposed by examining a functional of Nelson-Aalen type estimator in the context of counting processes. Consistency and asymptotic distribution of the proposed estimator are established by martingale inequalities and poisson approximation respectively. The performance of the proposed estimator is compared with that of a constrained maximum likelihood estimator using simulation. Robustness of the proposed estimator is also discussed.

Chu, C. K. (1994-5)

Estimation of change-points in a nonparametric regression function through kernel density estimation

Communications in Statistics: Theory and Methods, 23,3037-3062
Keywords: Central limit theorem

Abstract: In a stable system, the process under study usually has a constant mean. An action applied to the system may cause an effect instantly or gradually on the mean of the process. After a period of time, the effect of the action may become stable and it may drive the mean of the process to another constant instantly or gradually. In this case, the mean of the process before it is affected by the action and that after it becomes constant again are estimated. The times at which the mean of the process is affected by the action and when it becomes constant again are also estimated. The estimatores for these quantities are analyzed by central limit theorems(CLT) and strong convergence rates (SCR). The CLT for the estimators of the above two means are the same as those for the sample means constructed in the case that the above two times are known in advance. The smoothness of the change in the mean function has no effect on both the CLT and SCR for the estimators of the two means. But it does have an effect on the estimators for the above two times. This

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effect is quantified precisely through the order of the SCR of the estimators. Simulation studies demonstrate that the asymptotic results hold for reasonable sample sizes.

Cohen, Arthur and Kushary, Debashis (1994-6)
Adaptive and unbiased predictors in a change point regression model
Statistics and Probability Letters, 20, 131-138
Keywords: Generalized Bayes estimators; Uninformative prior distributions; Penalty factor; Bias; Mean squared error

Abstract: Consider the problem of prediction in a change point regression model. That is, assume a simple linear regression model holds for all x (independent variable) less than γ_k, and a different simple linear regression model holds for $x > \gamma_k$. However, γ_k, the change point, is unknown but can be one of m possible values ($\gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_m$). For $x = x_0 > \gamma_m$; we want to predict a future value of the dependent variable. Adaptive predictors (estimators) and unbiased predictors (estimators) are studied. The adaptive estimator is one which estimates by least squares for the kth model, provided the residual sum of squares for the kth model is smallest. Some theoretical justification for the adaptive estimator is given through a decision theory formulation. A small simulation study comparing the adaptive estimator and unbiased estimator is also offered.

Darkhovski, Boris S. (1994-7)
Nonparametric methods in change-point problems: A general approach and some concrete algorithms Change-point problems, 99-107
Carlstein, Edward G. (ed.), Müller, Hans-Georg (ed.), and Siegmund, David (ed.)
Institute of Mathematical Statistics (Hayward) Keywords: Dependence; Estimation; gradual change; Kolmogorov-Smirnov; Mixing; Optimality; Two-phase regression

Abstract: A general approach to change-point problems is proposed. This approach is based upon two ideas. The first idea is that any change-point problem can be reduced to the problem of detection of changes in the mean value of some new sequences. The second idea is that the nonparametric family of Kolmogorov Smirnov type statistics can be used for change-point detection in these sequences. This general approach is implemented in two cases: (a) the problem of gradual change-point detection, and (b) change-point detection in two-phase regression model.

Dmbgen, Lutz(1994-8)
Confidence sets for a change-point via randomization methods.
Change-point problems (South Hadley, MA, 1992), 108–129, IMS Lecture Notes Monogr. Ser., 23, Inst. Math. Statist., Hayward, CA, 1994.

Abstract: Let $X(i), i = 1, 2, \cdots, n$. be independent random variables with unknown distributions P for $i \leq n\theta$ and Q for $i > n\theta$. We investigate confidence sets for the unknown change-point $\theta \in (0, 1)$, which are based on randomization tests. In a simple parametric model for P and Q theses tests are chosen to be Bayes-optimal in a certain sense. Then we imitate this method in a nonparametric framework. Asymptotic properties of the confidence sets are derived under weak conditions allowing that θ tends to zero or one and P is getting closer to Q.

Eubank, R. L. and Speckman, P. L. (1994-9)
Nonparametric estimation of function with jump discontinuities.
Change point Problems (E. Carlstein, H. G. Müller and D. Siegmund, eds.) IMS, 130-144. Hayward, CA.

Abstract: The problem of estimating a function with a jump discontinuity in one of its derivative is considered. A semi-parametric framework is employed to formulate the problem, and a least-squares type estimator of the jump point is proposed for this setting. The asymptotic properties of this estimator are derived, including consistency and asymptotic distribution theory.

Eubank, R. L.; Speckman, P. L.(1994-10)
Nonparametric estimation of functions with jump discontinuities.
Change-point problems (South Hadley, MA, 1992), 130-144, IMS Lecture Notes Monogr. Ser., 23, Inst. Math. Statist., Hayward, CA, 1994.

Abstract: The problem of estimating a function with a jump discontinuity in one of its derivatives is concerned. A semi-parametric framework is employed to formulate the problem, and a least-squares type estimator of the jump point is proposed for this setting. The asymptotic properties of this estimator are derived, including consistency and asymptotic distribution theory.

Ferger, Dietmar(1994-11)
On the power of nonparametric changepoint-tests.
Metrika 41, 277-292.

Abstract: We consider sequence $X_{1n}, \cdots, X_{mn}, n \in N$, of independent random elements. Suppose there exists a $\theta \in [0, 1]$ such that $X_{1n}, \cdots, X_{[n\theta],n}$ have the distribution ν_1 and $X_{[n\theta]+1,n}, \cdots, X_{mn}$ have the distribution $\nu_2 \neq \nu_1$. We construct consistent level-α tests for $H_0 : \theta = 0$ verses $H_1 : \theta \in (0, 1)$, which are based on certain U-statistic type processes. A detailed investigation of the power function is also provided.

Ferger, Dietmar(1994-12)
Change-point estimators in case of small disorders
Journal of Statistical Planning and Inference, 40, 33-49
Keywords: Change in distribution; boundary effects; weight functions; maximal inequalities; invariance principle; Brownian motion with drift

Abstract: Let $X_1^n, \cdots, X_n^n$, $n \geq 2$, be a triangular array of row-wise independent random elements with values in a measurable space (χ, β) such that $X_1^n, , X_{[n\theta]}^n$ have distribution ν_1 and $X_{[n\theta]+1}^n, \cdots, X_n^n$ have distribution $\nu_{2,n}$ (both unknown) for some $\theta \in (0,1)$. We propose a large class of estimators θ_n of the unknown change-point θ, that are maximizers of weighted U-statistic-type processes. They overcome boundary effects, which typically occur when the change-point θ is close to zero or one. If $\nu_{2,n}$ converges in some sense to ν_1 at a rate γ_n, then $\xi_n = n\gamma_n^{-2}(\theta_n - \theta)$ converges in distribution to the (a.s.) unique maximizer of a two-sided Brownian motion with negative drift. The key step of the proof is the representation of ξ_n as the maximizer of a certain normalized increment process Y_n. The analysis of Y_n involves certain new maximal inequalities

Ferger, Dietmar(1994-13)
On the rate of almost sure convergence of Dmbgen's change-point estimators
Statistics and Probability Letters, 19, 27-31
Keywords: Change-point estimator; Maximal inequalities; Maximizer of stochastic processes

Abstract: Consider a triangular array of rowwise independent random elements with values in a measurable space. Suppose there exist $\theta_n \in T_n = i n_{-1} : 1 \leq i \leq n-1$ such that $X_{1n}, \cdots, X_{n,n\theta_n}$ have distribution P_n and $X_{n,n\theta_n+1}, \cdots, X_{nn}$ have distribution $Q_n \neq P_n$, where P_n, Q_n and θ_n are unknown. We investigate a large class of change-point estimators $\widehat{\theta_n}$ due to Dmbgen. Dmbgen proved that $\widehat{\theta_n} - \theta_n = O_p(\gamma_n^2 n^{-1})$, where the sequence (γ_n) measures the distance between P_n and Q_n. We show that $\widehat{\theta_n} - \theta_n = O_p(\gamma_n^2 n^{-1} \log n)$ with probability one.

Gombay, Edit(1994-14)
Testing for change-points with rank and sign statistics
Statistics and Probability Letters, 20, 49-55
Keywords: Change-point problem; Nonparametric statistics; Asymptotic distribution; Rank statistics; Sign statistics; Simple random walk

Abstract: Special tests are designed and their properties investigated for the change-point problem, when the alternative is of the epidemic or square-wave type. Rank statistics and sign statistics are used. The use of sign statistics leads to the study of the properties of the simple random walk.

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Gombay, Edit and Horváth, Lajos(1994-15)
An application of the maximum likelihood test to the change-point problem
Stochastic Processes and their Applications, 50, 161-171
Keywords: maximum likelihood; parameter estimation; standardized partial sums; limit
theorem; double exponential distribution

Abstract: A maximum-likelihood-type statistic is derived for testing a sequence of observations for no change in the parameter against a possible change. We prove that the limit distribution of the suitably normalized and centralized statistic is double exponential under the null hypothesis.

Gombay, Edit and Horváth, Lajos(1994-16)
Limit theorems for change in linear regression.
Journal of Multivariate Analysis, 48, 43-69

Abstract: To detect a change-point in a linear regression model with fixed or random design points, some tests based on the maxima of the weighted cumulative sum processes of residuals are considered. The limit distributions may be an extreme value distribution of the Gumbel type or maxima of Gaussian processes depending on the set where the maximum is taken. A few applications are also given. (Reviewed by Lin-Cheng Zhao)

Hartigan, J. A.(1994-17)
Linear Estimators in Change Point Problems
The Annals of Statistics, 22, 824-834.

Abstract: Observations X_i are uncorrelated with means $\theta_i, i = 1, \cdots, n$, and variances 1. The linear estimators $\widehat{\theta} = TX$, for some $n \times n$ matrix T, are widely used in smoothing problems, where it is assumed that neighbouring parameter values are similar. The smoothness assumption is violated in change point problems, where neighbouring parameter values are equal, except at some unspecified change points where there are jumps of unknown size from one parameter value to the next. In the case of a single change point in one dimension, for any linear estimator, the expected sum of squared errors between estimates and parameters is of order $\sqrt{n}$ for some choice of parameters, compared to order 1 for the least squares estimate. We show similar results for adaptive shift estimators, in which the linear estimator uses a kernel estimated from the data. Finally, for a change point problem in two dimensions, the expected sum of squared errors is of order $n^{3/4}$.

Inclan, Carla and Tiao, George C.(1994-18)
**Use of Cumulative Sums of Squares for Retrospective Detection of Changes of
Variance**

Journal of the American Statistical Association, 89, 913-923. Keywords: Cumulative sum of squares; Multiple change point; Variance change.

Abstract: This article studies the problem of multiple change points in the variance of a sequence of independent observations. We propose a procedure to detect variance changes based on an iterated cumulative sums of squares (ICSS) algorithm. We study the properties of the centered cumulative sum of squares function and give an intuitive basis for the ICSS algorithm. For series of moderate size (i.e., 200 observations and beyond), the ICSS algorithm offers results comparable to those obtained by a Bayesian approach or by likelihood ratio tests, without the heavy computational burden required by these approaches. Simulation results comparing the ICSS algorithm to other approaches are presented.

Kim, Hyune-Ju (1994-19)
Likelihood ratio and cumulative sum tests for a change-point in linear regression
Journal of Multivariate Analysis, 51, 54-70
Keywords: *P*-value; Power of a test

Abstract: Our concern in this paper is a detection of a change in regression coefficients of a linear model. First, we examine the null and alternative distributions of the likelihood ratio statistic and study its asymptotic behavior. We then propose analytic approximations for the p-value and power of the test and perform simulations to assess the accuracy of the analytic approximations. Also, the test based on the cusum of the recursive residuals is discussed and its power is compared with that of the likelihood ratio test. We conclude that the likelihood ratio yest is much more powerful than the cusum test of Brown et al. (1975, J. Roy. Statist. Soc. Ser. B37, 149-192) and propose a test based on the backward cusum to improve the power of the cusum test.

Kim, Hyune-Ju(1994-20)
Tests for a change-point in linear regression.
Change-point problems (South Hadley, MA, 1992), 170-176, IMS Lecture Notes Monogr. Ser., 23, Inst. Math. Statist., Hayward, CA, 1994

Abstract: The paper considers a problem of detecting a change-point in a linear model. We discuss analytic properties of the likelihood ratio statistic and study its asymptotic behavior. An approximation for the significance level of the test is provided assuming values of the independent variables are effectively random. We also discuss the power and the robustness of the likelihood ratio test.

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Lombard, F. and Hart, J. D.(1994-21)
The analysis of change-point data with dependent errors
Change-point problems (South Hadley, MA, 1992), 194-209, IMS Lecture Notes Monogr.
Ser., 23, Inst. Math. Statist., Hayward, CA, 1994

Abstract: We consider abrupt mean-change models for data with dependent, stationary, errors. No specific distributional assumptions, other than the existence and summability of cumulants, are made. A consistency property of the least squares estimator of the change-point is derived. The leads to the construction of consistent, asymptotically normal and efficient estimators of the error spectral density function and covariances. The application of theses results in testing for the existence of a change is discussed. A test for uncorrelatedness of the errors is given. An application is made to the detection of changes in the period of a variable star. The relationship between cusum charts used in statistics and O-C diagrams used in astronomy is pointed out.

Müller, H. G. and Wang, J.-L.(1994-22)
Change-point models for hazard functions.
Change-point problems (South Hadley, MA, 1992), 224-241, IMS Lecture Notes Monogr.
Ser., 23, Inst. Math. Statist., Hayward, CA, 1994

Abstract: A review is presented of parametric and nonparametric models and corresponding estimation procedures for change-points in hazard functions where the data are possibly subject to random censoring. In particular, we discuss nonparametric models and application of nonparametric smoothing techniques for change-point estimation and estimation of a hazard function when a change-point is presented. Preliminary theoretical results are mentioned and simulation study provides further insight.

Müller, H. G. and Song, Kai-Sheng (1994-23)
Maximin estimation of multidimensional boundaries
Journal of Multivariate Analysis, 50, 265-281
Keywords: Kernel estimator; Nonparametric regression; Change point

Abstract: We consider the problem of estimating the location and size of a discontinuity in an otherwise smooth multidimensional regression function. The boundary or location of the discontinuity is assumed to be a closed curve respective surface, and we aim to estimate this closed set. Our approach utilizes the uniform convergence of multivariate kernel estimators for directional limits. Differences of such limits converge to zero under smoothness assumptions, and to the jump size along the discontinuity. This leads to the proposal of a maximin estimator, which selects the boundary for which the minimal estimated directional difference among all points belonging to this boundary is maximized. It is shown that this estimated

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boundary is almost surely enclosed in a sequence of shrinking neighborhoods around the true boundary, and corresponding rates of convergence are obtained.

Rukhin, Andrew L.(1994-24)
Asymptotic minimaxity in the change-point problem
Change-point problems (South Hadley, MA, 1992), 284-291, IMS Lecture Notes Monogr. Ser., 23, Inst. Math. Statist., Hayward, CA, 1994

Abstract: A lower bound on the limit of the minimax risk under the zero-one loss function is established in the classical setting of the change-point estimation problem. This bound is attained by the maximum likelihood estimator in the situation when the two probability distributions before and after the change point are completely known. The nature of this bound is related to multiple decision problem and variety of inequalities relating to the information-type measures is deduced. Minimaxity of the maximum likelihood procedure is proved for normal observations with unknown means.

Sen, Pranab K.(1994-25)
Some change-point problems in survival analysis: relevance of nonparametrics in applications.
Journal of Applied Statistical Sciences, 1 , 425-444.

Abstract: In survival analysis, hazard functions often crop up in a natural way, and the scenario is dominated by the celebrated Cox proportional hazard models which provide the access to the so-called semi-parametric models. However, in the context of change-points models in survival analysis, the proportional hazard model may run into some conceptual obstructions, and time-dependent covariates as well as coefficients models appear to be more flexible. Therefore, such time-dependent coefficients models are incorporated in the formulation of suitable change-point models in survival analysis, and the relevant statistical methodology is presented in a unified manner. In this setup, with due emphasis on the scope of applications, various aspects of nonparametrics are studied thoroughly

Siegmund, David O. and Zhang, Heping(1994-26)
Confidence regions in broken line regression.
Change-point problems (South Hadley, MA, 1992), 292–316, IMS Lecture Notes Monogr. Ser., 23, Inst. Math. Statist., Hayward, CA, 1994

Abstract: The broken line regression model is viewed as a special case of nonlinear regression. Following the methodology of Knowles, Sigmunt, and Zhang(1991,Biometrica, 78, 15-31). We discuss procedures for constructing confidence region. Our method involves inversion of the likelihood ratio test. A slightly conservative bound is obtained for the level of the

test given the value of statistics which are sufficient for the nuisance parameters when the parameters of interest are fixed. We use a number of published data sets and simulations to compare our method with the approximately an F distribution, and with the Bayesian method of Smith and Cook (1980, Appl. Statist. 29, 180-189).

Stephens, A.(1994-27)
Bayesian Retrospective Multiple-Changepoint Identification
Applied Statistics, Vol. 43, No. 1., 159-178.
Keywords: Discrete and continous changepoint models; Gibbs sampler; Multiple changepoint models.

Abstract: Changepoint identification is important in many data analysis problems, such as industrial control and medical diagnosis–given a data sequence, we wish to make inference about the location of one or more points of the sequence at which there is a change in the model or parameters driving the system. For long data sequences, however, analysis (especially in the multiple-changepoint case) can become computationally prohibitive, and for complex non-linear models analytical and conventional numerical techniques are infeasible. We discuss the use of a sampling-based technique, the Gibbs sampler, in multiple-changepoint problems and demonstrate how it can be used to reduce the computational load involved considerably. Also, often it is reasonable to presume that the data model itself is continuous with respect to time, i.e. continuous at the changepoints. This necessitates a continuous parameter representation of the changepoint problem, which also leads to computational difficulties. We demonstrate how inferences can be made readily in such problems by using the Gibbs sampler. We study three examples: a simple discrete two-changepoint problem based on a binomial data model; a continuous switching linear regression problem; a continuous, non-linear, multiple-changepoint problem.

Tsybakov, A. B.(1994-28)
Multidimensional change-point problems and boundary estimation.
Change-point problems (South Hadley, MA, 1992), 317–329, IMS Lecture Notes Monogr. Ser., 23, Inst. Math. Statist., Hayward, CA, 1994

Abstract:We consider a multivariate extension of the change-point problem where one has to estimate a change curve (or surface). Three versions of this problem are considered.

1. the regression-type model of image segmention

2. the estimation of a discontinuity curve in an unknown density, and

3. the estimation of the edge of poission forest.

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For theses problems we give two approaches to the construction of estimators, study the rates of convergence of the proposed estimators, and show their optimality.

Antoch, J. and Hušková, M.(1993-1)
Change point problem. Computational aspects of model choice.
Contrib. Statist., Physica, Heidelberg, 1993.

Abstract: The main objective of this paper is to survey procedures connected with the change point problem and with testing the constancy of regression relationship over time. We focus on the case of independent observations. Detailed algorithms are presented for most of the methods discussed.

Antoniadis, A. and Grégoire, G.(1993-2)
Nonparametric estimation in change-point hazard rate models for censored data: a counting process approach.
Journal of Nonparametric Statistics, 3, 135-154

Abstract: This paper discusses a nonparametric method for estimating under random censorship a hazard rate function with a possible change-point. First, motivated by a practical example, we focus on the situation where the location of the change-point is known. Within the general framework of counting processes, using the kernel method of estimation in partial linear models, consistent estimators for the rate of change at the breakpoint and for the resulting hazard rate function are obtained. The case of an unknown location is also addressed. The performances of the estimators in both models are checked via simulations. (Reviewed by Hung Trung Nguyen)

Andrews, Donald W. K. (1993-3)
Tests for parameter instability and structural change with unknown change point
Econometrica,61, 821-856
Keywords: Asymptotic distribution; Change point; Bessel process; Brownian bridge; Brownian motion; Generatized method of moments estimator; Lagrange multiplier test; Likelihood ratio test; Parameter instability; Structural change; Wald test; Weak convergence.

Abstract: This paper considers tests for parameter instability and structural change with unknown change point. The results apply to a wide class of parametric models that are suitable for estimation by generalized method of moments procedures. The paper considers Wald, Lagrange multiplier, and likelihood ratio-like tests. Each test implicitly uses an estimate of a change point. The change point may be completely unknown or it may be known to lie

in a restricted interval. Tests of both "pure" and "partial" structural change are discussed. The asymptotic distributions of the test statistics considered here are nonstandard because the change point parameter only appears under the alternative hypothesis and not under the null. The asymptotic null distributions are found to be given by the supremum of the square of a standardized tied-down Bessel process of order $p \geq 1$, as in D. L. Hawkins (1987). Tables of critical values are provided based on this asymptotic null distribution. As tests of parameter instability, the tests considered here are shown to have nontrivial asymptotic local power against all alternatives for which the parameters are nonconstant. As tests of one-time structural change, the tests are shown to have some weak asymptotic local power optimality properties for large sample size and small significance level. The tests are found to perform quite well in a Monte Carlo experiment reported elsewhere.

Baiqi, Miao; Lincheng, Zhao and Krishnaiah, P. K.(1993-4)
On detection of change points using mean vectors
Acta Mathematicae Applicatae Sinica, 9, 193 - 203

Abstract: In this paper, the authors consider the problem of change points within the framework of model selection and propose a procedure for estimating the locations of change points when the number of change points is known. The strong consistency of this procedure is also established. The problem of detecting change points is discussed within the framework of the simultaneous test procedure. The case where the number of change points is unknown will be discussed in another paper. This project is supported by the National Natural Science Foundation of China and by the Air Office of Scientific Research of the United States.

Barry, Daniel and Hartigan, J. A.(1993-5)
A Bayesian analysis for change point problems.
Journal of the American Statistical Association, 88, 421, 309(11).
Keywords: Change points; Product partition models.

Abstract: A proposed Bayesian methodology for predicting sudden changes in a sequence of observations assumes the presence of an underlying sequence of parameters divided into adjoining blocks of equal parameter magnitude. Each block is assumed to be a change point such that, based from the sequence of parameters, the observations are independent for every block. The product partition models provide a framework through which the change points could be predicted based from the given data.

Brodskiĭ, B. E.; Darkhovskiĭ, B. S.(1993-6)
An algorithm for a posteriori detection of multiple change points in a random sequence.
Avtomat. i Telemekh. 1993, , no. 1, 62-67; translation in Automat. Remote Control 54 (1993), no. 1, part 1, 54-59

Abstract: The authors consider the problem of detection of multiple change points of the mathematical expectation of a random sequence based on the observation of its realization. They give a nonparametric approach, i.e., they do not use any a priori information on the distribution of the random sequence. The method leads to strongly consistent estimators of the number of change points and their coordinates. In comparison with earlier work of the authors, in this paper the method is based on a one-dimensional sample function. (Reviewed by H.-J. Engelbert)

Brodskiĭ, B. E.; Darkhovskiĭ, B. S.(1993-7)
Nonparametric methods in change-point problems.
Mathematics and its Applications, 243. Kluwer Academic Publishers Group, Dordrecht, 1993.

Abstract: The main goal of this excellent monograph is to provide a timely survey of developments in nonparametric methods of change-point detection. It is also the first one devoted to this rapidly developing area of statistical analysis of observations that form a random sequence. Since stochastic properties of a random sequence are determined by its distribution, a change in any such characteristics of the observations is, in fact, a change in their distribution. A full description of a change in the distribution of a random sequence of observations would, of course, be possible only with the help of all their finite-dimensional distributions. Hence, in order to extract relevant information from the data, one must concentrate on determining characteristics of a random sequence which would reliably reduce the complexity of change detection. It is convincingly argued by the authors in Section 1.2 that detecting changes in the mean (mathematical expectation) of a random sequence constitutes one basic situation to which other changes in distribution can be conveniently reduced. As a guideline for surveying this rapidly developing branch of mathematical statistics, this book concentrates on the authors' own results obtained in the last 15 or so years, which were among the first ones devoted to general problems of nonparametric change-point detection and diagnosis. Thus, it reflects these developments from a somewhat personal point of view. Nevertheless, due to the seminal nature of their many contributions, this work succeeds in conveying the essence of a vast number of essential publications in the areas involved. In addition to the already mentioned theme determining Section 1.2, Chapter 1 contains some necessary auxiliary results, without proofs but well referenced, from the theory of random processes and mathematical statistics.

Chapter 2 gives an excellent state-of-the-art review of the developments of a posteriori and sequential methods of change-point detection.

Chapter 3 is devoted to analysing some of the basic concepts and results of nonparametric a posteriori change-point detection theory. In this theory one is to study a given realization of a random sequence under the (null) hypothesis of its stochastic homogeneity. If this hypothesis is rejected, then estimates of change points have to be obtained. Section 3.1 summarizes the assumptions and the formulation of the problems. {Regarding Figure 3.1.1 on page 39, we note that, according to the description of these problems in the text, Problems

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IV and V should be interchanged.}In Section 3.2 some auxiliary results are formulated and proved. Almost everywhere in this chapter a posteriori change-point problems are formulated in terms of an unknown shift of the mean value of a random sequence. Limit theorems for corresponding nonparametric methods of change-point detection are formulated and proved in Section 3.3. In the same section a Cramér-Rao lower bound for the dispersion of change-point estimates and a lower bound for the probability of the error of estimation are obtained. The asymptotic optimality of the proposed detection methods is established in Section 3.5, where some results on the asymptotic analysis of a family of Kolmogorov-Smirnov-type change-point detection statistics are presented as well.

Chapter 4 is on the nonparametric sequential change-point detection theory of the authors. An impressive comparative asymptotic analysis of their proposed methods is summarized and thoroughly analyzed in five sections. Chapter 5 presents a theory for detecting homogeneous regions of a random field, an area again where contributions of the two authors have played a prominent role. The nonparametric methods of change-point detection developed in this book are used to analyze three practical problems in Chapter 6. These problems are concerned with (i) computer analysis of historical texts, (ii) computer-aided spatial data processing, and analysis of information obtained from an orbiting satellite, (iii) computer analysis of geophysical information. Chapter 7 contains proofs of some theorems and a description of the program package VERDIA for a posteriori change-point detection problems.

This book can be used, and it is highly recommended by the reviewer, as a graduate-level textbook on the subject, as well as an outstanding source of information for everyone doing research in these and related areas of random processes and mathematical statistics. (Reviewed by M. Csörgö)

Eastwood, Vera R.(1993-8)
Some Nonparametric Methods for Changepoint Problems
The Canadian Journal of Statistics / La Revue Canadienne de Statistique, Vol. 21, No. 2., 209-222.
Keywords:Projections of U-statistics; Limiting distributions; Distributions of Gaussian functionals.

Abstract: A general model for changepoint problems is discussed from a nonparametric viewpoint. The test statistics introduced are based on Cramr-von Mises functionals of certain processes and are shown to converge in distribution to corresponding Gaussian functionals (under the assumption of no change in distribution, $\mathcal{H}_0$). We also demonstrate how the distribution of the limiting Gaussian functionals may be tabulated. Finally, properties of the tests under the alternative hypothesis of exactly one changepoint occurring are studied, and some examples are given.

Foster, D. P. and George, E. I. (1993-9)

Estimation up to a Change-Point
The Annals of Statistics, 21, 625-644

Abstract: Consider the problem of estimating μ, based on the observation of $Y_0, Y_1, \ldots, Y_n$, where it is assumed only that $Y_0, Y_1, \ldots, Y_\kappa$ iid $N(\mu, \sigma^2)$ for some unknown κ. Unlike the traditional change-point problem, the focus here is not on estimating κ, which is now a nuisance parameter. When it is known that $\kappa = k$, the sample mean $\bar{Y}_k = \sum_0^k Y_i / (k+1)$, provides, in addition to wonderful efficiency properties, safety in the sense that it is minimax under squared error loss. Unfortunately, this safety breaks down when κ is unknown; indeed if $k > \kappa$, the risk of $\bar{Y}_k$ is unbounded. To address this problem, a generalized minimax criterion is considered whereby each estimator is evaluated by its maximum risk under $Y_0, Y_1, \ldots, Y_\kappa$ iid $N(\mu, \sigma^2)$ for each possible value of κ. An essentially complete class under this criterion is obtained. Generalizations to other situations such as variance estimation are illustrated.

Hawkins, D. L.(1993-10)
Detecting shifts in functions of multivariate location and covariance parameters
Journal of Statistical Planning and Inference, 33, 233-244

Abstract: Procedures are studied for monitoring for shifts in functions of the mean vector or covariance matrix of an arbitrary multivariate distribution. The procedures provide the capabilities, among others, to monitor a linear function (e.g. a contrast) of the mean vector, or to monitor the covariance matrix for shifts in variances, correlations, generalized variance or for shifts away from the conditions of sphericity or intra-class correlation structure. Two sampling schemes are considered: the retrospective and sequential settings. Test statistics are quadratic forms in split-sample mean and covariance estimates. The asymptotic theory of U-statistics is used to control null error rates. A small Monte Carlo study investigates non-null power and reaction time. A total of eight procedures are studied.

Henderson, R. and Matthews, J. N. S.(1993-11)
An Investigation of Changepoints in the Annual Number of Cases of Haemolytic Uraemic Syndrome

Abstract: There has been speculation that the number of cases of diarrhoea-associated haemolytic uraemic syndrome increased abruptly during the early part of the 1980s. To investigate this hypothesis, changepoint models for Poisson variables are applied to two series of data from regional referral units in Newcastle upon Tyne and Birmingham. The

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series are analysed both separately and jointly, with particular emphasis on determining the number and location of mean changes. The adequacy of the postulated models is considered.

Huanga, Wen-Tao and Chang, Yi-Ping(1993-12)
Nonparametric estimation in change-point models
Journal of Statistical Planning and Inference, 35, 335-347
Keywords: Change-point; Smooth mixture intervention; Least-squares type methods; Consistency; Convergence rate; Asymptotic distribution

Abstract: We consider a new change-point model with some smooth mixture intervention. Some least-squares type estimators for the parameters are proposed and some large sample properties for these estimators are shown. A Monte Carlo study shows that the proposed estimators behave satisfactorily for the normal case.

Jandhyala, V. K. and Minogue, C. D. (1993-13)
Distributions of Bayes-type change-point statistics under polynomial regression
Journal of Statistical Planning and Inference, 37, 271-290
Keywords: Change-point problem; Polynomial regression; Stochastic integral; Fredholm equation

Abstract: Under the assumption of a polynomial regression model, simple elegant expressions for the covariance kernels of residual partial sum limit processes are derived. A numerical method of solving Fredholm integral equations is derived, which is shown to provide solutions that are uniformly close to the analytical solutions. This numerical procedure is applied to compute quantiles for the asymptotic distributions of Bayes-type statistics derived to test for change in an arbitrary parameter of a general polynomial regression model. Finally, the methodology is applied to data on boys' weight/height ratio.

Jandhyala, V. K. (1993-14)
A property of partial sums of regression least squares residuals and its application
Journal of Statistical Planning and Inference, 37, 317-326
Keywords: Change-point; Linear regression; Limit process; singularity

Abstract: For a class of regression models, the sum of the sequence of partial sums of least squares residuals is shown to be zero. Simple and higher order polynomial regression models are included in the class for which this property holds. The result is applied to derive properties of the sample path behavior of residual partial sum processes. It is also applied

to show the singularity of the Bayes-type statistic derived to test for one-sided change at unknown time in the intercept parameter of a regression model.

Jiang, J. and Lou, S. X. C. (1993-15)
Parameter estimation of systems subject to random state changes
IEEE Transactions on Automatic Control, 38, 1532-1536
Keywords: Regression; Singular value decomposition

Abstract: Parameter estimation for a linear regression model subject to abrupt random state changes is formulated as an optimization problem. An identification algorithm comprising state regime classification and parameter identification is developed. The samples of the system are first partitioned into different groups called clusters, corresponding to different states. The standard linear-least-square method is then used to identify the parameters. The cluster control is a matrix of predetermined rank and can be computed by the singular-value-decomposition algorithm. Two iterative algorithms that ensure the decrease of the objective function are then proposed. An example is given to show the effectiveness of the method

Joseph, Lawrence and Wolfson, David B.(1993-16)
Maximum likelihood estimation in the multi-path change-point problem
Annals of the Institute of Statistical Mathematics 45, 511-530
Keywords: Change-point; Maximum likelihood estimation; EM algorithm ; Mixture distribution

Abstract: Maximum likelihood estimators of the parameters of the distributions before and after the change and the distribution of the time to change in the multi-path change-point problem are derived and shown to be consistent. The maximization of the likelihood can be carried out by using either the EM algorithm or results from mixture distributions. In fact, these two approaches give equivalent algorithms. Simulations to evaluate the performance of the maximum likelihood estimators under practical conditions, and two examples using data on highway fatalities in the United States, and on the health effects of urea formaldehyde foam insulation, are also provided.

Kim, Hyune-Ju (1993-17)
Two-phase regression with nonhomogeneous errors
Communications in Statistics: Theory and Methods, 22, 647-657
Keywords: Change point

Abstract: Likelihood ratio tests for a change in simple linear regression with unequal variances are studied. We first derive the likelihood ratio statistics assuming nonhomogeneous

error variance. (We call such statistics the weighted likelihood ratio statistics.) We obtain analytic approximations for the p-values of these weighted tests, and assess the accuracy of the approximations via simulation. Using numerical examples, we also study the robustness to heteroscedasticity of the unweighted likelihood ratio tests considered in Kim and Siegmund(1989).

MacNeill, I. B. and Mao, Y.(1993-18)
Change-point methods analysis for mortality and morbidity data
Journal of Applied statistical Science 1, 359-377
Keywords: Change-point statistics; Mortality, Morbidity; Monitoring; Growth models;
Age-cohort data; Spatial data.

Abstract: The problem of monitoring mortality data is considered. Models for time series of such data are discussed, and a brief review is presented of Bayes-type methods for identifying changes which occur at unspecified time in the parameters of regression models. Bivariate growth models are proposed for age-period and age-cohort data. Since age-period cohort data are analogous to spatial data, spatial models and change-point analogues for spatial data are discussed. The methods are applied to data which include those for Canadian female breast cancer mortality rates.

Powers, Daniel A. (1993-19)
Endogenous switching regression models with limited dependent variables
Sociological Methods and Research, 22, 248-273
Keywords: Binary data; Censored data

Abstract: Social research often involves estimating the effects of a categorical treatment on a dependent outcome variable. Endogenous switching regression models are natural extensions of classical experimental designs, which allow tests of assumptions about the exogeneity of treatment effects from survey data. Endogenous switching regression applications are pervasive in the econometric literature and are becoming widely used by sociologists. However, sociologists have devoted somewhat less attention to endogenous switching models involving limited dependent variables. Switching regression models for continuous variables can be generalized to account for binary and censored dependent variables. These models are applied to estimate the effect of family structure on early family formation.

Tang, S. M. and MacNeill, I. B. (1993-20)
The effect of serial correlation on tests for parameter change at unknown time
The Annals of Statistics, 21, 552-575
Keywords: Time series; Regression; Change point

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Abstract: It is shown that serial correlation can produce striking effects in distributions of change-point statistics. Failure to account for these effects is shown to invalidate change-point tests, either through increases in the type 1 error rates if low frequency spectral mass predominates in the spectrum of the noise process, or through diminution of the power of the tests when high frequency mass predominates. These effects are characterized by the expression $2\pi f(0)/\int_{-\pi}^{\pi} f(\lambda)d(\lambda)$, where $f(.)$ is the spectral density of the noise process; in sample survey work this is know as the design effect or "deff." Simple precise adjustments to change-point test statistics which account for serial correlation are provided. The same adjustment applies to all commonly used regression models. Residual processes are derived for both stationary time series satisfying a moment condition and for general linear regression models with stationary error structure.

Wang, China-Yuan and Lee, Chung-Bow (1993-21)
Bayesian analysis for a change in the intercept of simple linear regression
Communications in Statistics: Theory and Methods, 22, 3031-3050
Keywords: Change point; Monte Carlo method; Marginal posterior distribution.

Abstract: A Bayesian approach is considered to detect a change-point in the intercept of simple linear regression. The Jeffreys noninformative prior is employed and compared with the uniform prior in Bayesian analysis. The marginal posterior distributions of the change-point, the amount of shift and the slop are derived. Mean squares, mean absolute errors and mean Bayesian estimates are considered by Monte Carlo method and some numerical results are also shown.

Wu, J. S. and C. K. Chu(1993-22)
Kernel-Type Estimators of Jump Points and Values of a Regression Function
The Annals of Statistics, Vol. 21, No. 3. (Sep., 1993), pp. 1545-1566.

Abstract: In the fixed-design nonparametric regression model, kernel-type estimators of the locations of jump points and the corresponding sizes of jump values of the regression function are proposed. These kernel-type estimators are analyzed with almost sure results and limiting distributions. Using the limiting distributions, we are able to test the number of jump points and give asymptotic confidence intervals for the sizes of jump values of the regression function. Simulation studies demonstrate that the asymptotic results hold for reasonable sample sizes.

Barry, Daniel and Hartigan, J. A.(1992-1)
Product Partition Models for Change Point Problems
The Annals of Statistics, 20, 260-279.

Abstract: Product partition models assume that observations in different components of a random partition of the data are independent. If the probability distribution of random partitions is in a certain product form prior to making the observations, it is also in product form given the observations. The product model thus provides a convenient machinery for allowing the data to weight the partitions likely to hold; and inference about particular future observations may then be made by first conditioning on the partition and then averaging over all partitions. These models apply with special computational simplicity to change point problems, where the partitions divide the sequence of observations into components within which different regimes hold. We show, with appropriate selection of prior product models, that the observations can eventually determine approximately the true partition.

Carlstein, E. and Krishnamoorthy, C.(1992-2)
Boundary Estimation
Journal of the American Statistical Association, 87, 430-438.
Keywords: Change point; Cramer-von Mises; Empirical cummulative distribution function; Epidemic-change; Grid; Kolmogorov-Smirnov; Lipschitz; Partition; Template.

Abstract: A data set consists of independent observations taken at the nodes of a grid. An unknown boundary partitions the grid into two regions. All the observations coming from a particular region share a common distribution, but the distributions are different for the two different regions. These two distributions are entirely unknown and need not differ in their means, medians, or any other measure of "level." The grid is of arbitrary dimension, and its mesh is rectangular. Our objective is to estimate the boundary without making any distributional assumptions. We propose a class of estimators and obtain strong consistency for them (including rates of convergence and a bound on the error probability). The boundary estimate is selected from an appropriate collection of candidate boundaries, which must be specified by the user. The candidate boundaries as well as the true boundary must satisfy certain intuitively natural regularity assumptions, including a "smoothness" condition. The boundary estimation problem has applications in diverse fields, including quality control, epidemiology, forestry, marine science, meteorology, and geology. Our method provides (as special cases) estimators for the change point problem, the epidemic change model, templates, linear bisection of the plane, and Lipschitz boundaries. Each of these examples is explicitly analyzed. A simulation study provides numerical evidence that the boundary estimators work well; in this simulation, the two distributions actually share the same mean, median, variance, and skewness. Finally, as an illustration, a boundary estimate is calculated on a data grid of cancer mortality rates in the United States.

Deutsch, Joseph (1992-3)
Linear regression under two separate regimes: An empirical distribution for Quandt's log likelihood ratio
Applied Economics, 24, 123-127

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Keywords: Change point; CUSUM

Abstract: In the empirical analysis of structural changes, Quandt's test for detecting a single change in a regression relationship at an unknown point of time is frequently applied. A limitation regarding the applicability of the method is that the distribution of the Quandt's log likelihood ratio is unknown. The purpose of this work is to derive an empirical distribution for Quandt's statistic which enables the researcher to approximate the level of significance when testing the stability of regression relationships over time.

Ferger, Dietmar and Stute, Winfried(1992-4)
Convergence of changepoint estimators
Stochastic Processes and their Applications, 42, 345-351
Keywords: Changepoint estimator; Exponential tail bound; Almost sure convergence

Abstract: Let $X_1^n, \cdots, X_n^n$ be an array of independent random vectors such that $X_1^n, \cdots, X_{[n\theta]}^n$ have distribution function F, and $X_{[n\theta]+1}^n, \cdots, X_n^n$ have distribution function G with $F \neq G$. In this paper we propose an estimator θ_n of the changepoint θ and show that $n(\theta_n - \theta) = O(\ln n)$ with probability one

Hall, Peter and Titterington, D. M. (1992-5)
Edge-preserving and peak-preserving smoothing
Technometrics, 34, 429-440
Keywords: Change point; Nonparametric regression

Abstract: An alternative procedure is developed to the smoothed linear fitting method of McDonald and Owen. The procedure is based on the detection of discontinuities by comparing, at any given position, three smooth fits. Diagnostics are used to detect discontinuities in the regression function itself (edge detection) or in its first derivative (peak detection). An application in electron microscopy is discussed.

Hawkins, D. L.; Kochar, Subhash and Loader, Clive(1992-6)
Testing Exponentiality Against IDMRL Distributions with Unknown Change Point
The Annals of Statistics, 20, 280-290.

Abstract: Guess, Hollander and Proschan proposed tests for exponentiality versus IDMRL (increasing initially and then decreasing mean residual life) distributions when the change point, or corresponding quantile, is known. In this paper we propose two tests which do not require such knowledge of the change point. The tests are based on estimates of functionals of the cdf which discriminate between the exponential and IDMRL families.

Jandhyala, V. K. and MacNeill, I. B. (1992-7)

On testing for the constancy of regression coefficients under random walk and change-point alternatives

Econometric Theory, 8, 501-517

Abstract: In the context of linear models, the authors point out a close connection between sequential variation regression formulations and change-point formulations. They present a Bayesian-type statistic for testing the constancy of regression coefficients and demonstrate its equivalence to the locally best invariant statistic through a random walk alternative formulation. They then derive some important asymptotic results and discuss the asymptotic theory.

Jandhyala, V. K. and MacNeill, I. B.(1992-8)

On testing for the constancy of regression coefficients under random walk and change-point alternatives.

Econometric Theory 8, 501-517

Abstract: In the context of linear models, the authors point out a close connection between sequential variation regression formulations and change-point formulations. They present a Bayesian-type statistic for testing the constancy of regression coefficients and demonstrate its equivalence to the locally best invariant statistic through a random walk alternative formulation. They then derive some important asymptotic results and discuss the asymptotic theory. (Reviewed by V. K. Srivastava)

Kim, Hyune-Ju (1992-9)

Boundary crossing probabilities by nondifferentiable processes and applications to two-phase regression

Statistics and Probability Letters, 14, 97-102

Keywords: Random walk; Discrete approximation

Abstract: We derive a first order approximation for the probability that a general two-dimensional random walk crosses a circular boundary. Simulations are performed to assess the accuracy of the approximation, and statistical applications are discussed.

Loader, Clive R.(1992-10)

A Log-Linear Model for a Poisson Process Change Point

The Annals of Statistics, 20, 1391-1411.

Keywords: Nonhomogeneous Poisson process

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Abstract: Many methods have been proposed for modeling nonhomogeneous Poisson processes, including change point models and log-linear models. In this paper, we use likelihood ratio tests to choose which of these models are necessary. Of particular interest is the test for the presence of a change point, for which standard asymptotic theory is not valid. Large deviation methods are applied to approximate the significance level, and power approximations are given. Confidence regions for the change point and other parameters in the model are also derived. A British coal mining accident data set is used to illustrate the methodology. D.

Müller, Hans-Georg (1992-11)
Change-points in nonparametric regression analysis
The Annals of Statistics, 20, 737-761

Abstract: Estimators for location and size of a discontinuity or change-point in an otherwise smooth regression model are proposed. The assumptions needed are much weaker than those made in parametric models. The proposed estimators apply as well to the detection of discontinuities in derivatives and therefore to the detection of change-points of slope and of higher order curvature. The proposed estimators are based on a comparison of left and right one-sided kernel smoothers. Weak convergence of a stochastic process in local differences to a Gaussian process is established for properly scaled versions of estimators of the location of a change-point. The continuous mapping theorem can then be invoked to obtain asymptotic distributions and corresponding rates of convergence for change-point estimators. These rates are typically faster than $n^{-1/2}$. Rates of global L^p convergence of curve estimates with appropriate kernel modifications adapting to estimated change-points are derived as a consequence. It is shown that these rates of convergence are the same as if the location of the change-point was known. The methods are illustrated by means of the well known data on the annual flow volume of the Nile river between 1871 and 1970.

Ramirez-Beltran, Nazario D. (1992-12)
Optimal estimation for piecewise regression models
ASQC Technical Conference Transactions, 458-465 American Society for Quality Control (Milwaukee)
Keywords: Change point

Abstract: Two polynomial functions are pieced together so that a continuous function is developed. The joint point of these functions is determined so that the sum of square errors is minimized. Linear transformation is implemented into the original model to convert a constrained optimization problem into an unconstrained optimization problem. A single-variable search method is used to find the joint point. Assuming that SSE is a unimodal function of the joint point, an initial coarse search method is used to bound the optimum.

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Shanubhoguea, A.; Rajarshia, M. B.; Gorea, A. B. and Sitaramamb, V. (1992-13)
Statistical testing of equality of two break-points in experimental data
Journal of Biochemical and Biophysical Methods Volume 25, Issues 2-3 , 1992, Pages 95-112
Keywords: Arrhenius kinetics; Osmotic measurement; Two-phase regression; Break-point; (Rat erythrocyte)

Abstract: Two examples in quantitative biology are examined to emphasize the need for two-phase regression models: the osmotic behaviour of cells and the non-linear temperature kinetics of membrane-bound enzyme systems. Existing statistical techniques are inadequate to test the equality of break-points of two data sets for specific reasons. We suggest here a pragmatic solution by way of a computer programme useful in applying two-phase regression models to such data sets wherein a decision needs to be made whether the critical transition differs or not.

Stasinopoulos, M. and Rigby, R. A.(1992-14)
Detecting break points in generalized linear models
Computational statistics and Data Analysis, 13, 461-471
Keywords: Break point; Regression splines; Generalized Linear Models; Linear predictor

Abstract: This paper describes the use of regression splines as a means of detecting a single break point in one of the variables of the linear predictor in generalized linear models. Poisson data concerning counts of AIDS cases and binomial data from the National Child Development Study are used to show the usefulness of the models.

Thursby, Jerry G. (1992-15)
A comparison of several exact and approximate tests for structural shift under heteroscedasticity
Journal of Econometrics, 53, 363-386
Keywords: Regression

Abstract: It is known that the Chow test for equality of regression coefficients is not robust to heteroscedasticity. Much recent attention has been paid to this problem by way of new tests. The bulk of the new procedures are justified only asymptotically and little has been done to examine finite sample properties. Even less attention has been focused on comparisons of tests under either the null or alternative hypotheses. In this paper we redress these short-comings and also contribute by suggesting additional tests. The results indicate that there are tests available with good sampling characteristics, some of which are computationally easy to carry out from both the standpoint of the researcher's time and computation time.

Benzekri, S. and Brodeau, F. (1991-1)
Asymptotic results for parametric estimation in inadequate two phase regression models
Statistics, 22, 331-348
Keywords: Non linear regression; Least squares estimators; Inadequate models; Asymptotic properties.

Abstract: We study the asymptotic properties, consistency, asymptotic normality, of the least squares estimator in a non linear regression problem. The model uses a parametric class **M** of functions, but we do not assume that the unknown function belong to that class. **M** is here a class of continuous functions with a discontinuity in the first derivative. The problem of making a choice between two classes of that type is also studied.

Dumbgen, L.(1991-2)
The Asymptotic Behavior of Some Nonparametric Change-Point Estimators
The Annals of Statistics, 19, 3, 1471-1495.

Abstract: Consider a sequence $X_1, X_2, \ldots, X_n$ of independent random variables, where $X_1, X_2, \ldots, X_{n\theta}$ have distribution P, and $X_{n\theta+1}, X_{n\theta+2}, \ldots, X_n$ have distribution Q. The change-point $\theta \in (0, 1)$ is an unknown parameter to be estimated, and P and Q are two unknown probability distributions. The nonparametric estimators of Darkhovskh and Carlstein are imbedded in a more general framework, where random seminorms are applied to empirical measures for making inference about θ. Carlstein's and Darkhovskh's results about consistency are improved, and the limiting distributions of some particular estimators are derived in various models. Further we propose asymptotically valid confidence regions for the change point θ by inverting bootstrap tests. As an example this method is applied to the Nile data.

Ebrahimi, Nader(1991-3)
On estimating change point in a mean residual life function
Sankhyā Ser. A 53 (1991), no. 2, 206-219.

Abstract: Let F be the cumulative distribution function (c.d.f.) of a nonnegative random variable with probability density function (p.d.f.) f and mean residual life (m.r.l.) function $m(t)$. It is assumed that $m(t)$ is a truncated 'upside-down bathtub' model, that is, $m(t)$ is a nondecreasing continuous function for $t < \tau$ and is constant for $t \geq \tau$. In this paper a procedure for estimating τ is proposed, assuming that there exists a known p_0 with $0 < F(\tau) < p_0 < 1$. The estimator is shown to be consistent and the asymptotic distribution is given. Also considered is a specific parametric model $m(t) = (+ct)I(0 \leq t < \tau) + (+c\tau)I(t \geq \tau)$, with $I(A)$ denoting the indicator function of A. Two estimates for τ are proposed and their consistency is proved. Some simulations comparing the performance of these estimates are carried out and an example is given.(Reviewed by Prafulchandra N. Jani)

Eric J. Feuer, Larry G. Kessler, Stuart G. Baker, Helen E. Triolo and Donald T. Green(1991-4)

The impact of breakthrough clinical trials on survival in population based tumor registries

Abstract:Three statistical models are developed to study the impact that two breakthrough clinical trials (MOPP for Hodgkin's disease and PVB for disseminated testicular cancer) had on survival in the Connecticut tumor registry and the National Cancer Institute's Surveillance, Epidemiology, and End Results (SEER) registry program. A segmented regression model is used in conjunction with the Cox semi-parametric proportional hazards model, as well as the parametric Weibull and exponential cure models. These models allow us to determine approximately when survival first began to improve dramatically, indicating that improved treatments had become available, and how long it took for survival to level off again indicating that the full population survival impact had been realized. In addition, the degree to which the parametric models fit allows us to determine if the survival improvements occur within a parametric family. Results of the modeling indicate that dissemination took approximately 11 years in Hodgkin's disease while only 3 years in disseminated testicular cancer. In both disease sites survival first broke with prior trends between the time that the breakthrough trial started and its publication, indicating that earlier moderately successful precursor trials with combination chemotherapy may have initiated the improved population survival trends. Reasons for the difference in dissemination time in the two cancer sites are examined in order to understand what factors may be responsible for the speed of dissemination and effective utilization of new therapies.

Jandhyala, V. K. and MacNeill, I. B.(1991-5)

Tests for parameter changes at unknown times in linear regression models

Abstract:Statistics are derived for tests of changes at unknown times in the parameters of a general linear regression model. Asymptotic distribution theory for the tests is discussed. Simulations are carried out to compare power of the statistics derived in this paper with that of other statistics. The derived statistics are shown to have good power properties as compared to other statistics, particularly for the difficult problem of detecting small changes. The change-point methodology is then applied to data on the incidence of AIDS in the United States.

Kim, Dongcheol (1991-6)

A Bayesian significance test of the stationarity of regression parameters
Biometrika, 78, 667-675
Keywords: Behrens-Fisher problem; Change point

Abstract: This paper presents a Bayesian significance test for stationarity of a regression equation using the highest posterior density credible set. In addition, a solution to the Behrens-Fisher problem is provided. From a Monte Carlo simulation study, it has been shown that the Bayesian significance test has stronger power than the Cusum and the Cusum of squares tests. The Bayesian significance test may be useful in detecting individual parameter nonstationarity.

Kulasekera, K. B. and Lal Saxena, K. M.(1991-7)
Estimation of change point in failure rate models
Journal of Statistical Planning and Inference, 29, 111-124
Keywords: Change point; Increasing decreasing failure rate; Decreasing increasing failure rate

Abstract: Estimation of the change point is considered for a failure rate function $\gamma(x)$ which is an IDFR or a DIFR function. Kernel density estimators and emperical c.d.f. are utilized for estimating $f(x)$ and $F(x)$, respectively, which are then used to estimate $\gamma(x)$ and its change point. Asymptotic properties of the change point estimator are considered. Simulation results are given for different $\gamma(x)$ and kernel functions.

Loader, Clive R.(1991-8)
Inference for a Hazard Rate Change Point
Biometrika, 78, 749-757.
Keywords: Boundary crossing; Change point; Hazard rate; Poisson process.

Abstract: We discuss inference based on the likelihood ratio process for a hazard rate change point. A random change of time scale transforms the empirical process into a Poisson process which enables us to derive large deviation approximations to the significance level of the likelihood ratio test. We derive approximate confidence regions for the change point and joint confidence regions for the change point and size of change. The effect of censorship is also discussed. The methods are illustrated using Stanford heart transplant data, for which the 70 days following the transplant are found to be most critical.

Maiboroda, R. E.(1991-9)
Nonparametric detection of change points from observations with errors
Ukrainian Mathematical Journal, 43, Number 5, 658-661

Abstract: One considers the problem of the detection of the change points in a sequence of random variables, independent in their totality, from observations, representing the sum of the investigated data with an independent nonhomogeneous noise. One constructs a strongly consistent estimator of the change point and a confidence interval. Translated from Ukrainskii Matematicheskii Zhurnal, Vol. 43, No. 5, pp. 706709, May, 1991.

Phillips, Robert F. (1991-10)
A constrained maximum-likelihood approach to estimating switching regressions
Journal of Econometrics, 48, 241-262
Keywords: EM algorithm

Abstract: It is widely known that the likelihood function for the switching-regression model is unbounded if the error variances are unconstrained. This paper shows that a constrained maximum-likelihood formulation makes the likelihood function bounded. Relatively mild constraints are imposed on the parameters, and if the true parameters satisfy the constraints, there is a global maximizer of the likelihood function on the constrained parameter space which is consistent, asymptotically normal, and efficient. A well-known EM algorithm is modified in order to compute constrained maximizers of the likelihood function.

Barry, Daniel and Hartigan, J. A.(1990-1)
An Omnibus Test for Departures from Constant Mean
The Annals of Statistics, 18, 3, 1340-1357.

Abstract: Observations y_i are made at points x_i according to the model $y_i = F(x_i) + e_i$, where the e_i are independent normals with constant variance. In order to decide whether or not $F(x)$ is constant, a likelihood ratio test is constructed, comparing $F(x) \equiv \mu$ with $F(x) = \mu + Z(x)$, where $Z(x)$ is a Brownian motion. The ratio of error variance to Brownian motion variance is chosen to maximize the likelihood, and the resulting maximum likelihood statistic B is used to test departures from constant mean. Its asymptotic distribution is derived and its finite sample size behavior is compared with five other tests. The B-statistic is comparable or superior to each of the tests on the five alternatives considered.

Boukai, Benzion(1990-2)
The asymptotic power function of GLR tests for local change of parameter
Journal of Statistical Planning and Inference, 26, 291-303
Keywords: Change-point; Exponential family; Location parameter family; Scale parameter family; Local power.

Abstract: Retrospective tests are constructed to detect a local shift of parameter of a distribution function occuring at unknown point of time between consecutive independent observations. Three parametric models of distributions are considered; the one parameter exponential family, the location parameter family and the scale parameter family. The class of tests considered is based on the Generalized Likelihood Ratio (GLR) tests, appropriately adapted for such a change-point problem. Asymptotic techniques are used to obtain the limiting distribution of the test statistics, under both the null hypothesis of no change and the change-point alternative. The test statistics, being maximum likelihood type statistics, are shown to converge (in distribution) to the supremum of Brownian motion process, with or without a change-point according to the alternative and the null hypothesis respectively. Analytical expressions for the asymptotic power functions of the proposed tests are provided. These results are then used to provide power comparisons of the tests with those of the Chernoff-Zacks' quasi-Bayesian test.

Brodskiĭ, B. E. and Darkhovskiĭ, B. S.(1990-3)
Asymptotic analysis of some estimators in the a posteriori change-point problem.
(Russian) Teor. Veroyatnost. i Primenen. 35 (1990), no. 3, 551-557; translation in Theory Probab. Appl. 35 (1990), no. 3, 550-556 (1991)

Abstract: For parameters $0 \le \theta \le 1$, $m, h, \beta \in \mathbf{R}$ and sample sizes $N = 1, 2, \cdots$, the authors consider sample vectors $\mathbf{X}^N = (X_1^N, \cdots, X_N^N)$ with $X_n^N = m + h(I(n > [\theta N]) + e_n)$, $e_n \sim N(0, \beta^2)$ and i.i.d. Estimation of θ is considered in the presence of the nuisance parameters m, h, β using the family of statistics $\mathbf{Y}^{N,\delta} = (Y_1^{N,\delta}, \cdots, Y_N^{N,\delta})$, $0 \le \delta \le 1$, where $Y_n^{N,\delta} = p(n/N)^\delta D_n$, with $p(\theta) = \theta(1-\theta)$ and D_n the absolute difference between the sample means of the segments $(X_1^N, \cdots, X_n^N)$ and $(X_{n+1}^N, \cdots, X_N^N)$. A family of estimators θ_N^δ, $0 \le \delta \le 1$, is considered where, for δ, N fixed, $\theta_N^\delta = n_*/N$ for the least $1 \le n_* \le N$ maximizing $Y_n^{N,\delta}$. In particular, $\theta_N^{1/2}$ is the maximum likelihood estimator of θ. The main result is that, for every m, h, β, δ under consideration, and for every $0 < \theta < 1$, $x > 0$, the relation $P_\theta\{|\theta_N^\delta - \theta| > x(2\beta^2/N)\} = 4(1-)x(\lambda_1^x + \lambda_2^x) + o(N)$ holds as $N \to \infty$, where $= \delta\theta + (1-\delta)(1-\theta)$, $\lambda_1 = \lambda_1() = \exp\{-^2\}, \lambda_2 = \lambda_1(1-)$. It follows from here that if $\theta = \frac{1}{2}$ then all estimators from the family are equally efficient and that the overall minimax efficiency in this family is achieved by the maximum likelihood estimator $\delta_N^{1/2}$.

This result is derived using Brownian bridge approximations to the statistics $Y^{N,\delta}$. Analogously, the authors derive asymptotic formulas for probabilities of errors of the first and second kind when testing the hypothesis $\theta = 0$ against the alternative $a \le \theta \le b$ using the criterion $\max_{a \le [\theta N] \le b} Y_n^{N,\delta} > c$ for fixed $0 \le \delta \le 1$, $c > 0$ and $0 < a < b < 1$. (Reviewed by I. Vajda)

Cerrato, M. E. and Blackmer, A. M. (1990-4)
Comparison of models for describing corn yield response to nitrogen fertilizer

Abstract: For the analysis of binary data using logistic regression, there is a need for simple techniques for detecting model inadequacies and suggesting corrective actions. This paper proposes an approach based on the use of line segments to approximate the effect of a predictor. This provides the basis for easily implemented goodness-of-fit tests and graphical methods for identifying the nature of the non-linearities. Methods for selecting the join points are discussed, and their use is illustrated with data from breast cancer patients.

Fu, Yun-Xin and Curnow, R. N.(1990-5)
Maximum Likelihood Estimation of Multiple Change Points
Biometrika, 77, 563-573.
Keywords: Categorical variable; Changed segment; Multiple change point; Protein secondary structure.

Abstract: Maximum likelihood estimation of the locations of changes in sequences of independent categorical random variables when there are known lower bounds on the lengths of the sub-sequences between the change points is discussed. A method is developed which finds one of the maximum likelihood solutions. The problem of estimating the number of changed segments is discussed and some numerical results about the precision of the estimates of the locations of changes are presented. The method also allows the boundary distributions for the changed segments to be different from the distribution for the central region of the changed segments. An application to the prediction of protein helical regions is presented.

Fu, Yun-Xin and Curnow, R. N.(1990-6)
Locating a Changed Segment in a Sequence of Bernoulli Variables
Biometrika, 77, 295-304
Keywords: Bernolli variable; Change point; Changed segment; Markove chain; Protein structure.

Abstract: The theory underlying inferences about a single change point in a sequence of independent Bernoulli variables is extended to the two change points problem where the distance between the two change points is known, namely a changed segment. Both null and nonnull distributions of the log likelihood ratio for testing the hypothesis that there is no changed segment against the alternative that there is one of specified length are derived using recurrence equations. The distribution of the maximum likelihood estimator of the location of the changed segment is derived. Some numerical results for changed segments of length up to 20 are given and computational methods are presented.

Henderson, Robin(1990-7)

A Problem with the Likelihood Ratio Test for a Change-Point Hazard Rate Model

Abstract: A likelihood ratio test for constant hazard against a step change alternative has received considerable attention in recent years. Additional information in the maximum likelihood estimate of the change-point can seriously affect interpretation of test results. Some simple modifications are considered for which exact percentage points can be derived. Monte Carlo power and mean squared error comparisons are encouraging.

Hušková, M.(1990-8)

Some asymptotic results for robust procedures for testing the constancy of regression models over time.

Abstract: The aim of the paper is to derive the asymptotic distribution of test statistics connected with a robust version of the CUSUM (cumulative sums) procedure (the so-called CUSUM M-tests) used for testing the constancy of the regression relationship over time. The results obtained are certain extensions of the Darling-Erds theorem. Reviewed by Hannu Oja

Joseph, Lawrence and Wolfson, David B.(1990-9)

Estimating the change in a renewal process when the data are counts.

Abstract: Let $X_1, \cdots, X_n$ be the interarrival times of a renewal process. A change-point is said to occur in this sequence if $X_1, \cdots, X_\tau$ have a common distribution F while $X_{\tau+1}, \cdots, X_n$ have a distribution G, with $F \neq G$. Usually τ is unknown, and must be estimated from the data. In a renewal process, it is sometimes not possible to observe the renewal times accurately. Instead, the data consist of counts of renewals in successive time intervals. We investigate methods of estimation of the change-point when the only data available are these counts. Examples and simulations are included.

Krishnaiah, P. R.; Miao, Bai Qi and Zhao, Lin Cheng(1990-10)

Local likelihood method in the problems related to change points.

Abstract: Let $X(t)$ be an independent p-dimensional process on $(0, 1]$ such that $X(t) = (t) + V(t)$, $0 < t \leq 1$, where $V(t)$ is an independent normal process with mean 0 and covariances $\Lambda(t) > 0$, (t) is a p-dimensional real vector function. Suppose that (t) is a left-continuous function with the jump points $t_1 < t_2 < \cdots < t_q$, which are called the change points of the mean function of $X(t)$, and that $\Lambda(t) = \Lambda > 0$ for $t \in (0, 1]$, where Λ is an unknown matrix. Put $X = X(i/N)$. Take a positive integer m between $\log N$ and $N/2$ such that N/m and $m/\log N$ tend to infinity as N tends to infinity. Based on a likelihood ratio statistic for testing the null hypothesis of equality of mean vectors against all alternatives in two normal populations, calculated from the sample of $X_{k-m+1}, X_{k-m+2}, \cdots, X_k$ and $X_{k+1}, X_{k+2}, \cdots, X_{k+m}$, for $k = m, m+1, \cdots, N-m$, the authors propose a strongly consistent estimate of $(q, t_1, \cdots, t_q)$. Four other cases of similar models with (t) and $\Lambda(t)$ of a different structure are considered and strongly consistent estimates of the number of change points and the location of the change points are derived. (Reviewed by Nariaki Sugiura)

Maĭboroda, R. E.(1990-11)

A nonparametric method for searching for change points for multidimensional observations.
(Russian) Teor. Veroyatnost. i Primenen. 35 (1990), no. 3, 582-586; translation in Theory Probab. Appl. 35 (1990), no. 3, 590-593 (1991)

Abstract: Let $\xi_1, \cdots, \xi_N$ be a sequence of independent random vectors in R^d. Denote by F_j the distribution function of ξ_j. The problem of change point detection may be described as follows: (i) test the hypothesis H: there is an F such that $F_j = F$, $j = 1, \cdots, N$, against the alternative A: there are $t \in (0, 1)$ and $F \neq G$ such that $F_j = F$ for $j \leq tN$ and $F_j = G$ for $j > tN$; (ii) if A is true, find an estimator for t. If the dimension d is large, it is natural to use methods for reducing it and then study the data in spaces of lower dimension, looking for possible change points. In the present paper the problem of constructing such two-step tests and change-point estimators based on quantile methods is considered. (Reviewed by J. A. Melamed)

Maĭboroda, R. E.(1990-12)
A median test for the homogeneity of a sample.
(Russian) Teor. Veroyatnost. i Mat. Statist. No. 42 (1990), 82-87; translation in Theory Probab. Math. Statist. No. 42, (1991), 95-101

Abstract: Suppose that random variables $\xi_1, \cdots, \xi_n$, with distribution functions $F_1, \cdots, F_n$, respectively, are observed. Our aim is to test the hypothesis of homogeneity, i.e., $F_i = F_j$, $i, j = 1, \cdots, n$, against the alternatives of changes of the medians in the sequence $F_1, \cdots, F_n$ given by the following model: $F_i = G_k$ for $[nt_k] < i \leq [nt_{k+1}]$, where $G_0, G_1, \cdots, G_m$ ($G_{k-1} \neq G_k$) are some distribution functions, $t_1, \cdots, t_m$ some fixed numbers, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = 1$. Let M_n be the median of the whole sample, and let $\chi_k^+ =$ number of

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observations among $\xi_1, \cdots, \xi_k$ larger than M_n, $\chi_k^- =$ number of observations among $\xi_1, \cdots, \xi_k$ smaller than M_n. The set statistic is $L_n = \sup_{1 \le k \le n} |\chi_k^+ - \chi_k^-|$. For $n \to \infty$, it is shown that, under the alternatives, L_n has (stochastically) the rate of increase n, and that, under the alternative of only one change, the natural estimator of the change-point is consistent. (Reviewed by Z. Šidák)

Müller, Hans-Georg and Wang, Jane-Ling(1990-13)
Nonparametric Analysis of Changes in Hazard Rates for Censored Survival Data: An Alternative to Change-Point Models
Biometrika, 77, 305-314
Keywords: Asymptotic distribution; Change-point; Curve estimation; Estimation of derivatives; Kernal method; Leukaemia; Lo-Singh representation; Nelson estimator.

Abstract: As a nonparametric estimator for the point of the most rapid change of a hazard rate we propose the location of an extremum of a nonparametric estimate of the derivative, or equivalently, of a zero of a nonparametric estimate of the second derivative. Using the kernel method for the nonparametric estimation of derivatives of the hazard rate, the asymptotic local limiting distribution and uniform consistency are applied to prove consistency and to find the limiting distribution of these estimators under random censoring and to construct confidence intervals both for the derivatives of the hazard rate and for the point of most rapid change. An application to leukaemia data illustrates this concept, and we discuss its relations to change-point modeling. The Monte Carlo method is used to assess the reliability of finite sample analysis, and in particular of the given example.

Ng, Vee Ming (1990-14)
Bayesian analysis of linear models exhibiting changes in mean and precision at an unknown time point
Communications in Statistics: Theory and Methods, 19, 111-120
Keywords: Switching regression

Abstract: This paper analysis a linear model in which both the mean and the precision change exactly once at an unknown point in time. Posterior distributions are found for unknown time point at which the changes occurred and for the ratio of the precisions. The Bayesian predictive distribution of k future observations is also derived. It is shown that the unconditional posterior distribution of the ratio of precision is a mixture of F-type distributions and the predictive distribution is a mixture of multivariate t distribution.

Noura, A.A. and Read, K.L.Q.(1990-15)
Proportional Hazards Changepoint Models in Survival Analysis
Journal of the Royal Statistical Society: Series C (Applied Statistics) 39, 241-253

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Keywords: Changepoints; GLIM; Maximum likelihood; Proportional hazard; Survival analysis; Weibull

Abstract: This paper describes an approach to proportional hazards analysis of survival data with covariates by parametric modelling of the base-line hazard in terms of piece-wise distributions. Maximum likelihood estimation using GLIM and an iterative method is straightforward. Applications of the method and its use with competing risks are given, in which a two-piece Weibull fit is clearly superior to the simple Weibull model. Wide generality results from the fact that any given montonic increasing transformation may be applied to the base-line hazard parameter, it can be expected that piecewise models of this kind will usefully describe many proportional hazards survival processes involving changepoints at which the ruling conditions suddenly alter.

Pham, Tuan D. and Nguyen, Hung T.(1990-16)
Strong consistency of the maximum likelihood estimators in the change-point hazard rate model.
Statistics 21, 203-216

Abstract: A procedure for estimating the parameters in the change-point hazard rate model by maximizing the likelihood over suitable random compact sets is proposed. In particular, the parameters of a piecewise hazard function with a single unknown change point τ are estimated, the estimators are shown to be strongly consistent and their asymptotic distributions are given. (Reviewed by M. M. Siddiqui)

Ritov, Y.(1990-17)
Asymptotic Efficient Estimation of the Change Point with Unknown Distributions
The Annals of Statistics, 18, 1829-1839.

Abstract: Suppose $X_1, \cdots, X_n$ are distributed according to a probability measure under which $X_1, \cdots, X_n$ are independent, $X_1 \sim F_0$, for $i = 1, \cdots, [\theta_n n]$ and $X_i \sim F^{(n)}$ for $i = [\theta_n n] + 1, \cdots, n$ where $[x]$ denotes the integer part of x. In this paper we consider the asymptotic efficient estimation of θ_n when the distributions are not known. Our estimator is efficient in the sense that if $F^{(n)} = F_{\eta_n}, \eta_n \to 0$ and $\{F_\eta\}$ is a regular one-dimensional parametric family of distributions, then the estimator is asymptotically equivalent to the best regular estimator.

Yi-Ching Yao(1990-18)
On the asymptotic behavior of a class of nonparametric tests for a change-point problem

Abstract: A class of linear rank statistics is considered for testing a sequence of independent random variables with common distribution against alternatives involving a change in distribution at an unknown time point. It is shown that, under the null hypothesis and suitably normalized, this class of statistics converges in distribution to the double exponential extreme value distribution.

Antoch, J. and Hušková, M.(1989-1)
Some M-tests for detection of a change in linear models.
*Proceedings of the Fourth Prague Symposium on Asymptotic Statistics (Prague, 1988),
123-136, Charles Univ., Prague, 1989.*

Abstract: Consider independent random variables $X_1, \cdots, X_n$ observed at ordered time points $t_1 < \cdots < t_n$, respectively, with X_i having distribution function $F(x - c_i\theta_i')$, where $c_i = (c_{i1}, \cdots, c_{ip})$ are known regression vectors and $\theta_i = (\theta_{i1}, \cdots, \theta_{ip})$ are unknown regression vectors considered as parameters. Let $H_0 : \theta_1 = \cdots = \theta_n = \theta_0$ (usually unknown) against the change-point alternative $H_1 : \bigcup_{m=1}^{n-1} H_{m1}$, where $H_{m1} : \theta_1 = \cdots = \theta_m \neq \theta_{m+1} = \cdots = \theta_n$, $1 \leq m < n$. Under certain regularity conditions, some nonrecursive and recursive tests based on M-statistics are discussed and a few simulation results are presented. Reviewed by Sri Gopal Mohanty.

Darkhovskiĭ, B. S.(1989-2)
**Nonparametric methods in problems on the change point of a random
sequence.**
*(Russian) Statistics and control of random processes (Russian) (Preila, 1987), 57-70,
"Nauka", Moscow, 1989.*

Abstract: The following general model for analyzing asymptotic problems of change points in random sequences is adopted: Let $\theta = (\theta_1, \cdots, \theta_k)$ be the vector of "change points", $0 = \theta_0 < \theta_1 < \cdots < \theta_k < \theta_{k+1} = 1$, and let $\phi(t)$, $t \in [0,1]$, be a function expressing the "disorders" in the mean value. For fixed N, we observe random variables $x^N(n)$, $n = 1, \cdots, N$, having the form $x^N(n) = \phi(n/N) + \xi^i(n)$ if $[\theta_{i-1}N] < n \leq [\theta_i N]$, where $\xi^i(n)$ $(i = 1, \cdots, k+1; n = 1, 2, \cdots)$ are some random variables with zero mean values. The phrase "nonparametric" in the title means that no specific assumptions on the (joint) distributions of $\xi^i(n)$ are made; instead, only some types of mixing conditions are required for the results. Four special cases are considered: (1) "instant disorder" in which case $k = 1$, $\phi(t) = 0$ for $t \in [0, \theta_1]$, $\phi(t) = a \neq 0$ for $t \in (\theta_1, 1]$; (2) "gradual disorder" in which case $k = 2$, $\phi(t) = 0$ for $t \in [0, \theta_1]$, $\phi(t)$ is some Lipschitz function growing from 0 to $a \neq 0$ for $t \in (\theta_1, \theta_2]$,

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and $\phi(t) = a$ for $t \in (\theta_2, 1]$; (3) "disappearing disorder" in which case $k = 2$, $\phi(t) = 0$ for $t \in [0, \theta_1) \cup (\theta_2, 1]$, and $\phi(t) = a \neq 0$ for $t \in [\theta_1, \theta_2]$; (4) "multiple disorder" in which case $k \geq 2$, $\phi(t) = a_i$ for $t \in (\theta_{i-1}, \theta_i]$, $a_i \neq a_{i+1}$, $i = 1, \cdots, k+1$. Appropriate statistics for estimating θ are displayed for all these cases here, but for the construction of estimators $\hat{\theta}$ themselves the author refers to his previous papers [Teor. Veroyatnost. i Primenen. 29 (1984), no. 3, 464–473; ibid. 30 (1985), no. 4, 795–799].

The present paper gives, for $N \to \infty$, the rate of convergence for probabilities of deviations of $\hat{\theta}$ from θ, a functional limit theorem for statistics used for estimation, and an asymptotic minimax result for all estimators of θ, implying that the proposed estimators are asymptotically optimal. Finally, the problem in sequential testing of hypotheses is treated concerning the quickest detection of "disorder" in a sequence when only a finite memory is at our disposal, and the proposed method is shown to be asymptotically optimal. (Reviewed by Z. Šidák)

De Veaux, Richard D. (1989-3)
Mixtures of linear regressions
Computational Statistics and Data Analysis, 8, 227-245
Keywords: EM algorithm; Switching regression

Abstract: The purpose of this article is to develop the technology of models based on mixtures of linear regressions and, in particular, to draw out the relevance of the EM algorithm to the associated maximum likelihood equations. A n-consistent starting point for the EM algorithm is presented. The data from an experiment in music perception are analyzed using this technology. Performance of the estimators are examined via both simulated and actual data sets.

Hawkins, D. L.(1989-4)
A U-I approach to retrospective testing for shifting parameters in a linear model.
Comm. Statist. Theory Methods 18, 3117–3134.
Keywords: Union-intersection principle; Change point

Abstract:A union-intersection procedure is derived for detecting parameter shifts in a linear model. The procedure allows unlimited (and the author suggests possibly data-directed) investigation of any detected shift via testing of sub-hypotheses with controlled error rates. Use of weak convergence theory results in suitable critical values and a study of local power. Reviewed by Bert M. Steece

Hawkins, D. L.(1989-5)
Estimating changes in a multi-parameter exponential family.
Communication in Statistics Theory and Methods 18, 3595-3623

Abstract: Let $X_1, \cdots, X_n$ be i.i.d. random vectors with X_i distributed according to the probability density function $f_i(x) = f(x; \theta_0)\mathbf{I}(i < \tau) + f(x; \theta_1)\mathbf{I}(i \geq \tau)$, $i \geq 1$, where $f(x; \theta) = \exp\{\theta^T x - \psi(\theta)\}m(x)$, $x \in \mathbf{R}^p$, $\theta \in \Theta \subseteq \mathbf{R}^p$. Given n, the problem is to test whether a change in θ has occurred in the sequence $X_1, \cdots, X_n$, i.e., if $\tau \leq n$. A likelihood ratio test is considered. Upon rejection of this hypothesis, the change-point τ as well as the pre- and post-change values θ_0 and θ_1 are estimated by maximum likelihood. Asymptotic distributions of the underlying statistics are studied. Monte Carlo results for moderate samples are given. (Reviewed by Ryszard Zieliński)

Residual partial sum limit process for regression models with applications to detecting parameter changes at unknown times

Abstract: Limit processes for sequences of stochastic processes defined by partial sums of linear functions of regression residuals are derived. They are Gaussian and are functions of standard Brownian motion. Cramér-von Mises type functionals defined on the partial sum processes are shown to converge in distribution to the same functionals defined on the limit processes. This result is then applied to derive the asymptotic forms of two-sided change detection statistics for linear regression models. These are derived for a variety of weight sequences and are shown to involve sums of Cramér-von Mises type stochastic integrals. Finally a methodology is developed to derive distributions of these stochastic integrals for the case of harmonic regression. This methodology is applicable to more general situations.

The likelihood ratio test for a change-point in simple linear regression

Abstract: We consider likelihood ratio tests to detect a change-point in simple linear regression (a) when the alternative specifies that only the intercept changes and (b) when the alternative permits the intercept and the slope to change. Approximations for the significance level are obtained under reasonably general assumptions about the empirical distribution of the independent variable. The approximations are compared with simulations in order to assess their accuracy. For the model in which only the intercept is allowed to change, a confidence region for the change-point and an approximate joint confidence region for the change-point, the difference in intercepts, and the slope are obtained by inversion of the appropriate likelihood ratio tests.

Knowles, Mark and Siegmund, David(1989-8)

On Hotelling's Approach to Testing for a Nonlinear Parameter in Regression

Abstract: The method suggested by Hotelling (1939) to test for a nonlinear parameter in a regression model is reviewed. Using the method of Weyl (1939), we obtain a simple expression for the volume of a tube about a two dimensional manifold with boundary embedded in the unit sphere in $\mathbf{R}^n$. Applications to testing for a single harmonic of undetermined frequency and phase and to testing for a change-point in linear regression are discussed.

Lee, Jae Chang and Song, Il Seong (1989-9)

A test procedure for change in level occurring at unknown points

Abstract : A procedure is considered for the problem of testing whether there exist changes in location at possibly two points in a sequence of independent random variables which are successively drawn from a normal population. A test statistic based on a modified likelihood ratio is proposed and its asymptotic null distribution is derived through a stochastic process representation. A small-sample power comparison is made by the Monte Carlo method.

Nyblom, Jukka(1989-10)

Testing for the Constancy of Parameters Over Time

Abstract: Tests are proposed for detecting possible changes in parameters when the observations are obtained sequentially in time. While deriving the tests the alternative one has in mind specifies the parameter process as a martingale. The distribution theory of these tests relies on the large-sample results; that is, only the limiting null distributions are known (except in very special cases). The main tool in establishing these limiting distributions is weak convergence of stochastic processes. Suppose that we have vector-valued observations $\mathbf{x}_1, \ldots, \mathbf{x}_n$ obtained sequentially in time (or ordered in some other linear fashion). Their joint distribution is described by determining the initial distribution for $\mathbf{x}_1$ and the conditional distribution for each $\mathbf{x}_k$ given the past up to $\mathbf{x}_{k-1}$. Suppose further that these distributions depend on a p-dimensional parameter vector θ. At least locally (i.e., in a short time period) this may be more or less legitimate. In the long run, however, the possibility of some changes in the observation-generating process should be taken into account. Specifically,

it is assumed here that those changes occur through a parameter variation in the form of a martingale. The martingale specification has an advantage of covering several types of departure of constancy: for example, a single jump at an unknown time point (the so-called change-point model) or slow random variation (typically random walk). The tests are derived by first finding the locally most powerful test against a martingale-type alternative when the starting value of the parameter process is known. After some simplification a test having a known numerically tractable limiting distribution is developed. When the starting point is unknown an efficient estimate is substituted for it. In addition, the corresponding limiting distribution is established. The proposed tests turn out to be based on cumulative sums of the score function (the derivative of the log-likelihood).

Ploberger, W, Krmer, W., Kontrus, K. (1989-11)
A new test for structural stability in the linear regression model
Journal of Econometrics, 40, 2 , 307-318

Abstract: We propose a new test for the constancy of regression coefficients in linear models. The test does not require that possible change points be known. We derive the limiting null distribution of the test statistic, prove that the test has non-trivial power against many local alternatives, and show that it compares favourably to both the CUSUM and CUSUM of squares tests.

Schulze, U. (1989-12)
Switching regression models
Nonlinear Regression, Functional Relations and Robust Methods. Statistical Methods of Model Building. Volume 2, 73-105
Bunke, Helge (ed.) and Bunke, Olaf (ed.)
John Wiley and Sons (New York; Chichester)
Keywords: Segmented regression; Change point

Basu, A. P.; Ghosh, J. K. and Joshi, S. N.(1988-1)
On estimating change point in a failure rate.
Statistical decision theory and related topics, IV, Vol. 2 (West Lafayette, Ind., 1986), 239-252, Springer, New York, 1988.

Abstract: In reliability theory a widely accepted procedure is to apply "burn-in" techniques to screen out defectives and increase the life of the remaining surviving items. It is assumed that the hazard rate of the life distribution is a truncated "bathtub" model, i.e., $r(t) = \lambda(t)$ if $0 \leq t \leq \tau$, $r(t) = \lambda_0$ if $t > \tau$, where $\lambda(t)$ is nonincreasing and $\lambda(t) \geq \lambda_0$ with equality only if $\lambda(t)$ is strictly decreasing in $(\tau - \delta, \tau]$ for some $\delta > 0$. The threshold τ is to be

estimated. In this paper two estimators are derived for τ and a proof of their consistency is presented. In the special case of the hazard rate mentioned above, the authors introduce a restricted maximum likelihood estimate for τ and compare it with a consistent estimate proposed by H. T. Nguyen et al. [Biometrika 71 (1984), no. 2, 299-304]. If the estimate for τ is known, items could be tested up to time τ and only survivors sold. This would be one way of screening. Screening is usually provided in a different way by subjecting items to thermal or electrical stress. (Reviewed by A. Balogh)

Bennett, G. W. (1988-2)
Determination of anaerobic threshold
The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 16, 307-316
Keywords: Segmented regression; Nonlinear regression

Abstract: This report describes the estimation of parameters for a model of ventilation(y) by human subjects as a two-segmente linaer function of oxygen uptake(x). Experiment data were supplied by R. L. Hughsen of the Department of kinesoilogy. The analysis of one of nine available sets of data is described in data.

Carlstein, E.(1988-3)
Nonparametric Change-Point Estimation
The Annals of Statistics, 16, 188-197

Abstract: Consider a sequence of independent random variables $\{X_i : 1 \leq i \leq n\}$ having cdf F for $i \leq \theta n$ and cdf G otherwise. A class of strongly consistent estimators for the change-point $\theta \in (0,1)$ is proposed. The estimators require no knowledge of the functional forms or parametric families of F and G. Furthermore, F and G need not differ in their means (or other measure of location). The only requirement is that F and G differ on a set of positive probability. The proof of consistency provides rates of convergence and bounds on the error probability for the estimators. The estimators are applied to two well-known data sets, in both cases yielding results in close agreement with previous parametric analyses. A simulation study is conducted, showing that the estimators perform well even when F and G share the same mean, variance and skewness.

Chen, Xi Ru(1988-4)
Inference in a simple change-point model
Scientia Sinica. Series A, Mathematical, physical, astronomical and technical sciences,31, 6, 654 -667

Abstract: The author considers the change-point problem in the following situation: $X_1, \cdots, X_n$ are independent random variables, X_i has the distribution function $F(x - \theta_i)$, $i = 1, \cdots, n$,

where $\theta_1 = \cdots = \theta_t = a$ and $\theta_{t+1} = \cdots = \theta_n = a + \theta$ with a, t, θ unknown. He proposes test procedures for testing the hypothesis $H_0: t \geq n$ against $H_1: t < n$ and estimators of θ and t. Asymptotic properties are derived. As examples of the distribution function F the author considers (a) the normal distribution with parameters 0 and σ^2 (known), (b) the normal distribution with parameters 0 and σ^2 (unknown), and (c) F satisfying certain regularity conditions but otherwise unknown. (Reviewed by M. Huškov)

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Csörgö, M. and Horváth, L.(1988-5)

Nonparametric Methods for Changepoint Problems

P.R. Krishnaiah and C. R. Rao, eds., Handbook of statistics, Vol. 7, Elsevier Science Publishers B. V.

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Abstract: Sequential as well as non-sequential procedures are usually based on parametric or nonparametric models for changepoint problems, allowing at most one change (AMOC) or, possibly, more than one change.

In order to simplify the discussion, this exposition is concerned with the nonparametric AMOC setting only. In section 2 we deal with non-sequential nonparametric AMOC procedures. In particular, Section 2.1 is on change in the mean problems, where we illustrate that a large number of nonparametric as well as parametric modelling of AMOC problems result in the same test statistic whose asymptotic theory we describe in some detail. Section 2.2 is concerned with general rank statistics with quantile and Wilcoxon type scores whose asymptotics are described in terms of a two-time parameter stochastic process. The asymptotic distributions of functionals of this stochastic process which are of interest in changepoint problems are identified in this section, where references are also given to their available tabulations in the literature. In Section 2.3 U-statistics type processes are considered for the nonparametric AMOC problem. Their astmptotics yields a Gaussian process whose variance is that of a Brownian. Section 2.4 deals with detecting change in the intensity parameter of a renewal process. In chapter 3 we consider the problem of sequential detection of change in a random sequence.

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Csörgö, M. and Horváth, L.(1988-6)

Invariance principles for changepoint problems

Journal of Multivariate Analysis, 27, 151-168

Keywords: U-statistics; Wiener process; Weighted metrics; Weak approximations; Consistency

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Abstract: We study the asymptotic behaviour of U-statistics type processes which can be used for detecting a changepoint of a random sequence. Invariance principles are proved for these processes.

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Erasmus, C. M. and Lombard, F.(1988-7)
Asymptotic Distributions of Quadratic Forms Occurring in Changepoint Problems
The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 16, 259-268
Keywords: Cusum process; Ito integral; Multiple changepoints.

Abstract: A stochastic-process approach is used to derive the asymptotic distributions of quadratic forms occurring in the analysis of changepoint data

Hušková, M.(1988-8)
Adaptive procedures for detection of change.
Statist. Decisions 6, 137-148.

Abstract: An adaptive test procedure based on ranks is developed for the detection of small changes in a regression model at an unknown time point when the distribution of the errors is unknown.

Hušková, M.(1988-9)
Recursive M-test for detection of change.
Sequential Anal. 7, 75-90.

Abstract: P. K. Sen(1984) proposed recursive and nonrecursive procedures based on M-statistics to test for a possible change in the regression relationships occurring at an unknown time point. Here two modifications of Sen's recursive procedure for the location model are considered. These procedures are based on recursive M-estimators and a recursive one-step version of M-estimators. They are shown to be asymptotically equivalent to Sen's procedure; further, they are easily computable

Hušková, M.(1988-10)
Nonparametric tests for change in regression models at an unknown time point.
International Workshop on Theory and Practice in Data Analysis (Berlin, 1988), 101-124, Rep. MATH, 89-01, Akad. Wiss. DDR, Berlin, 1989

Abstract: This paper concerns test procedures based on M- and R-estimators for a possible change in the regression relationships occurring at an unknown time point. Some asymptotic properties of the test procedures are presented.

James, Barry; James, Kang Ling and Siegmund, David(1988-11)
Conditional Boundary Crossing Probabilities, with Applications to Change-Point Problems
The Annals of Probability, 16, 825-839

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Abstract: For normal random walks $S_1, S_2, \ldots$, formed from independent identically distributed random variables $X_1, X_2, \ldots$, we determine the asymptotic behavior under regularity conditions of

$$P(S_n > mg(n/m) \text{ for some } n < m \mid S_m = m\xi_0, U_m = m\lambda_0), \quad \xi_0 < g(1),$$

where $U_m = X_1^2 + \cdots + X_m^2$. The result is applied to a normal change-point problem to approximate null distributions of test statistics and to obtain approximate confidence sets for the change-point.

Leipus, R. (1988-12)

Weak convergence of two-parameter empirical fields in change-point problems

LitovskiĭMatematicheskiĭSbornik , 28, 716-723
Keywords: Hypothesis testing

Abstract: In this paper two-parameter empirical processes appearing in the problem of testing hypotheses about the existence of a change-point in a sequence of independent random variables are considered and functional limit theorems in the Skorokhod space $D[0,1]^2$ are proved under a sequence of null hypotheses and for contiguous alternatives. (Reviewed by Zuzana Prškov)

Lombard, F. (1988-13)

Detecting Change Points by Fourier Analysis

Technometrics, Vol. 30, No. 3. (Aug., 1988), pp. 305-310.
Keywords: Commulative sum; Fourier expantion; Orthogonal components; Rank statistics; Smooth CUSUM.

Abstract: The cumulative sum (CUSUM) is a basic diagnostic tool in the analysis of change-point data. It is shown that Fourier analysis of the CUSUM can be a useful supplementary tool in such analyses. The technique is applied to three data sets that have appeared previously in the statistical literature.

MacLean, Charles J. (1988-14)

Assessing changes in risk factor effect over multiple levels of severity

American Journal of Epidemiology, 127, 663-673
Keywords: Logistic regression

Abstract : Use of the logistic function has become standard for regression of proportions onto risk factors, but where multiple levels of response are recorded, several different techniques are common. For most purposes, the "inclusive" method is most appropriate. However, if

investigation is focused directly on the pattern of risk factor effects over levels of severity, the inclusive method is too conservative, because its overlapping categories tend to minimize any real differences among risk factor effects at separate levels. Several alternative techniques maximize these estimated differences, but interpretation of their results is much more sensitive to the assumed model than is estimation of category frequencies. In most cases, it is not possible to know precisely what model actually generated the data at hand. Therefore, a procedure is recommended which combines the safety of the inclusive method with an adjuvant technique called the "continuation ratio" method which has greater power to disclose differences between risk factor effects at different levels of response.

Miao, B. Q.(1988-15)
Inference in a model with at most one slope-change point
Journal of Multivariate Analysis, 27, 375-391
Keywords: Asymptotic distribution; Change point; Detection; Gaussian process; Interval estimate

Abstract: In this paper the problem of slope-change point in linear regression model is discussed with the help of the theory of Gaussian process. The distribution of the estimators of the change point proposed in this paper can be approximated by the first type of extremal distribution. Based on this fact, the detection and interval estimation of a change-point in various situations are discussed.

Miao, B. Q.; Zhao, L. C.(1988-16)
Detection of change points using rank methods
Communications in statistics. theory and methods, 17, 3207 -3217

Abstract: In this paper, the authors consider the detection and estimation of change points of local parameters using nonparametric methods. Specifically, localization and rank statistics are used to obtain estimators which are shown to be strongly consistent when the distribution function from which the observed data came is continuous and has a unique median. The method can be extended to change points of scale parameters and to location parameters of directional data. (Reviewed by Margaret P. Gessaman)

Nabeya, S. and Tanaka, K. (1988-17)
Asymptotic Theory of a Test for the Constancy of Regression Coefficients Against the Random Walk Alternative
The Annals of Statistics, 16, 1, 218-235.

Abstract: The LBI (locally best invariant) test is suggested under normality for the constancy of regression coefficients against the alternative hypothesis that one component of

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the coefficients follows a random walk process. We discuss the limiting null behavior of the test statistic without assuming normality under two situations, where the initial value of the random walk process is known or unknown. The limiting distribution is that of a quadratic functional of Brownian motion and the characteristic function is obtained from the Fredholm determinant associated with a certain integral equation. The limiting distribution is then computed by numerical inversion of the characteristic function.

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van de Geer, S. A.(1988-18)

Regression analysis and empirical processes.

CWI Tract, 45. Stichting Mathematisch Centrum, Centrum voor Wiskunde en Informatica, Amsterdam, 1988

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Abstract: This monograph is another impressive addition to this excellent series. Given n observations $(\mathbf{x}_k, \mathbf{y}_k)$, $k = 1, \cdots, n$, which are assumed to satisfy the equations $\mathbf{y}_k = g(\mathbf{x}_k) + \epsilon_k$ $(k = 1, \cdots, n)$, where ϵ_k are independent random errors with mean zero and finite variance and the $\mathbf{x}_k$ are vectors in some Euclidean space, the aim of this exposition is to investigate the least squares method for estimating the function g. The problem is posed as follows: find a $\widehat{g}_n$ such that $n^{-1} \sum_{k=1}^{n} (\mathbf{y}_k - g(\mathbf{x}_k))^2$ is minimized, where the minimization is over the class G of regression functions that one considers feasible. Linear regression is a special case, where g is known up to a finite-dimensional parameter. This more general formulation is usually called nonlinear regression. The class G is $G = \{g = g(\cdot, \theta) : \theta \in \Theta\}$ with $\Theta \subset \mathbf{R}^r$, $r \geq 1$. Due to the possible nonlinearity of $q(\cdot, \theta)$ the approach taken to study this least squares problem is mostly asymptotic, and the distributions of the random errors $epsilon_k$ are not specified. The function g itself is regarded as an unknown parameter. The concepts and techniques of empirical process theory (mainly of the Vapnik-Chervonenkis, Dudley, and Pollard type theories) provide the asymptotic tools. Chapter 2 gives an overview of the history of the asymptotics that led to the uniform law of large numbers, which goes from sets via bounded functions to integrable functions. This type of result is helpful for proving the consistency of $\widehat{g}_n$. Hence the just mentioned uniform law of large numbers is extended here to the case of nonidentically distributed random variables. Using these tools, a general consistency theorem is proved in Chapter 3, followed by some applications to nonlinear and nonparametric regression, and to two-phase regression. Chapter 4 summarizes some results on uniform central limit theorems, which are then used in Chapter 5 to prove the asymptotic normality of some specific least squares estimators. In Chapter 6 the techniques for proving uniform central limit theorems are exploited to obtain rates of convergence for $\widehat{g}_n$ in case of finite-, as well as infinite-dimensional models. Here it is shown to what extent the rate at which the estimation error goes to zero can be deduced from the entropy of the class of regression functions G. This general theory is then applied to two-phase regression, and the results are compared with those of Chapter 5. Two-phase regression is closely related to change-point models, and Chapter 7 is devoted to studying some of these models via likelihood ratio methods. In Chapter 8 the least squares estimators of a specific model are computed, using simulated as well as real data.

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This monograph is highly recommended to mathematical statisticians, to probabilists who are interested in techniques of modern asymptotics, and to students of these disciplines and of their interface. (Reviewed by M. Csörgö)

Praagman, Jaap(1988-19)
Bahadur Efficiency of Rank Tests for the Change-Point Problem
The Annals of Statistics, 16, 198-217

Abstract: A sequence of independent random variables $X_1, X_2, \cdots, X_N$ is said to have a change point if $X_1, X_2, \cdots, X_n$ have a common distribution F and $X_{n+1}, \cdots, X_N$ have a common distribution $G, G \neq F$. Consider the problem of testing the null hypothesis of no change against the alternative of a change $G < F$ at an unknown change point n. Two classes of statistics based upon two-sample linear rank statistics (max- and sum-type) are compared in terms of their Bahadur efficiency. It is shown that for every sequence of sum-type statistics a sequence of max-type statistics can be constructed with at least the same Bahadur slope at all possible alternatives. Special attention is paid to alternatives close to the null hypothesis.

Siegmund, David(1988-20)
Confidence Sets in Change-Point Problems
International Statistical Review / Revue Internationale de Statistique, Vol. 56, No. 1. (Apr., 1988), pp. 31-48.
Keywords: Boundray crossing problem; Change-point; Likelihood ratio.

Abstract: Several methods are discussed for confidence set estimation of a change-point in a sequence of independent observations from completely specified distributions. The method based on the likelihood ratio statistic is extended to the case of independent observations from an exponential family. Joint confidence sets for the change-point and the parameters of the exponential family are also considered.

Worsley, K. J.(1988-21)
Exact Percentage Points of the Likelihood-Ratio Test for a Change-Point Hazard-Rate Model
Biometrics, Vol. 44, 1, 259-263

Abstract:We wish to test that the hazard rate of survival or failure-time data is constant against the alternative of a change in hazard after an unspecified time. The likelihood ratio is unbounded but the exact null distribution of a restricted likelihood-ratio test statistic is found. This distribution is not affected by Type II censoring but it does depend very strongly on the interval in which the unknown change-point is assumed to lie. Some exact percentage

points are given which are much larger than simulated points that have been reported in the literature.

Yao, Yi-Ching(1988-22)

Estimating the number of change-points via Schwarz' criterion

Statistics and Probability Letters, 6, 181-189

Keywords: Change-points; Dimension of a model; Schwarz' criterion

Abstract: An estimator of the number of change-points in an independent normal sequence is proposed via Schwarz' criterion. Weak consistency of this estimator is established.

Bhattacharya, P. K.(1987-1)

Maximum likelihood estimation of a change-point in the distribution of independent random variables: General multiparameter case

Journal of Multivariate Analysis, 23, 183-208

Keywords: Change-point; Maximum likelihood estimator; Weak convergence; Brownian motion

Abstract: In a sequence of n independent random variables the *pdf* changes from $f(x, \theta)$ to $f(x, \theta + \delta\nu_n^{-1})$ after the first $n\lambda$ variables. The problem is to estimate $\lambda \in (0, 1)$, where θ and δ are unknown d-dim parameters and $\nu_n \to \infty$ slower than $n^{1/2}$. Let $\widehat{\lambda_n}$ denote the maximum likelihood estimator (mle) of λ. Analyzing the local behavior of the likelihood function near the true parameter values it is shown under regularity conditions that if $n\nu_n^2(\widehat{\lambda_n} - \lambda)$ is bounded in probability as $n \to \infty$, then it converges in law to the time $T_{(\delta j \delta)}1/2$ at which a two-sided Brownian motion (B.M.) with drift $1/2(\delta' J\delta)^{1/2}\|t\|$ on $(-\infty, \infty)$ attains its *a.s.* unique minimum, whereJ denotes the Fisher-information matrix. This generalizes the result for small change in mean of univariate normal random variables obtained by Bhattacharya and Brockwell **(1976,Z. Warsch. Verw. Gebiete37, 5175)** who also derived the distribution of T for $\mu > 0$. For the general case an alternative estimator is constructed by a three-step procedure which is shown to have the above asymptotic distribution. In the important case of multiparameter exponential families, the construction of this estimator is considerably simplified.

Csörgö, M. and Horváth, L.(1987-2)

Nonparametric tests for the changepoint problem

Journal of Statistical Planning and Inference, 17, 1-9

Keywords: Changepoint; Quantile score statistics; Kiefer process

Abstract: Let $X_1, \cdots, X_{[n\lambda]}, X_{[n\lambda]+1}, \cdots, X_n$ be independent, continuous random variables such that $X_i, I = 1, \cdots, [n\lambda]$, have distribution H, and $X_i, I = [n\lambda] + 1, \cdots, n$, have

distribution function G. The integer $[n\lambda]$ refers to the changepoint of the random sequence. We propose nonparametric procedures for testing $\lambda = 1$(no changepoint) versus $\lambda \in (0,1)$. An asymptotically strongly consistent estimator of λ is also given.

Davies, Robert B.(1987-3)

Hypothesis testing when a nuisance parameter is present only under the alternative

Biometrika, 74, 33-43

Keywords: Chi-squared process; Frequency component; Hypothesis test ; Maximum; Nuisance parameter; Quick test; Two-phase regression; Time series; Upcrossing

Abstract:We wish to test a simple hypothesis against a family of alternatives indexed by a one-dimensional parameter, θ. We use a test derived from the corresponding family of test statistics appropriate for the case when θ is given. Davies (1977, Biometrika, 64, 247-254) introduced this problem when these test statistics had normal distributions. The present paper considers the case when their distribution is chi-squared. The results are applied to the detection of a discrete frequency component of unknown frequency in a time series. In addition quick methods for finding approximate significance probabilities are given for both the normal and chi-squared cases and applied to the two-phase regression problem in the normal case.

Florens-Zmirou, Daniéle (1987-4)

Parameter estimation of a diffusion by sign changes of the discretized process

C. R. Acad. Sci. Paris Sér. I Math, 305, 661-664

Keywords: Brownian motion

Abstract: Let X_t be a recurrent diffusion on **R**. The drift depends on an unknown parameter θ. Let $\overline{X}_\Delta$ be the process obtained through linear interpolation using the values $X_{k\Delta}$, $\Delta > 0$. One observes the sequence sign $X_{k\Delta}$, where Δ is the sampling interval. We construct asymptotically efficient estimators based on the number $N_\Delta(t)$ of zeros of $\overline{X}_\Delta$ in the interval $[0,t]$. It is shown that $\sqrt{\frac{1}{2}\pi\Delta}/N_\Delta(t)$ converges in L^2 as $\Delta \to 0$ to the local time of X at zero.

Haccoua, Patsy; Meelisa, Evert and van de Geerb, Sara(1987-5)

The likelihood ratio test for the change point problem for exponentially distributed random variables

Stochastic Processes and their Applications, 27, 121-139

Keywords: Bahadur efficiency; change point problem; exponential distribution; likelihood ratio test; normed uniform quantile process; power properties

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Abstract: Let $x_1, \cdots, x_{n+1}$ be independent exponentially distributed random variables with intensity λ_1 for $\leq i\tau$ and λ_2 for $i > \tau$, where τ as well as λ_1 and λ_2 are unknown. By application of theorems concerning the normed uniform quantile process it is proved that the asymptotic null-distribution of the likelihood ratio statistic for testing $\lambda_1 = \lambda_2$ (or, equivalently, $\tau = 0$ or $n + 1$) is an extreme value distribution.

Change point problems occur in a variety of experimental sciences and therefore have considerabla attention of applied statisticians. The problems are non-standard since the usual regularity conditions are not satisfied. Explicit asymptotic distributions of likelihood ratio tests have until now only been derived for a few cases. The method of proof used in this paper is based on the strong invariance principle.

Furthermore it is shown that the test is optimal in the sense of Bahadur, although the Pitman efficiency is zero. However, simulation results indicate a good power for values of n that are relevant for most applications.

The likelihood ratio test is compared with another test which has the same asymptotic null-distribution. This test has Bahadur efficiency zero. The simulation results confirm that the likelihood ratio test is superior to the latter test.

Hawkins, D. L.(1987-6)
A test for a change point in a parametric model based on a maximal Wald-type statistic.
Sankhyā Ser. A 49, no. 3, 368-376

Abstract: A large-sample test for a change in the parameter of a parametric model is studied. The test statistic is a maximal Wald-type statistic based on the difference in the maximum likelihood estimator calculated from $X_1, \cdots, X_k$ and $X_{k+1}, \cdots, X_n$. The asymptotic null distribution is obtained via weak convergence of an empirical process to the standardized tied-down Bessel process. The local power function is obtained in terms of the first-crossing probability of a drifted version of this process

Hinkley, David and Schechtman, Edna (1987-7)
Conditional bootstrap methods in the mean-shift model
Biometrika, 74, 85-93
Keywords: Density estimation; Logistic regression; Change point

Abstract: Bootstrap methods are not inherently conditional, but they can be made so by appropriate stratification of the simulated samples which bootstrap produces. We show how stratification can work in a bootstrap analysis of mean-shift in Nile river flow data. The results are compared with both parametric and semiparametric likelihood analyses. The paper ends with some general remarks on conditional bootstraps.

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James, Barry; James, Kang Ling and Siegmund, David(1987-8)
Tests for a change-point
Biometrika 74, 71-83
Keywords: Boundary crossing probabilities; Change-point; Likelihood ratio test; Power approximations; Recursive residual

Abstract: The problem considered is that of testing a sequence of independent normal random variables with constant, known or unknown, variance for no change in mean versus alternatives with a single change-point. Various tests, such as those based on the likelihood ratio and recursive residuals, are studied. Power approximations are developed by integrating approximations for conditional boundary crossing probabilities. A comparison of several tests is made.

Lombard, F.(1987-9)
Rank tests for changepoint problems
Biometrika, 74, 615-624
Keywords: Abrupt change; Changepoint; Multiple change; Rank test; Smooth change

Abstract: We consider procedures based on quadratic form rank statistics to test for one or more changepoints in a series of independent observations. Models incorporating both smooth and abrupt changes are introduced. Various test statistics are suggested, their asymptotic null distributions are derived and tables of significance points are given. A Monte Carlo study shows that the asymptotic significance points are applicable to moderately sized samples.

Solow, Andrew R. (1987-10)
Testing for climate change: An application of the two-phase regression model
Journal of Climate and Applied Meteorology, 26, 1401-1405

Abstract: A statistical test for detecting a change in the behavior of an annual temperature series is presented. The test is based on the two-phase regression model. By trading the hypothesized time of change as an unknown parameter, the approach allows an inference to be made about the time of change. The approach also avoids a serious problem, called data-dredging, that can arise in testing for change occurring at a specified time. The test is applied to a series of Southern Hemisphere temperatures, and the hypothesis of no change cannot be rejected.

Yao, Yi-Ching(1987-11)
Approximating the Distribution of the Maximum Likelihood Estimate of the Change-Point in a Sequence of Independent Random Variables
The Annals of Statistics, 15, 3, 1321-1328.

Abstract: The problem of estimating the change-point in a sequence of independent random variables is considered. As the sample sizes before and after the change-point tend to infinity, Hinkley (1970, Biometrika, 57, 1-17) showed that the maximum likelihood estimate of the change-point converges in distribution to that of the change-point based on an infinite sample. Letting the amount of change in distribution approach 0, it is shown that the distribution, suitably normalized, of the maximum likelihood estimate based on an infinite sample converges to a simple one which is related to the location of the maximum for a two-sided Wiener process. Numerical results show that this simple distribution provides a good approximation to the exact distribution (with an infinite sample) in the normal case. However, it is unclear whether the approximation is good for general nonnormal cases.

Yao, Yi-Ching(1987-12)
A note on testing for constant hazard against a change-point alternative
Annals of the Institute of Statistical Mathematics, 39, 377 -383

Abstract: The problem of testing for constant hazard against a change-point alternative is considered. It is shown that this problem is related to another one in quality control. Based on this relationship, a test is proposed. The main advantages of this test are its computational simplicity and the ready availability of small and large sample distribution theory

Akman, V. E. and Raftery, A. E.(1986-1)
Asymptotic Inference for a Change-Point Poisson Process
The Annals of Statistics, 14, 1583-1590.

Abstract: Easily implemented asymptotic off-line procedures for the change-point Poisson process with $\lambda(t)$, the intensity at time t, equal to λ_1 if $t \leq \tau$ and to λ_2 if $t > \tau$, are developed. They may also be applied to a problem of estimation of the location of a discontinuity in density discussed by Chernoff and Rubin (1956, Proc. 3rd Berkekey Symp. Math. Stat. Probab. 1 19-37. Univ. California Press.). A test for change is noted, a test of the hypothesis that $\tau = \tau_0$ is proposed, and point and interval estimates of τ, λ_1, and λ_2 are provided. The small-sample performance of the proposed procedures is studied using simulation, and an example is given.

Ebrahimi, Nader(1986-2)
A parametric approach to theory of competing risks when the time of death from each cause follows a change-point hazard rate model.
Communication in Statistics: Theory and Methods, 15, 215-229

Abstract: The author considers survival models, with different causes of death, which are "sectionally" exponential, i.e., have exponential behaviour with parameters α_i [resp. β_i]

when the cause of death is c_i and the time of death is less [resp. greater] than θ_i (the change-point). In Section 2 he considers the special case where all the change-points θ_i are equal to θ, and obtains first estimators for the α_i and β_i when θ is known and then for the α_i and β_i and θ when θ is unknown. In Section 3 the same program is carried out without assuming the equality of the change-points. Finally, he gives a likelihood ratio test for the equality of the change-points. He also gives an example. (Reviewed by J. Tiago de Oliveira)

Freeman, J. M.(1986-3)
An Unknown Change Point and Goodness of Fit
The Statistician, Vol. 35, 335-344.

Abstract: In this paper we describe a procedure for detecting a systematic change in mean for a series of normal random variables. The procedure is based on a goodness-of-fit argument. Tests for an unknown change point are given. The procedure is found to be appropriate to problems in which the data series has been subject to a single discrete change in level or where there have been cumulative changes in level after an unknown point.

Hawkins, D. L.(1986-4)
A simple least squares method for estimating a change in mean.
Communication in Statistics B, Simulation Computation, 15, 655-679

Abstract: A simple least squares method for estimating a change in the mean of a sequence of independent random variables is studied. The method first tests for a change in the mean based on the regression principle of constrained and unconstrained sums of squares. Conditionally on a decision by this test that a change has occurred, least squares estimates are used to estimate the change point, the initial mean level (prior to the change point) and the change itself. The estimates of the initial level and change are functions of the change point estimate. All estimates are shown to be consistent, and those for the initial level and change are shown to be asymptotically jointly normal. The method performs well for moderately large shifts (one standard deviation or more), but the estimates of the initial level and change are biased in a predictable way for small shifts. The large-sample theory is helpful in understanding this problem. The asymptotic distribution of the change-point estimator is obtained for local shifts in the mean, but the case of nonlocal shifts appears analytically intractable.

Henderson, Robin(1986-5)
Change-Point Problem with Correlated Observations, with an Application in Material Accountancy
Technometrics, Vol. 28, No. 4., 381-389.
Keywords: Correlation; Locally most powerful testing; Bayesian estimation; Nuclear safeguards.

Abstract:Techniques are developed for the change-point problem when the data are a series of correlated normal variables, combining locally most powerful testing with Bayesian estimation. The approach is applied to a nuclear materials accounting problem. The effect of correlation on change-point tests is examined.

McDonald, John Alan and Owen, Art B. (1986-6)
Smoothing with split linear fits
Technometrics, 28, 195-208
Keywords: Edge detection; Nonparametric regression; Image processing; Change point

Abstract: We introduce a family of smoothing algorithms that can produce discontinuous output. Unlike most commonly used smoothers, that tend to blur discontinuities in the data, this smoother can be used for smoothing with edge detection. We cite examples of other approaches to (two-dimensional) smoothing with edge detection in image processing, and apply our one-dimensional smoother to sea surface temperature data where the discontinuities arise from changes in ocean currents.

Raftery, A. E. and Akman, V. E.(1986-7)
Bayesian Analysis of a Poisson Process with a Change-Point
Biometrika, 73, 85-89.
Keywords: Bayes factor; Coal mining disasters; Imaginary observation; Improper prior; Log linear intensity function

Abstract: A Bayesian approach to estimation and hypothesis testing for a Poisson process with a change-point is developed, and an example given.

Schulze, U.(1986-8)
Identification of the change-point of a distribution in a sequence of random variables.
(Polish) Translated from the German by Tomasz Rychlik. Mat. Stos. 28 (1986), 33-89

Abstract: This is an expository paper on the problem of detecting a change-point. The author assumes that $X_1, \cdots, X_n$ are independent random variables observed at ordered time points t_i, $t_1 < \cdots < t_n$, the variable X_i has distribution function F_i, $i = 1, \cdots, n$. She considers the problem of testing the hypothesis $H_0\, F_1 = \cdots = F_n$ against H_1: there exists $1 \leq m < n$ such that $F_1 = \cdots = F_m \neq F_{m+1} = \cdots = F_n$, and the problem of estimating the change-point m. Test procedures based on the likelihood, the partial sums and the Bayesian approach are presented. Further, the related methods of estimation of m are explained. The emphasis is on the cases when F_i is either the binomial or a normal distribution. The case where F_i is a one-parameter exponential type distribution is mentioned briefly. There is also a brief

section devoted to nonparametric procedures. The paper provides a good introduction to the field, and gives a survey of various approaches to the solution of the problem; however, it does not contain any illustrative numerical examples. (Reviewed by M. Hušková)

Siegmund, David(1986-9)
Boundary Crossing Probabilities and Statistical Applications
The Annals of Statistics, 14, 361-404.

Abstract: This paper surveys recent results involving boundary crossing probabilities and related statistical applications. The first part is concerned with problems of sequential analysis, especially repeated significance tests and their application to sequential clinical trials involving survival data. The second part develops the probability theory motivated by the problems of Part 1. A method for computing first passage distributions of Brownian motion to linear boundaries is introduced and then modified to handle problems problems in discrete time and those involving nonlinear boundaries. The third part is concerned with fixed sample statistical problems, especially change-point problems, which involve boundary crossing probabilities. Examples are given of problems for which the methods of Part 2 appear adequate and of problems which require new methods.

Srivastava, M. S. and Worsley, K. J.(1986-10)
Likelihood Ratio Tests for a Change in the Multivariate Normal Mean
Journal of the American Statistical Association, 81, 199-204
Keywords: Change point; Binary segmentation; Bonferroni inequality; Extramultinomial variation

Abstract: A sequence of independent multivariate normal vectors with equal but possibly unknown variance matrices are hypothesized to have equal mean vectors, and we wish to test that the mean vectors have changed after an unknown point in the sequence. The likelihood ratio test is based on the maximum Hotelling T^2 for the sequences before and after the change point. The main result is a conservative approximation for its null distribution based on an improved Bonferroni inequality. If the change is judged significant, then further changes are estimated by splitting the two subsequences formed by the first change point. The methods can also be used to test for a change in row probabilities of a contingency table, allowing for extramultinomial variation. The results are used to find changes in a set of geological data previously analyzed by Chernoff (1973) by the "faces" method and to find changes in the frequencies of pronouns in the plays of Shakespeare.

Worsley, K. J.(1986-11)
Confidence Regions and Tests for a Change-Point in a Sequence of Exponential Family Random Variables

Abstract: Maximum likelihood methods are used to test for a change in a sequence of independent exponential family random variables, with particular emphasis on the exponential distribution. The exact null and alternative distributions of the test statistics are found, and the power is compared with a test based on a linear trend statistic. Exact and approximate confidence regions for the change-point are based on the values accepted by a level α likelihood ratio test and a modification of the method proposed by Cox & Spjotvoll (1982). The methods are applied to a classical data set on the time intervals between coal mine explosions, and the change in variation of stock market returns. In both cases the confidence regions for the change-point cover historical events that may have caused the changes.

Yao, Yi-Ching and Davis, Richard A.(1986-12)
The asymptotic behavior of the likelihood ratio statistic for testing a shift in mean in a sequence of independent normal variates.

Abstract: Let $X_1, \cdots, X_n$ be independent and normally distributed with common variance σ^2, $X_1, \cdots, X_r$ having mean μ and $X_{r+1}, \cdots, X_n$ having mean $\mu + \theta$, $\theta \neq 0$. For the (two-sided) testing problem H_0: no change-point $r < n$, the likelihood ratio statistic is investigated. It is shown that, under H_0, this statistic, suitably standardized, converges weakly to the double exponential extreme value distribution. The asymptotic power of the likelihood ratio test is computed for alternatives with change-point $r = t_0 n$ and θ converging to zero at the rate $n^{-1/2} \log \log n$. At these alternatives, the likelihood ratio test is compared with a Bayes test. It turns out that the Bayes test has a better performance than the likelihood ratio test. The proofs are based on the Darling-Erdös results on the maximum of a Wiener process. (Reviewed by Christian Hipp)

Yao, Yi-Ching(1986-13)
Maximum likelihood estimation in hazard rate models with a change-point.

Abstract: The author considers the following hazard model. Assume that the hazard function $h(t)$ satisfies $h(t) = aI(0 \leq t < \tau) + bI(t \geq \tau)$, where I is the indicator function, a and b are unknown parameters and τ is an unknown change-point parameter. The density function associated with the model is

$$f(t) = aI(0 \leq t \leq \tau) \exp[-at] + bI(t \geq \tau) \exp[-at - b(t - \tau)].$$

Let $T_{(1)} < \cdots < T_{(n)}$ be n ordered observations from the population. The author considers the problem of estimating the parameters. The likelihood function is unbounded. It

approaches infinity when a is fixed, $b = 1/(T_{(n)} - \tau)$ and $\tau \uparrow T_{(n)}$. The global maximum likelihood estimator for τ does not exist. Under the constraint $t \geq T_{(n-1)}$, the author obtains the maximum likelihood estimator for a, b and τ. He shows that the maximum likelihood estimator of τ subject to the above constraint is consistent. The limiting distributions of the constrained maximum likelihood estimators for a, b and τ are derived. It follows that these three estimators are asymptotically independent. (Reviewed by James C. Fu)

Ali, Mukhtar M. and Silver, J. Lew (1985-1)
Tests for equality between sets of coefficients in two linear regressions under heteroscedasticity

Abstract: Structural shift is a common problem in a relationship dealing with time series data. Chow (1960, Econometrca, 28, 591-605) developed a test to detect such a shift under the assumption that observations both before and after the shift have the same variance. Structural shifts, however, often accompany changes in variance as well, and the Chow test is not robust to such changes. Two relatively robust tests are proposed and are found to be highly powerful.

Brown, Keith C., Lockwood, Larry J. and Lummer, Scott L. (1985-2)
An examination of event dependency and structural change in security pricing models

Abstract: This paper considers two aspects of the tendency for systematic risk to change during the period surrounding a firm-specific event. First, a statistic allowing for heteroskedasticity is presented as a means of more precisely testing for the incidence of structural change in the market model. Secondly, the bias resulting from the imposition of a single, arbitrary event period on every firm in a market efficiency study is formally demonstrated. Using a sample based upon stock splits, the switching regression technique of Quandt is then adapted to show that event intervals are more appropriately considered on a case-by-case basis. A comparison of alternative residual measures illustrates these procedures.

Halvorson, Alan L. (1985-3)
Switching regression estimates of a sequential production process: The case of underground coal mining

Abstract: Underground coal mining is modeled as a two-stage sequential process in which raw coal is mined in the first stage, and hauled to the surface in the second. This approach allows for constraints on mine output due to both limited mining capacity and haulage capacity. The process is estimated by a switching regression methodology previously applied to the "markets in disequilibrium" model. Estimation results yield evidence that the two-stage sequential model is an appropriate alternative model for underground coal mining.

Lubrano, Michel (1985-4)
Bayesian analysis of switching regression models
Journal of Econometrics, 29, 69-95

Abstract: The paper considers a class of switching regression models where the change of regime is represented by a min operator. After reviewing a maximum likelihood method, a local identification criterion is proposed. Contrary to classical analysis, it is shown that Bayesian analysis requires here local identification everywhere on the parameter space. The paper then proposes a Monte Carlo procedure to compute posterior moments with an importance function based on an approximate posterior density of the model. Feasibility of the method is shown in a numerical example and then in an economic example.

Matthews, D. E. and Farewell, V. T.(1985-5)
On a Singularity in the Likelihood for a Change-Point Hazard Rate Model
Biometrika, 72, 3, 703-704
Keywords: Change-point hazard rate; Likelihood function; Singularity.

Abstract:A singularity identified by Nguyen, Rogers & Walker (1984, Biometrka, 71, 299-304) in the log likelihood function for a change-point hazard rate model is eliminated if the likelihood function is appropriately defined

Matthews, D. E.; Farewell, V. T. and Pyke, R.(1985-6)
Asymptotic Score-Statistic Processes and Tests for Constant Hazard Against a Change-Point Alternative
The Annals of Statistics,13, 583-591

Abstract: The problem of testing for a constant failure rate against alternatives with failure rates involving a single change-point is considered. The asymptotic significance level for tests based on maximal score statistics are shown to involve the solution to a first passage time problem for an Ornstein-Uhlenbeck process. An example illustrates the methodology.
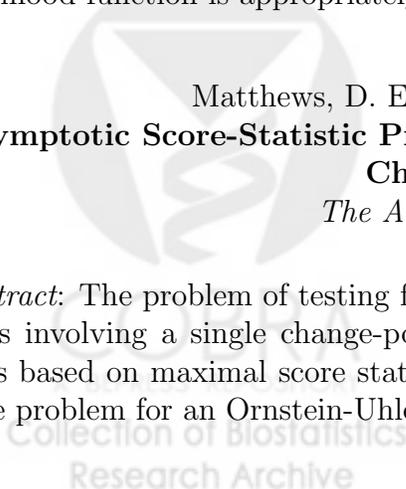
Moen, David H., Salazar, Diego and Broemeling, Lyle D. (1985-7)
Structural changes in multivariate regression models
Communications in Statistics: Theory and Methods, 14, 1757-1768
Keywords: Posterior distribution of change points; Posterior distribution of the regression matrices; posterior distribution of the precision matrix.

Abstract: This study generalizes the work of Chin Choy and Broemeling(1980, Technometrics, 22(1), 71-78) who investigated the change in the regression parameters of univariate linear models. The marginal posterior distributions of the change point, the regression matrices, and the precision matrix are found with the use of a proper multivariate normal-Wishart distribution for the parameters of the model. A numerical study is undertaken in order to gain some insight into the effect that changes in sample size and certain parameter values have on these marginal posterior distributions.

Muliere, Pietro and Scarsini, Marco(1985-8)
Change-point problems: a Bayesian nonparametric approach.
Apl. Mat. 30 (1985), no. 6, 397–402, ii, iv

Abstract: Suppose $X_1, \cdots, X_n$ are independent random variables and, given r, X_j for $1 \leq j \leq r$ has distribution function F_1, and X_j for $r < j \leq n$ has distribution function F_2. The authors consider making inferences for r, the change-point, and F_1, F_2 assuming that r, F_1 and F_2 have independent prior distributions. The prior distributions for F_1 and F_2 are taken to be Dirichlet processes, $D(\alpha_1)$ and $D(\alpha_2)$, respectively [T. S. Ferguson, Ann. Statist. 1 (1973), 209-230]. The marginal posterior distribution of r is found, as are the marginal distributions of F_1 and F_2. (Reviewed by A. N. Pettitt)

Teeter, Rebecca A. (1985-9)
The application of linear piecewise regression to basal body temperature data
Biometrical Journal, 27, 759-773
Keywords: Measurement error; Join point estimation; Ovulation.

Abstract: In many situations one wish to fit piecewise regression model which enables one to obtain estimates of the join points as well as the slopes and intercepts of the fitted sub models. This study develops a technique for fitting piecewise models to the data which contain measurement error in an independent variable. The technique developed here combines the Hudson(1966) procedure for estimating parameters in piecewise regression and the Wald(1940) Grouping Technique which obviates the problem of measurement error. If one assumes some knowledge of the position of the join point in relation to the data, methodology has been developed to estimate the parameters and study the asymptotic properties of the means and variances of the parameter estimates. However, in the more realistic case, when additional knowledge is limited, it is only possible to obtain the parameter estimates

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using an iterative technique (Teeter, 1982). The general technique for obtaining the joint point estimate in the presence of measurement error is presented here and an example is given using data on woman's basal body temperature during menstrual cycles.

Thursby, Jerry G. (1985-10)
The relationship among the specification tests of Hausman, Ramsey, and Chow
Journal of the American Statistical Association, 80, 926-928
Keywords: Structural shift; Regression

Abstract: This article describes a relationship between the tests for specification error in regression models introduced by Ramsey (1969, Journal of Royal Statistical Society, Ser. B, 32, 350-371) and Hausman (1978, Econometrica, 46, 1251-1271). It also shows that the Chow (1960, Econometrica, 28, 561-605) test for structural shift can be viewed as a special case of specification error tests. This article shows the common structure of these tests. The common basis is the comparison of alternative estimators of regression coefficients. Under the null hypothesis, one estimator is efficient and consistent and the other is consistent. The probability limits of the two estimators are different under the alternative hypothesis.

Vaman, H. J. (1985-11)
Optimal online detection of parameter changes in two linear models
Stochastic Processes and their Applications, 20, 343-351
Keywords:Disorder problem; Sequential detection; Autoregressive process; Regression model; Optimal stopping; Change point

Abstract: Shiryaev has obtained the optimal sequential rule for detecting the instant of a distributional change in an independent sequence using the theory of optimal stopping of Markov processes. This paper considers the problem of sequential detection of certain parameter changes in two dependent sequences: an autoregressive process, and a regression model with serially correlated error terms. It is shown that the rule that is optimal in the sense of minimizing the expected positive delay is the one which declares a change to have occured as soon as the posterior probability of a change crosses a threshold. This rule also permits control of the probability of a false-declaration of change, just as in the independent sequence case.

Darkhovskiĭ, B. S.(1984-1)
Two problems of the estimation of change-points of probability characteristics of a random sequence.
Teor. Veroyatnost. i Primenen. 29, no. 3, 464-473

Abstract: Consider a sequence of observations $x_1, \cdots, x_{n_0}, \cdots, x_{n_1}, \cdots, x_N$ such that the one-dimensional distribution of x_j is F_0 for $j \leq n_0$ and $j \geq n_1$ $(n_1 > n_0 + 1)$ and F_1 for

$n_0 < j < n_1$. The author discusses the estimation of the unknown change-points n_0 and n_1. Using nonparametric methods, he finds consistent estimators and, in some cases, derives lower bounds for the variance of the estimators. (Reviewed by A. Balogh)

Lee, L.-F. and Porter, R. H. (1984-2)
Switching regression models with imperfect sample separation information: With an application on cartel stability (STMA V27 2258)
Econometrica, 52, 391-418

Abstract: An exogenous switching regression model with imperfect regime classification information is specified and applied to a study of cartel stability. An efficient estimation method is proposed which takes this imperfect information into account. The consequences of misclassification are analyzed. The direction of the least squares bias is derived. An optimal regime classification rule is obtained and compared theoretically and empirically with other classification rules. We then examine the Joint Executive Committee, a railroad cartel in the 1880's. The econometric evidence indicates that reversions to noncooperative behavior did occur for the firms in our sample, and these reversions involve a significant decrease in market price.

Moen, David H. and Broemeling, Lyle D.(1984-3)
Testing for a change in the regression matrix of a multivariate linear model.
Communication in Statistics: Theory and Methods, 13, 1521-1531

Abstract: A Bayesian test procedure is developed to test the null hypothesis of no change in the regression matrix of a multivariate linear model against the alternative hypothesis of exactly one change. The resulting test is based on the marginal posterior distribution of the change point. To illustrate the test procedure a numerical example using a bivariate regression model is considered

Nguyen, H. T.; Rogers, G. S. and Walker, E. A.(1984-4)
Estimation in Change-Point Hazard Rate Models
Biometrika, 71, 299-304.
Keywords: Change-point parameter; Consistent estimator; Geometric method; Hazard rate; Quantile.

Abstract: This paper discusses the estimation of parameters in hazard rate models with a change-point. Due to the irregularity of the models, the classical maximum likelihood method and the method of moments cannot be used. A consistent estimator of the change-point is obtained by examining the properties of the density represented as a mixture. The performance of the estimator is checked via simulation

Schulze, U. (1984-5)
A method of estimation of change points in multiphasic growth models
Biometrical Journal, 26, 495-504
Keywords: Segmented regression; Nonlinear regression(Math Library)

Abstract: The least-squares estimation (LSE) of change points and the phase parameters is considered for two growth models with piecewise continuously connected exponential and stationary phases. The calculation of a LSE is reduced to the solution of a finite number of simpler least-squares problems for which methods of calculation are well known.

Sen, Pranab Kumar (1984-6)
Recursive M-tests for the constancy of multivariate regression relationships over time
Sequential Analysis, 3, 191-211
Keywords: Change-point; CUSUM; Sequential detection; Stopping rule

Abstract: For a general multivariate linear model, based on a general class of recursive M-estimators of regression parameters and allied M-statistics, some testing procedures for a possible change in the regression relationships occurring at an unknown time point are considered. The (asymptotic) theory of the proposed tests rests on some invariance principles for some recursive M-estimators and related (residual) M-statistics, and these are also studied. (Reviewed by H. J. Engelbert)

Wolfe, Douglas A. and Schechtman, Edna(1984-7)
Nonparametric statistical procedures for the changepoint problem
Journal of Statistical Planning and Inference
Keywords: At most one changepoint; Mann-Whitney statistics; Monte Carlo study

Abstract: Let $X_1,...,X_{r-1}, X_r, X_{r+1},...,X_n$ be independent, continuous random variables such that X_i, i = 1,,r, has distribution function $F(x)$, and X_i, $i = r + 1,...,n$, has distribution function $F(x - \Delta)$, with $-\infty < \Delta < \infty$ When the integer r is unknown, this is reffered to as a change point problem with at most one change. The unknown parameter Δ represents the magnitude of the change and r is called the changepoint. In this paper we present a general review discussion of several nonparametric approaches for making inferences about r and Δ.

Zacks, S.(1984-8)
Estimating the Shift to Wear-Out of Systems Having Exponential-Weibull Life Distributions
Operations Research, 32, 3, Reliability and Maintainability. 741-749

Abstract: We develop a new family of life distributions, called the exponential-Weibull wear-out distributions, for systems whose failure rate function is a constant up to a change-point (wear-out point) and strictly increasing afterward. We derive properties of these wear-out distributions and develop a Bayes adaptive procedure for estimating the change point. Recursive formulas are given for determining the posterior probability that the change has occurred and its Bayes estimator. Results of numerical simulations are given to illustrate the properties of the adaptive procedure.

Awan, Hayat M.; Chaudhry, Abdul Hafeez(1983-1)
Some Bayesian inferences for changing multivariate regression model.
Riazi J. Karachi Math. Assoc. 5, 35-44.

Abstract: A multivariate regression model including a change-point is considered. Let $y_1, \cdots, y_n$ be the p-dimensional observations on the dependent variable at points $x_1, \cdots, x_n \in R^k$ of the independent variable. Let $y_i = x_i\beta_1 + e_i$, $i = 1, \cdots, m$, and $y_i = x_i\beta_2 + e_i$, $i = m + 1, \cdots, n$, where β_1 and β_2 are matrices of unknown parameters, and the random errors e_i, $i = 1, \cdots, n$, are i.i.d. $N_p(0, \Sigma)$ with an unknown positive definite Σ. Also, the change-point $m \in \{1, \cdots, n-1\}$ is unknown. A prior distribution for $m, \beta = (\beta_1, \beta_2)$ and Σ is assumed, such that (i) m is uniformly distributed over $1, \cdots, n-1$, (ii) the conditional distribution of each row of β_i given Σ is multivariate normal for $i = 1, 2$, (iii) the marginal distribution of Σ is a Wishart distribution, and (iv) m and (β, Σ) are stochastically independent. As a result, the posterior distributions of m, β and Σ are found. This provides a multivariate analogue to the univariate case, considered by J. H. Chin Choy and L. D. Broemeling [Technometrics 22 (1980), no. 1, 71-78]. The paper ends with numerical examples. (Reviewed by N. Gaffke)

Carter, R. L. and Blight, B. J. N(1983-2)
Correction: A Bayesian Change-Point Problem with an Application to the Prediction and Detection of Ovulation in Women
Biometrics, 39, 4, 1137

Correction: There is an ommision of a multiplier in the $(1/2)\sigma^2 n(\overline{X_n} - \mu_1)^2$ term of (3.7). This term should be $(1/2)\sigma^2(1 + nP^{11})^{-1}(\overline{X_n} - \mu_1)^2$, where P^{11} is the first-row, first-column element of $\mathbf{P}^{-1}$. This error accounts for problems encountered in the paper's applications when an informative prior on θ_1 is used.

Fisher, G. R.(1983-3)
Tests for two separate regressions
Journal of Econometrics, 21, 117-132

Abstract: Several recently proposed tests for separate regressions in econometrics are re-examined in the light of recommendations by Cox (1961). This re-examination points to simplified criteria and emphasizes the unity underlying the tests. The exact distributions of some of the tests are developed under the tested hypothesis. These are given a geometrical characterization which is helpful in exploring relations with the classical F-test. An orthogonal decomposition is proposed which provides a direct link between the F-test and tests based upon artificial nesting.

Heath, A. B. and Anderson, J. A.(1983-4)
Estimation in a multivariable two-phase regression
Communications in Statistics: Theory and Methods, 12, 809-828
Keywords: Change point; Changepoint; Monte-Carlo testing; Likelihood; Support.

Abstract: A likelihood approach is considered for the problems of estimating the changepoint and other parameters in a multivariable two-phase regression. Methods for finding the maximum likelihood estimates are given for the cases when the covariance matrix is known, and unknown. The distribution of the usual likelihood ratio tests statistics is investigated using simulations, and a Monte-Carlo approach is suggested for testing for the existence of a change-point. Numerical illustrations are provided.

Schechtman, Edna(1983-5)
A conservative nonparametric distribution-free confidence bound for the shift in the changepoint problem.
Communication in Statistics. A—Theory Methods 12, no. 21, 2455-2464

Abstract: Let $X_1, \cdots, X_n$ be independent random variables such that up to (unknown) r, the variables $X_1, \cdots, X_r$ have the same distribution F, while $X_{r+1}, \cdots, X_n$ have that obtained from F by shifting it by the amount θ, which is also unknown. The author obtains confidence bounds for θ. The difficulty arising out of r being a nuisance parameter is overcome by techniques used in the corresponding hypothesis testing problem treated by A. K. Sen and M. S. Srivastava [Ann. Inst. Statist. Math. 27 (1975), no. 3, 479-486] and the author [A nonparametric test for detecting changes in location, Comm. Statist. A—Theory Methods, to appear]. The basic statistic used is a Mann-Whitney two-sample analogue. The confidence bounds obtained by using this statistic are shown to be conservative, i.e., the coverage probability is at least as large as the specified confidence coefficient. Simulation studies indicate that the actual coverage probability comes closer to the desired confidence coefficient when the nuisance parameter r is close to $n/2$. (Reviewed by Kumar Joag-Dev)

Sen, Pranab Kumar(1983-6)
On some recursive residual rank tests for change-points.
Recent advances in statistics, 371–391, Academic Press, New York, 1983.

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Abstract: Let $X_1, \cdots, X_n$ be independent random variables with the distribution of X_i given by $F_i(\cdot) = F(\cdot - \beta_i c_i)$ where F is an unspecified continuous distribution function, the c_i are q-vectors ($q \geq 1$) of known constants, and the β_i are the unknown parameters of interest. The author develops some tests for testing the hypothesis $H_0 \colon \beta_1 = \beta_2 = \cdots = \beta_n = \beta$ (unknown) against the alternative $\beta_1 = \beta_2 = \cdots = \beta_m \neq \beta_{m+1} = \cdots = \beta_n$ for some $m \in \{1, \cdots, n\}$. The proposed tests are shown to be asymptotically distribution-free under H_0 under certain conditions on, among others, the score function φ involved in the definition of the test statistic. The paper provides also the nonnull asymptotic distribution of the test statistic under certain conditions on, among others, the design matrix φ, and on the asymptotic behavior of the change-point sequence. (Reviewed by V. Susarla)

Worsley, K. J. (1983-7)
Testing for a two-phase multiple regression
Technometrics, 25, 35-42
Keywords: Switching regression; Segmented regression; Change point

Abstract: Maximum likelihood methods are used to estimate the parameters of two separate multiple regressions that switch at an unknown point in the data. Normal errors with constant variance are assumed and likelihood ratio statistics are used to test for the presence of two separate regressions. Our main result is a conservative bound on the null distribution function of the test statistic. This bound is based on an improved Bonferroni inequality, and a simple power-series approximation is provided. Similar bounds are given for likelihood ratio statistics that test for a shift in the constant term of the regression only. The accuracies of the bounds and approximations are evaluated for a number of examples.

Zacks, S.(1983-8)
Survey of classical and Bayesian approaches to the change-point problem: fixed sample and sequential procedures of testing and estimation.
Recent advances in statistics, 245-269, Academic Press, New York, 1983.

Abstract: The author takes stock of existing results concerning the change-point problem: tests, estimate of the change-point, stopping rule in the case of sequential observation. In the section dedicated to tests, H_0 is the "no change" hypothesis, and the alternative H_1 means that "at a certain point, the distribution shifts". The observations are always independent and the shift parameter is generally unknown. The distribution F_0 may be known, Gaussian or more generally of exponential type, or unknown. The tests include Bayes tests, tests based on the likelihood ratio and nonparametric tests. The estimators of the change-point are Bayes or are based on the maximum likelihood method; in the latter case the distribution f_1 is assumed to be known. Concerning sequential control procedures, the author seeks a stopping rule that minimizes the risk and does not provoke too many false alarms: the Bayes

procedures and Page's CUSUM procedure are studied, as is an application to a reliability problem. (Reviewed by Franoise Garcia-Brouaye)

Bootha, N. B. and Smith, A.F.M.(1982-1)
A Bayesian approach to retrospective identification of change-points
Journal of Econometrics, 19, 7-22

Abstract: The authors develop a complete bayesian analysis of structural change for a wide variety of linear models including normal sequences, multiple linear regression models, and the $ARMA$ class of time series processes. This paper is an excellent review of the Bayesian approach to structural change in the parameters of those models which are often used in statistical practice. Also, the authors give many new results about time series models. The main emphasis of the paper is on the shift point, which indexes the observation where the change in parameters will occur. (Reviewed by Lyle D. Broemeling)

Daz, Joaqun(1982-2)
Bayesian detection of a change of scale parameter in sequences of independent gamma random variables.
Journal of Econometrics 19, 23-29

Abstract: This paper is a Bayesian analysis of structural change in a sequence of independent exponential random variables. The first m observations have a scale parameter θ_1 while the remaining have a scale parameter of θ_2, where $\theta_2 \neq \theta_1$. The change is represented by a change-point parameter m, and the author finds its posterior distribution using both proper and improper prior densities of the parameters. He illustrates his formulas with two examples. (Reviewed by Lyle D. Broemeling)

Gans, Daniel J. (1982-3)
A simple method based on broken-line interpolation for displaying data from long-term clinical trials
Statistics in Medicine, 1, 131-137
Keywords: Window method

Abstract: In displaying data from long-term clinical studies, variation across patients in the times at which readings are recorded often presents a problem. A simple method for treating such data is proposed, which avoids arbitrary conventions and methodological difficulties inherent in other approaches. It is based on within-patient broken-line interpolation, and is easy to implement on a computer. The method is tested and compared to others with computer-simulated data whose time-pattern is known. Though the comparison is of limited

scope, the method appears to compare favourably with the 'closest-reading' and 'window' approaches.

Guttman, Irwin and Menzefricke, Ulrich(1982-4)
On the use of loss functions in the changepoint problem
Annals of the Institute of Statistical Mathematics, 34, 319-326

Abstract: We consider a sequence of independent random variable whose densities depend on a parameter which is subjected to a change at unknown time point. A Byesian decision-theoretic approach is used to obtain an optimal choice of changepoint. The exponential and multivariate normal models are analyzed, and some numerical examples are given

Matthews, David E. and Farewell, Vernon T.(1982-5)
On Testing for a Constant Hazard against a Change-Point Alternative
Biometrics, 38, 2, 463-468.

Abstract: A frequently recurring question posed by leukemia researchers concerns a test of a constant failure rate against the alternative of a failure rate involving a single change-point. In answer to this question, a likelihood ratio test appropriate for the stated alternative is derived and simulated. Consideration is given also to tests based on alternatives in the log gamma family, which perform quite well when the change-point model is correct. A practical application is given.

Ohtani, Kazuhiro (1982-6)
Bayesian estimation of the switching regression model with autocorrelated errors
Journal of Econometrics Volume 18, Issue 2 , February 1982, Pages 251-261

Abstract: In this paper, we make a Bayesian analysis of the switching (two-phase) regression model when the subset of the regression coefficients shifts and the error terms are generated by a first-order autoregressive process. The posterior distributions of the shift point and other parameters are derived, and some numerical studies are performed. From the numerical studies, we see that the shift point is accurately estimated when the shift of the regression coefficient is relatively large. Also, the conditional distributions of the autocorrelation and regression coefficients on the shift point are compared with the marginal ones.

Ohtani, Kazuhiro (1982-7)
Bayesian estimation of the switching regression model with autocorrelated errors
Journal of Econometrics, 18, 251-261

Abstract: In this paper, we make a Bayesian analysis of the switching (two-phase) regression model when the subset of the regression coefficients shifts and the error terms are generated by a first-order autoregressive process. The posterior distributions of the shift point and other parameters are derived, and some numerical studies are performed. From the numerical studies, we see that the shift point is accurately estimated when the shift of the regression coefficient is relatively large. Also, the conditional distributions of the autocorrelation and regression coefficients on the shift point are compared with the marginal ones.

Ohtani, K. (1982-8)
A bayesian analysis of a random coefficient model in a simple keynesian system.
J. Econometrics, 18, 239-249.

Abstract: The marginal propensity to consume in a simple Keynesian model is treated as a random coefficient. This gives rise to the problem of quotient of random variables, i.e., the Fieller-Creasy problem. The Bayesian and maximum likelihood estimators are compared in sampling experiments. The Bayesian estimators have smaller mean squared errors than the maximum likelihood estimators. Marginal posterior probability density functions for a given sample are also presented.

Schmidt, Peter (1982-9)
An improved version of the Quandt-Ramsey MGF estimator for mixtures of normal distributions and switching regressions
Econometrica, 50, 501-524

Abstract: Quandt and Ramsey have suggested an estimator for normal mixtures and switching regressions, which minimizes a sum of squared differences between empirical and theoretical values of the moment generating function. This paper demonstrates how their estimator can be improved by minimizing a generalized sum of squares rather than an ordinary sum of squares. When this is done, more points of evaluation (moments) are unambiguously better than less. Most of the results presented are also applicable to method of moments estimators in general.

Schulze, U. (1982-10)
Estimation in segmented regression: Known number of regimes
Mathematische Operationsforschung und Statistik, Series Statistics, 13, 295-316
Keywords: Maximum likelihood; Least squares; Changepoint

Abstract: We give a review of methods for estimating the change points in regression models with abrupt or continuous changes. These estimates are based on maximum likelihood, least

squares and Bayesian analysis. Restrictions in applicability are discussed. Some examples are presented.

Sen, P. K.(1982-11)

Invariance Principles for Recursive Residuals

The Annals of Statistics, 10, 307-312.

Abstract: A general class of recursive residuals is defined by means of lower-triangular, orthonormal transformations. For these residuals, some weak invariance principles are established under appropriate regularity conditions. The theory is then incorporated in the study of robustness of some tests for change of parameters occurring at unknown time points.

Zacks, S.(1982-12)

Classical and Bayesian appoaches to the change-point problem: fixed sample and sequential procedures.

Statist. Anal. Données 7, no. 1, 48-81

Abstract: The change-point problem can be described in the following terms. Consider a sequence of independent random variables $X_1, X_2, \cdots$ and a sequence of positive, integer-valued parameters $2 \leq \tau_1 < \tau_2 < \tau_3 < \cdots$. The points $\tau_j (j = 1, 2, \cdots)$ are epochs of change in the distribution laws of the random variables; i.e., $X_1, \cdots, X_{\tau_1 - 1}$ have an identical distribution F_1; $X_{\tau_1}, \cdots, X_{\tau_2 - 1}$ have an identical distribution F_2, etc. The distributions $F_1, F_2, \cdots$ may be known or partially known, but the points of change, τ_j, are unknown. The problem is to estimate the unknown parameters τ_j or to test hypotheses concerning these points of change. This class of problems is a very broad one. It embraces essentially all problems which test the stationarity of a sequence of random variables versus the possibility of abrupt changes in the location, scale or shape of the distributions. Thus, all problems of statistical control fall in this domain. In the literature there are various formulations of the problem and different approaches. There are static or dynamic formulations of the problem with a possibility of only one point of change or many points of change. The sampling procedures are either fixed sample or sequential sampling. The inference framework is either classical or Bayesian. The present paper reviews the various formulations and approaches and provides an extensive bibliography

Carter, Randy L. and Blight, Barry J. N.(1981-1)

A Bayesian Change-Point Problem with an Application to the Prediction and Detection of Ovulation in Women

Biometrics, 37, 743-751

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Abstract: Under the assumptions of independent normally distributed and sequentially observed responses, a Bayesian rule for detecting a change from a constant mean response is derived. It is known that both basal body temperature (BBT) and preovulatory estrogen values undergo such a change in mean value at some random time during the menstrual cycle. The Bayesian rule is applied to estrogen to predict ovulation and to BBT to detect ovulation. Data from an aggregate of women are used to obtain prior information about the change-points and the parameters that define the changes in estrogen and BBT. A method is proposed by which the accumulation of information for a specific woman can be incorporated into the aggregate prior information

Esterby, R. S. and El-Shaarawi, A. H.(1981-2)
Inference about the Point of Change in a Regression Model
Applied Statistics, 30, 277-285.
Keywords:Segmented polynomial regression; Change-point; Estimation of degree; Marginal and conditional likelihood.

Abstract: Consider a sequence of (n_1+n_2) independent ordered pairs of observations for which the relationship between variables can be represented by a segmented polynomial regression model with unknown point of change n_1. The relative marginal likelihood function for n_1 is derived and the expressions for the relative conditional and maximum likelihood functions are given. Either of the first two likelihoods, which account for the uncertainty about the value of the other parameters, are to be preferred to the maximum likelihood function, with the relative marginal likelihood function being examined more extensively here. In the case where the segmented regression model can be represented by two polynomials of unknown degrees p and q, a procedure is described for estimating p and q. The use of these methods is illustrated using two observed sets of data and three artificially generated sets.

Menzefricke, Ulrich(1981-3)
A Bayesian Analysis of a Change in the Precision of a Sequence of Independent Normal Random Variables at an Unknown Time Point
Applied Statistics, 30, 141-146.
Keywords: Change in precision; Change in variance; Change point problem.

Abstract: This paper analyses a sequence of independent normal random variables in which the precision (inverse of the variance) may have been subjected to one change at an unknown point in time. Posterior distributions are found both for the unknown point in time at which the change occurred and for the magnitude of the change. Two examples are given.

Pettitt, A. N.(1981-4)
Posterior Probabilities for a Change-Point Using Ranks

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Abstract: Simple approximate methods of analysis are given for change-point models using ranked observations. The models can be applied to continuous observations, providing a robust analysis in this case. For ranked observations, the models provide analyses for association. Numerical examples, involving continuous and ranked observations, are given.

Schmidt, Peter(1981-5)
Further results on the value of sample separation information

Abstract: The asymptotic distribution of prediction is derived for the general simultaneous equation model with lagged endogenous variables and vector autoregressive errors. The results turn out to be particularly simple when no lagged endogenous variables are present.

Srivastava, M. S.(1981-6)
On tests for detecting change in the multivariate mean.

Abstract: We consider tests based on one observation on each of $N \geq 2$ random vectors $x_1, \cdots, x_N$ to decide whether the mean vectors μ_i of the x_i are all equal against the alternative hypothesis that a change has occurred at some unknown point r (i.e., $\mu_1 = \mu_2 = \cdots = \mu_r \neq \mu_{r+1} = \cdots = \mu_N$). The x_i are assumed to be normally distributed with a common unknown covariance. An estimate of the change point r is also given.

Tishler, Asher and Zang, Israel (1981-7)
A new maximum likelihood algorithm for piecewise regression

Abstract: This paper presents a piecewise regression method for continuous models containing max or min operators, or both. This method does not require knowledge of the zone in which a shift in regimes occurs. Moreover, it allows the application of analytical derivatives to maximize the likelihood function, which greatly simplifies the estimation of the model.

The method proposed exhibits fast convergence and can be used for an arbitrary number of regimes and variables.

Choy, J. H. Chin and Broemeling, L. D. (1980-1)
Some Bayesian inferences for a changing linear model
Technometrics, 22, 71-78
Keywords: Shift point; Mixture of distributions; Two-phase regression

Abstract: This paper is a generalization of earlier studies by Ferreira (1975, J. Amer. Statist. Assoc., 70, 370-374) and Holbert and Broemeling (1977, Comm. in Statist., A6(3), 265-275), who used improper prior distributions in order to make informal Bayesian inferences for the shift point and other parameters of a changing linear model. In this study, normal-gamma distributions are employed as prior distributions for the regression parameters of the model and, as a result, the posterior distribution of the regression parameters are mixtures of t distributions, while a mixture of gamma distributions is the posterior distribution of the precision parameter. Point and interval estimators of the regression parameters and the residual precision are based on the appropriate marginal and conditional posterior distributions and are illustrated with data generated from a known model.

Hawkins, Douglas M. (1980-2)
A note on continuous and discontinuous segmented regressions
Technometrics, 22, 443-444
Keywords: Nonlinear regression; Change-point; Likelihood ratio test

Abstract: It is pointed out that the inferential theory of the segmented regression model depends strongly on whether or not continuity at the change-point is assumed. In particular, if it is not, the likelihood ratio test for the presence of two segments tends to infinity.

Monti, K. L., Koch, G. G. and Sawyer, J.(1980-3)
Segmented linear regression models applied to the analysis of data from a cross-sectional growth experiment
Biometrical Journal. 22, 1, 23-39
Keywords: Segmented linear regression; Maximum likelihood method; Variance heterogeneity; Weighted least squares; Growth data.

Abstract: Several different methods of analysis are applied to data consisting of weight measurements, taken at specified post-treatment times, of harvested thyroids from rats given one of four treatments. Previous studies of this type of data indicated that the growth is initially rapid, and that a second phase of less rapid growth is followed by a final phase in which little additional growth occurs. The data are further characterized by increasing

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variance through time. The primary purpose of the analysis is to study the effect of the treatments at the end of the study period. One-way analysis of variance tests among groups are performed on each day, but results are not partially helpful. However, results from two-way analyses of variance (over subsets of days and groups) are consistent with the three phase model and accordingly indicate significant group differences during each. Finally, maximum likelihood methods are used to fit a three part segmented linear regression model.

Pettitt, A. N.(1980-4)
A Simple Cumulative Sum Type Statistic for the Change-Point Problem with Zero-One Observations
Biometrika, 67, 1, 79-84.

Abstract: A simple cumulative sum type statistic for the change-point with zero-one observations is introduced. A conditional test of no change against change is introduced and compared with a likelihood ratio test. The estimation of the change-point is also considered, using the simple statistic, and the method is shown to be asymptotically equivalent to the maximum likelihood estimator in certain circumstances and almost equivalent in others. To investigate the small sample behavior, simulation experiments were carried out and these showed the new estimator to be generally superior to the maximum likelihood estimator.

Pettitt, A. N.(1980-5)
A Simple Cumulative Sum Type Statistic for the Change-Point Problem with Zero-One Observations
Biometrika, 67, 79-84
Keywords: Asymptotic distribution; Binary data; Change-point problem; Cumulative sum; Estimation; Hypothesis testing; Zero-one observation.

Abstract: A simple cumulative sum type statistic for the change-point with zero-one observations is introduced. A conditional test of no change against change is introduced and compared with a likelihood ratio test. The estimation of the change-point is also considered, using the simple statistic, and the method is shown to be asymptotically equivalent to the maximum likelihood estimator in certain circumstances and almost equivalent in others. To investigate the small sample behavior, simulation experiments were carried out and these showed the new estimator to be generally superior to the maximum likelihood estimator.

Shaban, S. A. (1980-6)
Change point problem and two-phase regression: An annotated bibliography
International Statistical Review, 48, 83-93

Abstract: Recemment, beaucoup de travaux ont ete faits sur le probleme de l'estimation de la moyenne d'une distribution statistique quand elle est sujette a un changement a un moment

non connu. Un autre probleme est celui de l'estimation d'une regression, quand le modele prend des formes differentes dans differentes regions du domaine de la variable independante. Le but de ce travail est de compiler et de classer les travaux disponibles concernant ces problemes. La plupart des resumes ont ete choisis parmi ceux prepares par l'auteur lui-meme. La classification dependra de l'approche adoptee par l'auteur: Nous la mentionnerons par des lettres entre parentheses selon le schema suivant: (1) La premiere lettre sera soit (T) pour un test d'hypothese, soit (E) pour une estimation. (2) Pour les travaux sur les tests d'hypothese, une deuxieme classification est (B) pour bayesien, et (S) pour la theorie de l'echantillonnage; la sous-classification pour ces derniers est (P) pour parametrique et (NP) pour non-parametrique. (3) Pour les travaux sur l'estimation, une deuxieme classification est (B) pour Bayesien, (ML) pour maximum de vrausemblance, (LS) pour moindres carres, (MS) pour minimum de l'erreur quadratique, (CS) pour sommes cumulatives, et (O) pour: autres methodes. Par exemple, (T-S-NP) veut dire: test non-parametrique, du point de vuede la theorie de l'echantillonnage.

Smith, A. F. M.(1980-7)
Change-point problems: approaches and applications.
Bayesian statistics (Valencia, 1979), pp. 83–98, Univ. Press, Valencia, 1980.

Abstract: Problems of making inferences about abrupt changes in the mechanism underlying a sequence of observations are considered in both retrospective and on-line contexts. Among the topics considered are the Lindisfarne scribes problem; switching straight lines; manoeuvering targets, and shifts of level or slope in linear time series models. Summary analyses of data obtained in studies of schizophrenic and kidney transplant patients are presented

Smith, A. F. and Cook, D. G.(1980-8)
Straight Lines with a Change-Point: A Bayesian Analysis of Some Renal Transplant Data
Applied Statistics, 29, 180-189
Keywords: Stright line; Change-point; Posterior distribution; Renal transplants.

Abstract: A Bayesian analysis is made of a straight line model with a single change-point and, for both constrained and unconstrained versions of the model, posterior distributions are obtained for the time of change and the intersection point of the lines. The results are applied to the problem of inferring the time of rejection of transplanted kidneys.

Kumar, K. D., Nicklin, E. H. and Paulson, A. S. (1979-1)
Comments on "Estimating mixtures of normal distributions and switching regressions"
Journal of the American Statistical Association, 74, 52-55

Pettitt, A. N.(1979-2)
A Non-Parametric Approach to the Change-Point Problem
Applied Statistics, 28, 126-135
Keywords: Change-point problem; CUSUM; Kolmogorov-Smirnov d statistics;
Mann-Whitney U Statistic; Non-parametric tests.

Abstract: Non-parametric techniques are introduced for the change-point problem. Exact and approximate results are obtained for testing the null hypothesis of no change. The methods are illustrated by the analysis of three sets of data illustrating the techniques for zero-one observations, Binomial observations and continuous observations. Some comparisons are made with methods based on CUSUMS

Quandt, Richard E. and Ramsey, James B. (1979-3)
Reply to comments on "Estimating mixtures of normal distributions and switching regressions"
Journal of the American Statistical Association, 74, 56-56

Tishler, Asher and Zang, Israel (1979-4)
A switching regression method using inequality conditions
Journal of Econometrics, 11, 259-274

Abstract: This paper presents three simple approximations to the likelihood function of a switching regression model with inequality conditions. These approximations, which leave the likelihood function unchanged almost everywhere, have analytical derivatives that are continuously differentiable, and hence, allow the use of efficient gradient techniques.

Worsley, K. J.(1979-5)
On the Likelihood Ratio Test for a Shift in Location of Normal Populations
Journal of the American Statistical Association, 74, 366. , 365-367.
Keywords: Segmentation; Location shift; Testing for change-point.

Abstract: An alternative to the hypothesis that the sequence $X_1, \ldots, X_n$ are independent and identically distributed normal random variables, with mean μ and variance σ^2, is that the location parameter μ shifts at some unknown instant. The null distributions of likelihood ratio test statistics are given by Hawkins (1977) for the two cases of known and unknown σ^2. Unfortunately, the null distribution for unknown σ^2 obtained in that article is incorrect. In this article the correct null distribution is found and a numerical integration technique is used to obtain standard percentage points for $n = 3(1)10$. A Monte Carlo method is used to obtain additional standard percentage points for $n = 15(5)50$.

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Binder, David A. (1978-1)
Comments on "Estimating mixtures of normal distributions and switching regressions"
Journal of the American Statistical Association, 73, 746-747

Bryant, Peter (1978-2)
Comments on "Estimating mixtures of normal distributions and switching regressions"
Journal of the American Statistical Association, 73, 748-749

Clarke, B. R. and Heathcote, C. R. (1978-3)
Comments on "Estimating mixtures of normal distributions and switching regressions"
Journal of the American Statistical Association, 73, 749-750

Cobb, George W. (1978-4)
The problem of the Nile: Conditional solution to a changepoint problem
Biometrika, 65, 243-252
Keywords: Ancillary statistic; Hydrology

Abstract: Inference is considered for the point in a sequence of random variables at which the probability distribution changes. An approximation to the conditional distribution of the maximum likelihood estimator of the changepoint given the ancillary values of observations adjacent to the estimated changepoint is derived and shown to be numerically equal to a Bayesian posterior distribution for the changepoint. A hydrological example is given to show that inferences based on the conditional distribution of the maximum likelihood estimator can differ sharply from inferences based on the marginal distribution.

Fowlkes, Edward B. (1978-5)
Comments on "Estimating mixtures of normal distributions and switching regressions"
Journal of the American Statistical Association, 73, 747-748

Hartley, Michael J. (1978-6)
Comments on "Estimating mixtures of normal distributions and switching regressions"
Journal of the American Statistical Association, 73, 738-741

Hosmer, David W. (1978-7)

Comments on "Estimating mixtures of normal distributions and switching regressions"

Journal of the American Statistical Association, 73, 741-744

Johnson, Norman L. (1978-8)

Comments on "Estimating mixtures of normal distributions and switching regressions"

Journal of the American Statistical Association, 73, 750-750

Kiefer, Nicholas M. (1978-9)

Comments on "Estimating mixtures of normal distributions and switching regressions"

Journal of the American Statistical Association, 73, 744-745

Kiefer, Nicholas M. (1978-10)

Discrete parameter variation: Efficient estimation of a switching regression model

Econometrica, 46, 427-434

Abstract: An efficient estimator for regressions in which the parameter vector can take any of several values is devised. It is shown that although the likelihood function is unbounded, the likelihood equations have a consistent root. An initial consistent estimator is provided. One Newton step provides efficient estimates. Applications to nonlinear models and contaminated normal models are suggested.

Lindgren, Georg (1978-11)

Markov regime models for mixed distributions and switching regressions

Scandinavian Journal of Statistics, 5, 81-91

Abstract: Let $\{X_t\}_{t=1}^{\infty}$ be a Markov chain with state space $\{1, \cdots, r\}$, stationary transition probabilities $\pi_{jk} = P[X_{t+1} = k | X_t = j]$ and stationary distribution $\pi_j = P[X_t = j]$. Let $f_j(; \theta_j), j = 1, \cdots, r$, be probability density functions (pdf's) with joint continuous or discrete space, and let $\{Y_t\}_{t=1}^{\infty}$ be a sequence of random variables, such that $f_{Y_t | X_t = j}(y) = f_j(y; \theta_j)$, $j = 1, \cdots, r$. The marginal parameters θ_j may be multidimensional. It is further assumed

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that $f_{Y_1,\cdots,Y_T|X_1=j_1,\cdots,X_T=j_T}(y_1,\cdots,y_T) = \prod_{t=1}^{T} f_{j_t}(y_t;\theta_{j_t})$, which implies that the marginal pdf of Y_t is the mixture $f(y) = \sum_{j=1}^{r} \pi_j f_j(y;\theta_j)$, $t = 1,\cdots,T$. The objective is to estimate θ_j, π_j and π_{jk} on the basis of observations $y_1,\cdots,y_T$ from $f(y)$, when the so-called regime variables $X_1,\cdots,X_T$ are unobservable. The author first assumes that π_j and π_{jk} are known, and uses a maximization technique due to L. E. Baum, T. Petrie, G. Soules and N. Weiss [Ann. Math. Statist. 41 (1970), 164–171] in order to derive the maximum likelihood estimator (MLE) of θ_j, $j = 1,\cdots,r$. He then considers π_j and π_{jk} unknown, as is the case in real situations, and obtains their MLEs. Finally, he compares the MLEs of θ_j, π_j and π_{jk} via a simulation study with their MLI estimators, i.e., their MLEs obtained under the assumption that the regime variables are independent. (Reviewed by A. N. Philippou)

MacNeill, Ian B.(1978-12)
Limit Processes for Sequences of Partial Sums of Regression Residuals
The Annals of Probability, 6, 4. 695-698.

Abstract: Linear regression of a random variable against several functions of time is considered. Limit processes are obtained for the sequences of partial sums of residuals. The limit processes, which are functions of Brownian motion, have covariance kernels of the form:

$$K(s,t) = \min(s,t) - \int_0^t \int_0^s g(x,y)dxdy.$$

The limit process and its covariance kernel are explicitly stated for each of polynomial and harmonic regression.

MacNeill, Ian B.(1978-13)
Properties of Sequences of Partial Sums of Polynomial Regression Residuals with Applications to Tests for Change of Regression at Unknown Times
The Annals of Statistics, 6, 2, 422-433.

Abstract: Limit processes are obtained for the sequences of partial sums of polynomial regression residuals. Properties of linear and quadratic functionals on the sequences are discussed. Distribution theory for Cramér-von Mises type functionals is obtained. An indication is given of the relevance of these results to the problem of testing for change of regression at unknown times.

Maronna, Ricardo and Yohi, Victor J.(1978-14)
A Bivariate Test for the Detection of a Systematic Change in Mean
Journal of the American Statistical Association, 73, 363., 640-645.

Abstract: Let $(x_i, y_i), i = 1, \cdots, n$, be a sequence of observations such that $y_i = b_i + cx_i + u_i$, where b_i and c are unknown parameters, and $\{u_i\}$ and $\{x_i\}$ are independent sequences of independent, identically distributed random variables. The likelihood ratio test is derived for the hypothesis that $b_i = b(i = 1, \cdots, n)$, against the alternative that $b_i = b(i \leq i_0)$ and $b_i = b + d(i > i_0)$ for some b, i_0, and $d \neq 0$, assuming the u_i's are normal. Quantiles of the test statistic are computed by simulation, and the consistency of the test is proved. Some asymptotic properties of the test statistic are shown.

Park, Sung H. (1978-15)
Experimental designs for fitting segmented polynomial regression models
Technometrics, 20, 151-154
Keywords: Spline; D-optimality; Response surface; Nonlinear estimation; Join point.

Abstract: In many applications a response variable, y, may not be adequately represented by a polynomial function of the input variable, x, over the entire experimental space. Often a desirable choice of a regression model is one which consists of grafted polynomial submodels. This paper mainly considers the problem of finding minimum point experimental designs to estimate the coefficients in segmented polynomial regression. For the efficiency of estimation, the D-optimality design criterion (which minimizes the generalized variance of the least squares estimates of the unknown parameters) is adopted.

Quandt, Richard E. and Ramsey, James B. (1978-16)
Estimating mixtures of normal distributions and switching regressions
Journal of the American Statistical Association, 73, 730-738
Keywords: Moment generating function; Method of moments; Maximum likelihood

Abstract: Since the likelihood function corresponding to finite mixtures of normal distributions is unbounded, maximum likelihood estimation may break down in practice. The article introduces the "moment generating function estimator" defined as the estimator which minimizes the sum of squares of differences between the theoretical and sample moment generating functions. The consistency and asymptotic normality of the estimator are proved and its finite sample behavior is compared to that of the standard method of moments estimator by Monte Carlo experiments. The estimator is applied to the Hamermesh model of wage bargain determination.

Quandt, Richard E. and Ramsey, James B. (1978-17)
Reply to comments on "Estimating mixtures of normal distributions and switching regressions"
Journal of the American Statistical Association, 73, 751-752

Austin F. S. Lee; Sylva M. Heghinian(1977-1)
A Shift of the Mean Level in a Sequence of Independent Normal Random Variables: A Bayesian Approach

Abstract: In this article, a study is made about a shift in the mean of a set of independent normal random variables with unknown common variance. The marginal and joint posterior distributions of the unknown time point and the amount of shift are derived. Small and large sample results are presented.

Bhattacharya, P. K.; Brockwell, P. J.(1977-2)
The minimum of an additive process with applications to signal estimation and storage theory.

Abstract: The authors consider a real-valued additive process $\{X(t), t \geq 0\}$ with no negative jumps, right-continuous sample paths with left limits, and satisfying the condition $P(X(t) < 0) > 0$ for all $t > 0$. A representation for the joint distribution of M_t and T_t is derived, where $M_t = \inf_{0 \leq u \leq t} X(u)$, the minimum in $[0, t]$, and $T_t = \inf\{u : \inf_{0 \leq y \leq u} X(y) = M_t\}$, the first passage time to M_t. Asymptotic results as $t \to \infty$ and specific formulas for this joint distribution are given when X is Brownian motion and when X is a stable process of exponent $\frac{1}{2}$. This joint distribution is used to study a problem of faint signal detection. Specifically, it is assumed that observations $\{Y_i^{(n)}, 1 \leq i \leq n : n > 1\}$ are made, where $Y_j^{(n)} = X_j^{(n)} + a_j^{(n)}$, $\{X_j^{(n)}, 1 \leq j \leq n, n \geq 1\}$ being i.i.d. with mean and variance σ^2. Furthermore, the signal $\{a_j^{(n)}\}$ satisfies the relation $a_j^{(n)} = 0$ if $1 \leq j \leq [n\lambda_1]$, $[n\lambda_2] + 1 \leq j \leq n$, $a_j^{(n)} = \delta v_n^{-1}$ if $[n\lambda_1] + 1 \leq j \leq [n\lambda_2]$ with $\delta, v_n^{-1}, \lambda_1, \lambda_2$, and σ^2 unknown. The authors wish to test for the presence of a signal ($H_0 : a_j^{(n)} = 0, 1 \leq j \leq n$) in the weak signal case ($v_n \to \infty$, $n^{-1/2} v_n \to 0$ as $n \to \infty$). They provide approximate maximum likelihood estimators for the change points λ_1 and λ_2 using asymptotic methods. The result is then applied to a problem in image processing. The results on change point inference generalize those of H. Chernoff and S. Zacks [Ann. Math. Statist. 35 (1964), 999-1018] and D. Hinkley [Biometrika 57 (1970), 1-17]. The authors also apply the results on the minimum and its location to a storage process and to the integral of a Markov chain. (Reviewed by John P. Lehoczky)

Davies, Robert B.(1977-3)
Hypothesis Testing When a Nuisance Parameter is Present Only Under the Alternative

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Abstract: Suppose that the distribution of a random variable representing the outcome of an experiment depends on two parameters ξ and θ and that we wish to test the hypothesis $\xi = 0$ against the alternative $\xi > 0$. If the distribution does not depend on θ when $\xi = 0$, standard asymptotic methods such as likelihood ratio testing or $C(\alpha)$ testing are not directly applicable. However, these methods may, under appropriate conditions, be used to reduce the problem to one involving inference from a Gaussian process. This simplified problem is examined and a test which may be derived as a likelihood ratio test or from the union-intersection principle is introduced. Approximate expressions for the significance level and power are obtained.

Dent; W. T. and Hildreth, C(1977-4)
Maximum Likelihood Estimation in Random Coefficient Models
Journal of the American Statistical Association, Vol. 72, 69-72.
Keywords: Random coefficient models; Maximum likelihood estimation; Nonlinear optimization; Numerical accuracy.

Abstract: Previous Monte Carlo studies examining properties of estimators in random coefficient models have been hindered in part by computational difficulties. In particular, determination of maximum likelihood estimators appears sensitive to the computational algorithm used. In a small Monte Carlo experiment, several distinctly motivated algorithms are examined with respect to accuracy and cost in searching for global and local maximum likelihood parameter estimates. A noncalculus oriented approach offers promise. When compared with other estimators, maximum likelihood estimators, so determined, appear to be statistically relatively efficient.

Draper, Norman R., Guttman, Irwin and Lipow, Peter (1977-5)
All-bias designs for spline functions joined at the axes
Journal of the American Statistical Association, 72, 424-429
Keywords: Response surface; First order design; Second-order design

Abstract: Often, a sensible approach to a general design problem is to slightly expand the appropriate all-bias designs. This idea is applied to situations involving polynomial spline models in which it is necessary to (i) guard against second-degree models in some part, or all, of a cuboidal region of interest, while fitting first-degree models; (ii) guard against third-degree models, while fitting second-degree models. All joins are made on the axes. For (i), suitable designs include 2_{IV}^{k-p} factorial designs in each 2^k-tant. For (ii), consideration of symmetrical designs leads to cuboidal designs of the type previously considered in Draper and Lawrence (1965).

Gallant, A. Ronald (1977-6)

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Testing a nonlinear regression specification: A nonregular case
Journal of the American Statistical Association, 72, 523-530
Keywords: Hypothesis testing; Segmented regression(pdf file is saved)

Abstract: A statistical test of whether an additive nonlinear term in the response function should be omitted from a nonlinear regression specification is considered. The regularity conditions used to obtain the asymptotic distributions of the customary test statistics are violated when the null hypothesis of omission is true. Moreover, standard iterative algorithms are likely to perform poorly when the data support the null hypothesis. Methods designed to circumvent these mathematical and computational difficulties are described and illustrated.

Garbade, Kenneth(1977-7)
Two Methods for Examining the Stability of Regression Coefficients
Journal of the American Statistical Association, 72, 357, 54-63.
Keywords: Kalman filter; Recursive residuals; Parameter stability; CUSUM test; CUSUM of squre test; Demand for money.

Abstract: This paper investigates the power of two methodologies, the tests of Brown, Durbin, and Evans(1975, J. Roy. Statist. Soc., Ser. B, 37, No. 2, 149-192) and variable parameter regression, to detect several varieties of instability in the coefficients of a linear regression model. The study reported by Khan (1982, J. of Political Economy, 82, 1205-1219) on the stability of the demand for money is replicated with variable parameter regression, and his results are in part questioned and in part sharpened.

Hawkins, Douglas M.(1977-8)
Testing a Sequence of Observations for a Shift in Location
Journal of the American Statistical Association, 72, 357, 180-186.
Keywords: Segmentation; Location shift; Testing for change-point.

Abstract: A possible alternative to the hypothesis that the sequence $X_1, X_2, \ldots, X_n$ are iid $N(\xi, \sigma^2)$ random variables is that at some unknown instant the expectation ξ shifts. The likelihood ratio test for the alternative of a location shift is studied and its distribution under the null hypothesis found. Tables of standard fractals are given, along with asymptotic results.

Holbert, Donald and Broemeling, Lyle Bayesian(1977-9)
inferences related to shifting sequences and two-phase regression.
Communications in statistics. theory and methods, 6, 265-275

Abstract: The problem of estimating the switch point in a sequence of independent normal random variables is studied from a Bayesian viewpoint. The method appears to provide a valuable alternative to the standard cusum techniques. The authors consider changes in a normal sequence of random variables, and show that it is possible to obtain the posterior distribution of the change-point using a one-dimensional integration. The results are extended to a two-phase regression model. Two numerical examples are included. A parallel approach by A. F. M. Smith [Biometrika 62 (1975), 407-416] also copes with non-normal random variables. (Reviewed by Tom Leonard)

Hušková M.(1977-10)
The Rate of Convergence of Simple Linear Rank Statistics Under Hypothesis and Alternatives
The Annals of Statistics, 5, 658-670.

Abstract: Convergence rates for distributions of simple linear rank statistics are investigated. Both the null hypothesis and near alternatives are considered. The method of proof consists in approximating the characteristic function of the statistic by that of a sum of independent random variables and then applying standard tools.

Sen, P.K.(1977-11)
Tied-Down Wiener Process Approximations for Aligned Rank Order Processes and Some Applications
The Annals of Statistics, 5, 6, 1107-1123.

Abstract: For independent random variables distributed symmetrically around an unknown location parameter, aligned rank order statistics are constructed by using an estimator of the location parameter based on suitable rank statistics. The sequence of these aligned rank order statistics is then incorporated in the construction of suitable stochastic processes which converge weakly to some Gaussian functions, and, in particular, to tied-down Wiener processes in the most typical cases. The results are extended for contiguous alternatives and then applied in two specific problems in nonparametric inference. First, the problem of testing for shift at an unknown time point is treated, and then, some sequential type asymptotic nonparametric tests for symmetry around an unknown origin are considered.

Box, G. E. P. and Tiao, G. C. (1976-1)
Comparison of forecast and actuality
Applied Statistics, 25, 195-200
Keywords: Parameter changes; Surveillance; Intervention analysis; Ozone level; Pollution control

Abstract: The paper shows how possible change in a system generating a time series may be studied by comparing forecasts made from a model built on data prior to the suspected change with data actually occurring. An environmental example illustrates the decomposition of the overall criterion into relevant components and shows how difficulties can occur in distinguishing alternative models for change. The relation to surveillance problems and to intervention analysis is briefly discussed.

Darkhovskii, B. S. (ed.) and Rabindranathan, M. (ed.) (1976-2)

A nonparametric method for the a posteriori detection of the "disorder" time of a sequence of independent random variables

Theory of Probability and its Applications, 21, 178-183

Ertel, James E. and Fowlkes, Edward B.(1976-3)

Some Algorithms for Linear Spline and Piecewise Multiple Linear Regression

Journal of the American Statistical Association, 71, 640-648.

Abstract :This paper develops some efficient algorithms for linear spline and piecewise multiple linear regression. A plotting procedure that shows the existence and location of changes in linear regression models is developed as an adjunct to one of the algorithms. The algorithms are compared with other presently available algorithms both in terms of efficiency and in terms of performance on sets of artificial data. An example shows how the algorithms, implemented in portable FORTRAN IV, can be used profitably in the analysis of data.

Hawkins, Douglas M. (1976-4)

Point estimation of the parameters of piecewise regression models

Applied Statistics, 25, 51-57

Keywords: Nonlinear estimation

Abstract: Two methods of fitting piecewise multiple regression models are presented. One, based on dynamic programming, yields maximum-likelihood estimators and is suitable for sequences of moderate length. A second, hierarchical, procedure yields approximations to the maximum-likelihood estimators and is suitable for very long sequences of data. Both methods have computational requirements that are linear in the number of segments.

Hines, W. G. S. (1976-5)

A simple monitor of a system with sudden parameter changes

IEEE Transactions on Information Theory, 22, 210-216

Abstract: Performance of a monitor using a geometric moving-average (GMA) of observations of a system subject to sudden changes is related to parameters describing some aspects of the changes and to parameters of the monitor. Monitor parameters to optimize performance are determined numerically for an example of practical interest.

Schweder, Tore (1976-6)
Some "optimal" methods to detect structural shift or outliers in regression
Journal of the American Statistical Association, 71, 491-501
Keywords: CUSUM; Sequential residuals; Power; Random walk; *t*-test

Abstract: A Neyman-Pearson approach is taken to the problem of detecting structural shifts in naturally ordered regression problems. When the variance is known, backwards CUSUM methods are shown to maximize average power, and their application is discussed. Two methods with optimality properties for outlier detection are developed, assuming that the observations may be divided into two parts, where the first part satisfies the model assumptions, while outliers may be present in the other.

Singh, Balvir; Nagar, A. L.; Choudhry, N. K. and Raj,Baldev (1976-7)
On the Estimation of Structural Change: A Generalization of the Random Coefficients Regression Model
International Economic Review, 17, 2, 340-361.

Abstract: In this paper we extend the random regression coefficient model [**20, 25, 36, 39**], herein termed the constant mean response (**CMR**) model to a more general case in which the regression coefficients change systematically with calender time, herein termed the variable mean response (**VMR**) model. Two estimators are proposed for such a model. The first following Theil and Mennes [**45**], is a four step generalization of the Hildreth-Houck estimator [**20**], in view of the fact that the residuals in the second stage of the Hildreth-Houck (**H-H**) esimator are not homoscedastic. Second we derive a maximum likelihood estimator for **VMR** model. The last section in this paper presents an empirical illustration of the comparative performance of the estimators under focus in this paper. The paper contains three appendices. Appendix A obtains covariance matrix of the second stage **H-H** residuals for use in the four step estimator to which we allude above. In Appendix B we prove the consistency of the estimator proposed for **VMR** model in view of the fact that as a consequence of the inclusion of calender time among the regressors the probability limit of the moments matrix of the regressors is no longer finite. Lastly, since the maximizing equations are highly non-linear, Appendix C outlines an algorithm for their solution.

Brown, R. L., Durbin, J. and Evans, J. M. (1975-1)
Techniques for testing the constancy of regression relationships over time

Abstract: Methods for studying the stability over time of regression relationships are considered. Recursive residuals, defined to be uncorrelated with zero means and constant variance, are introduced and tests based on the cusum and cusum of squares of recursive residuals are developed. Further techniques based on moving regressions, in which the regression model is fitted from a segment of data which is moved along the series, and on regression models whose coefficients are polynomials in time are studied. The Quandt log-likelihood ratio statistic is considered. Emphasis is placed on the use of graphical methods. The techniques proposed have been embodied in a comprehensive computer program, TIMVAR. Use of the techniques is illustrated by applying them to three sets of data.

El-Sayyad, G. M. (1975-2)
A Bayesian analysis for the change-point problem
The Egyptian Statistical Journal, 19, 1-13

Abstract: A Bayesian method for estimating a change in a parameter at an unknown point is given. The method is applied to cases of normal and exponential distribution.

Feder, Paul I. (1975-3)
The log likelihood ratio in segmented regression
The Annals of Statistics, 3, 84-97
Keywords: Asymptotic distribution

Abstract: This paper deals with the asymptotic distribution of the log likelihood ratio statistic in regression models which have different analytical forms in different regions of the domain of the independent variable. It is shown that under suitable identifiability conditions, the asymptotic chi square results of Wilks and Chernoff are applicable. It is shown by example that if there are actually fewer segments than the number assumed in the model, then the least squares estimates are not asymptotically normal and the log likelihood ratio statistic is not asymptotically χ^2. The asymptotic behavior is then more complicated, and depends on the configuration of the observation points of the independent variable.

Feder, Paul I. (1975-4)
On asymptotic distribution theory in segmented regression problems –
Identified case
The Annals of Statistics, 3, 49-83
Keywords: Least squares estimation; Spline

Abstract: This paper deals with the asymptotic distribution theory of least squares estimators in regression models having different analytical forms in different regions of the domain of the independent variable. An important special case is that of broken line regression, in which each segment of the regression function is a different straight line. The residual sum of squares function has many corners, and so classical least squares techniques cannot be directly applied. It is shown, however, that the problem can be transformed into a new problem in which the sum of squares function is locally smooth enough to apply the classical techniques. Asymptotic distribution theory is discussed for the new problem and it is shown that the results are also valid for the original problem. Results related to the usual normal theory are derived.

Ferreira, Pedro E. (1975-5)

A Bayesian analysis of a switching regression model: Known number of regimes

Journal of the American Statistical Association, 70, 370-374

Keywords: Estimation; Segmented regression

Abstract: Some aspects of Bayesian methods of inference relative to switching regression models are analyzed. It is shown that Bayesian confidence intervals and tests may be obtained by using Student's t and χ^2 distributions. Mean biases and MSE of some Bayesian estimates are compared by Monte Carlo methods with those of the ML estimate. The MSE of the Bayesian estimates were uniformly smaller than those of the ML estimate. An experiment designed by Quandt [11] is also analyzed.

Sen, Ashish and Srivastava, Muni S.(1975-6)

On Tests for Detecting Change in Mean

The Annals of Statistics, 3, 1 , 98-108

Abstract: Procedures are considered for testing whether the means of each variable in a sequence of independent random variables can be taken to be the same, against alternatives that a shift might have occurred after some point r. Bayesian test statistics as well as some statistics depending on estimates of r are presented and their powers compared. Exact and asymptotic distribution functions are derived for some of the Bayesian statistics.

Sen, Ashish and Srivastava, Muni S. (1975-7)

Some one-sided tests for change in level

Technometrics, 17, 61-64

Keywords: Shift in mean; Hypothesis testing; Monte Carlo; Bayes; Maximum likelihood

Abstract: We consider tests based on one observation on each of N 2 random variables $x_1, \cdots, x_N$ to decide if the means μ_i of the x_i's are all equal against the one-sided alternative

that a shift has occurred at some unknown point r,(i. e. $\mu_1 = \mu_2 = \cdots = \mu_r < \mu_{r+1} = \cdots = \mu_N$). The x_i's are considered to be normally distributed with a common unknown variance. Bayesian tests as well as a test based on the maximum likelihood estimate of r are considered and their powers are compared by Monte Carlo methods. The exact distribution of a Bayesian test statistic is derived. A simple application using traffic accident data is presented.

Smith, A. F. M.(1975-8)

A Bayesian Approach to Inference about a Change-Point in a Sequence of Random Variables

Biometrika, 62, 407-416

Keywords: Bayesian inference; Change-point; Informative priors.

Abstract: A Bayesian approach is considered to the problem of making inferences about the point in a sequence of random variables at which the underlying distribution changes. Inferences are based on the posterior probabilities of the possible change-points. Detailed analyses are given for cases in which the distributions are binomial and normal, and numerical illustrations are provided. An informal sequential procedure is also noted

Swamy, P. A. V. B. and Mehta, J. S. (1975-9)

Bayesian and non-Bayesian analysis of switching regressions and of random coefficient regression models

Journal of the American Statistical Association, 70, 593-602

Keywords: Estimation; Maximum likelihood; Panel data

Abstract: Quandt [20] analyzed the problem of discontinuous shifts in regression regimes at unknown points in the data series. We note that Quandt's statistical approach based solely on the likelihood function can be misleading, whereas the Bayesian method based on a proper prior distribution of the unknown parameters yields sensible results. However, the exact evaluation of the posterior distribution is unusually burdensome and cannot be simplified even in large samples. To avoid this difficulty, we suggest an alternative formulation and provide an approximate Bayesian solution. In this alternative formulation, the coefficient vectors are treated as random drawings from a continuous multivariate distribution.

Atkinson, C.(1974-1)

A Wiener-Hopf Integral Equation Arising in Some Inference and Queueing Problems

Biometrika, 61, 277-283

Keywords: Change-point; Inference; Queues; Wiener-Hopf integral equation.

Abstract: The solution is presented to an integral equation of Wiener-Hopf type which has been recently treated numerically by Hinkley in connection with the problem of inference about the change-point in a sequence of random variables. The closed form solution given here enables results to be obtained easily in situations where the numerical method fails. A similar integral equation arises in a queueing problem where the difference between inter-arrival and service time of a customer is distributed normally. Explicit expressions are derived for this situation also.

Guthery, Scott B. (1974-2)
Partition regression
Journal of the American Statistical Association, 69, 945-947
Keywords: Dynamic programming; Segmented regression; Cluster analysis; Spline regression

Abstract : This article considers an approach to partition or unconstrained piece-wise regression which begins with an ordering of the observations and applies a dynamic program from cluster analysis to greatly reduce the number of calculations involved in its application.

MacNeill, I. B.(1974-3)
Tests for Change of Parameter at Unknown Times and Distributions of Some Related Functionals on Brownian Motion
The Annals of Statistics, 2, 5, 950-962.

Abstract: Statistics are derived for testing a sequence of observations from an exponential-type distribution for no change in parameter against possible two-sided alternatives involving parameter changes at unknown points. The test statistic can be chosen to have high power against certain of a variety of alternatives. Conditions on functionals on $C[0, 1]$ are given under which one can assert that the large sample distribution of the test statistic under the null-hypothesis or an alternative from a range of interesting hypotheses is that of a functional on Brownian Motion. We compute and tabulate distributions for functionals defined by nonnegative weight functions of the form $\psi(s) = as^k, k > -2$. The functionals for $-1 \geq k > -2$ are not continuous in the uniform topology on $C[0, 1]$.

Watts, Donald G. and Bacon, David W.(1974-4)
Using an Hyperbola as a Transition Model to Fit Two-Regime Straight-Line Data
Technometrics, 16, 3, 369-373.

Abstract: In a recent paper (Bacon, D. W. and Watts D. Q.(1970)) a general form of transition model was suggested to describe data which appear to follow two different straight

line relationships on opposite sides of an undetermined join point. An alternative model is now considered, the familiar hyperbola, parameterized in a geometrically meaningful form. The two models are fitted to two sets of experimental data for purposes of comparison. In one of the examples account is taken of autocorrelated errors using a procedure suggested by Sredni, J.(1970, Ph.D. Thesis, University of Wisconsin, Madison, Wisconsin.)

Andrews, D. F. and Herzberg, Agnes M. (1973-1)
A simple method for constructing exact tests for sequentially designed experiments
Biometrika, 60, 489-497
Keywords: Two-stage design; Regression; Change-point; Inverse estimation

Abstract: A particular method is given for constructing exact tests for sequentially designed experiments when the treatments for the experimental units may be chosen. A simple artificial example of the form of test is presented. This method is applied to inverse estimation in a simple linear model, to testing a simple linear hypothesis about a linear model and to finding the change-point when a response is described by two linear models. In all cases the designs and analysis are less than optimal. However, the potential gains for such sequential designs can outweigh the losses and justify the simple techniques, which may be easily implemented.

Froehlich, B. R.(1973-2)
Some Estimators for a Random Coefficient Regression Model
Journal of the American Statistical Association, 68, 329-335

Abstract: The justification of applying a random coefficient regression model in econometric work has been discussed by numerous econometricians. Hildreth and Houck have derived a set of consistent estimators for such a model. Alternatives to these are developed but analytic attempts to ascertain the small sampling properties of these alternative estimators have not been very successful so far. A Monte Carlo experiment is made and the relative performance of these estimators is described.

Gallant, A. R. and Fuller, Wayne A. (1973-3)
Fitting segmented polynomial regression models whose join points have to be estimated
Journal of the American Statistical Association, 68, 144-147
Keywords: Hartley theorem; Gauss-Newton method

Abstract : The study considers the problem of finding the least squares estimates for the unknown parameters of a regression model which consists of grafted polynomial submodels.

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The abscissae of the join points are a subset of the unknown parameters. Examples are given to illustrate how continuity and differentiability conditions on the model can be used to reparameterize the model so as to allow Modified Gauss-Newton fitting. A slightly generalized version of Hartley's theorem is stated to extend the Modified Gauss-Newton method to this problem.

Goldfeld, Stephen M. and Quandt, Richard E.(1973-4)
A Markov model for switching regressions
Journal of Econometrics, 1, 3-16.

Poirier, Dale J. (1973-5)
Piecewise regression using cubic splines (Corr: V69 p288)
Journal of the American Statistical Association, 68, 515-524
Keywords: Structural change; Segmented regression

Abstract : Spline theory and piecewise regression theory are integrated to provide a framework in which structural change is viewed as occurring in a smooth fashion. Specifically, structural change occurs at given points through jump discontinuities in the third derivative of a continuous piecewise cubic estimating function. Testing procedures are developed for detecting structural change as well as linear or quadratic segments. Finally, the techniques developed are illustrated empirically in a learning-by-doing model.

Sen, Ashish K. and Srivastava, Muni S.(1973-6)
On multivariate tests for detecting change in mean
Sankhya, 35, Series A, Pt. 2, 173–186

Abstract: We consider the problem of testing the hypothesis that the means of a sequence of independent multivariate normal random variables $x_1, \cdots, x_N$ are equal against the alternative that after an unknown point $r(1 \leq r \leq N-1)$, the means have shifted. Asymptotic and exact distributions of the test statistics under the hypothesis are derived and some tables are presented.

Duncan, D. B. and Horn, S. D.(1972-1)
Linear Dynamic Recursive Estimation from the Viewpoint of Regresion Analysis
Journal of the American Statistical Association, 67, 815-821.

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Abstract: A large class of useful multivariate recursive time series models and estimation methods has appeared in the engineering literature. Despite the interest and utility which this recursive work has when viewed as an extension of regression analysis, little of it has reached statisticians working in regression. To overcome this we (a) present the relevant random-β regression theory as a natural extension of conventional fixed-β regression theory and (b) derive the optimal recursive estimators in terms of the extended regression theory for a typical form of the recursive model. This also opens the way for further developments in recursive estimation, which are more tractable in the regression approach and will be presented in future papers.

Hinkley, D. V.(1972-2)
Time-Ordered Classification
Biometrika, 59, 3, 509-523.

Abstract: Classification for the special situation where observations after a certain time no longer come from the initial population is considered. The discussion focuses on inference about that time, often called the change-point. Some useful approximations are derived for the distributions of change-point statistics, and large-sample results are established for nuisance-parameter situations.

Quandt, Richard E. (1972-3)
A new approach to estimating switching regressions
Journal of the American Statistical Association, 67, 306-310
Keywords: Maximum likelihood; Spline regression; Time-varying parameter

Abstract: In recent years much attention has been focussed on the problem of discontinuous shifts in regression regimes at unknown points in the data series. This article approaches this problem by assuming that nature chooses between regimes with probabilities λ and $1 - \lambda$. This allows formulation of the appropriate likelihood function maximized with respect to the parameters in the regression equations and λ. The method is compared to another recent procedure in some sampling experiments and in a realistic economic problem and is found satisfactory.

Rao, Poduri S. R. S. (1972-4)
On two phase regression estimator
Sankhy, Series A, 34, 473-476
Keywords: Mean square error; Sampling without replacement; Finite population; Minimum variance linear unbiased estimator

Abstract: When the samples at the two phases are drawn independently, regression estimators that have smaller mean square error then the one given in the literature are suggested.

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Bacon, David W. and Watts, Donald G.(1971-1)
Estimating the transition between two intersecting straight lines
Biometrika, 58, 525-534
Keywords: Two-phase regression; Bayesian approach

Abstract: For experimental data which appear to behave according to two different distinct linear relationships, a general model is proposed which allows for a smooth transition from one linear regime to the other. The transition is accomplished by a curve incorporating a transition parameter. The special case of two intersecting straight lines is included in this model. A Bayesian estimation procedure is used to determine the plausibility of different parameter values. The analysis procedure may be extended to any number of join points and for any linear intersecting functions.

Hinkley, D.V.(1971-2)
Inference about the Change-Point from Cumulative Sum Tests
Biometrika, 58, 509-523.

Abstract: The point of change in mean in a sequence of normal random variables can be estimated from a cumulative sum test scheme. The asymptotic distribution of this estimate and associated test statistics are derived and numerical results given. The relation to likelihood inference is emphasized. Asymptotic results are compared with empirical sequential results, and some practical implications are discussed

Hinkley, David V. (1971-3)
Inference in two-phase regression
Journal of the American Statistical Association, 66, 736-743
Keywords: Maximum likelihood estimator; Confidence region

Abstract: Procedures are outlined for obtaining maximum likelihood estimates and likelihood confidence regions in the intersecting two-phase linear regression model. The procedures are illustrated on a small set of data, and the distributional properties are examined empirically.

Hinkley, D.V.; Hinkley Elizabeth A.(1971-4)
Correction and Amendments: Inference about the Change-Point in a Sequence of Binomial Variables
Biometrika, 58, 3, 687

Barten, A. P. and Bronsard, Lise Salvas (1970-1)
Two-stage least-squares estimation with shifts in the structural form
Econometrica, 38, 938-941

Abstract: we consider the estimation of linear models when the coefficients of the structural form are not the same for all observations for which the model is postulated to be valid. An example of such a model is given in Van Den Beld, C. A.(1968, Mathematical Model Building in Economics and Industry. Griffn, London, pp. 31-48.), where some structural relations have a piecewise linear form. Another example is the water melon market model of Suits, D. B.(1955, J. of Farm Economy, Vol. 37, pp. 237-251) where there are two alternative harvest supply schedules. Also discussed here is the case where for one part of the sample period one or more variables are endogenously determined while for another part they are exogenous, for instance, the wage rate or the rate of exchange. Such a change in the nature of the model can also be interpreted as a change in the coefficients of the...

Burnett, Thomas D. and Guthrie, Donald(1970-2)
Estimation of Stationary Stochastic Regression Parameters
Journal of the American Statistical Association, 65, 1547-1553.

Abstract:This article considers repeated regression experiments wherein the regression parameters vary according to a stationary stochastic process with known covariance structure. Expressions are derived for best linear estimators and predictors of linear functions of the regression parameters.

Farley, John U. and Hinich, Melvin J. (1970-3)
A test for a shifting slope coefficient in a linear model
Journal of the American Statistical Association, 65, 1320-1329
Keywords: Time series; Power; Time-varying parameter

Abstract: A locally most powerful test is developed for the hypothesis that a slope coefficient in a linear time series model is stable, against the alternative that the slope shifts exactly once somewhere in the series. Analysis of the procedure using artificial data indicates good power characteristics even when the ratio of the shift size to the error variance is moderate–especially if the shift does not occur very near either end of the series. Power also depends on the pattern of the independent variables and on whether the error variance is known or must be estimated using the residuals about the regression line.

Fisher, F. M.(1970-4)
Tests of equality between sets of coefficients in two linear regressions
Econometrica, 38, 2, 361-366.

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Abstract: The problem of testing the equality of sets of regression coefficients in two or more regressions arises with considerable frequency in econometrics. While the tests involved can all be found in the statistical literature, the derivation thereof is usually very difficult for students in econometrics courses to follow. Accordingly, the present note attempts to derive the relevant results in a unified and relatively simple way and to show the close relation of the tests involved to the standard F test of the hypothesis that a subset of the regression coefficients in a single regression has all its elements zero.

Hinkley, V.D.(1970-5)
Inference About the Change-Point in a Sequence of Random Variables
Biometrika, 57, 1., 1-17.

Abstract: Inference is considered about the point in a sequence of random variables at which the probability distribution changes. In particular, we examine a normal distribution with changing mean. The asymptotic distribution of the maximum likelihood estimate is derived and also the asymptotic distribution of the likelihood ratio statistic for testing hypotheses about the change-point. These asymptotic distributions are compared with some finite sample empirical distributions.

Hinkley, D.V.; Hinkley, E.A.(1970-6)
Inference About the Change-Point in a Sequence of Binomial Variables
Biometrika, 57, 3., 477-488.

Abstract: The problem of making inferences about the point in a sequence of zero-one variables at which the binomial parameter changes is considered. The asymptotic distribution of the maximum likelihood estimate of the change-point is derived in computable form using random walk results. The asymptotic distributions of likelihood ratio statistics for testing hypotheses about the change-point are also obtained. Some exact numerical results for these asymptotic distributions are given and their accuracy as finite sample approximations is discussed.

McGee, Victor E. and Carleton, Willard T. (1970-7)
Piecewise regression
Journal of the American Statistical Association, 65, 1109-1124
Keywords: Hierarchical clustering

Abstract: A difficult regression parameter estimation problem is posed when the data sample is hypothesized to have been generated by more than a single regression model. To find the best-fitting number and location of underlying regression systems, the investigator must specify both the statistical criterion and the search-estimation procedure to be used. The

approach outlined in this article is essentially a wedding of hierarchical clustering and standard regression theory. As the name suggests, piecewise regression may be described as a method of finding that piecewise continuous function which best describes the data sample. Computational procedures and a fully-worked example, together with possible extensions, are provided.

Yakowitz, S. J. (1970-8)
Unsupervised learning and the identification of finite mixtures.
IEEE Trans. Inf. Theory, IT-16, 330-338.

Abstract: The first portion of this paper is tutorial. Beginning with a standard definition of an abstract pattern-recognition machine, "learning" is given a mathematical meaning and the distinction is made between supervised and unsupervised learning. The bibliography will help the interested reader retrace the history of learning in pattern recognition. The exposition now focuses attention on unsupervised learning. Carefully, it is explained how problems in this subject can be viewed as problems in the identification of finite mixtures, a statistical theory that has achieved some maturity. From this vantage point, it is demonstrated that identification theory implies unsupervised learning is possible in many important cases. The remaining sections present a general method for achieving unsupervised learning. Other authors have proposed schemes having greater computational convenience, but no method previously published is as inclusive as the one revealed here, which we demonstrate to be effective for all the many cases wherein unsupervised learning is known to be possible.

Bellman, Richard and Roth, Robert(1969-1)
Curve Fitting by Segmented Straight Lines
Journal of the American Statistical Association, 64, 327, 1079-1084

Abstract: In many situations, approximation of a set of data by a polygonal curve is more advantageous than approximation by a polynomial. If the join points of the polygonal curve are known, the problem is quite simple. If, however, they are to be chosen in some expeditious fashion, considerable numerical difficulties can arise if the curve-fitting problem is approached directly. In this paper it is shown that dynamic programming offers a simple direct approach to the determination of an optimal fit.

Dagenais, Marcel G. (1969-2)
A threshold regression model
Econometrica, 37, 193-203
Keywords: Change point; Step function

Abstract: A special regression model is suggested to analyze economic variables possessing step-like time paths. The dependent variable is assumed not to move until the concerted

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action of the independent variables and the error term induces it to overcome its reaction threshold. After the theoretical solution is presented, the model is shown to be applicable both to time series, such as newsprint prices, and to cross sections, such as household durable good purchases and plant capacity increases. Finally, computational problems are briefly discussed.

Gardner, L. A., Jr (1969-3)
On detecting changes in the mean of normal variates
The Annals of Mathematical Statistics, 40, 116-126

Hinkley, D. V. (1969-4)
Inference about the intersection in two-phase regression
Biometrika, 56, 495-504
Keywords: Maximum likelihood estimation

Abstract: We study the problem of estimating and making inferences about the intersection in a two-phaser regression model with one independent variable. In particular we derive an asymptotic distribution for the maximum likelihood estimate of the intersection which is a good finite sample approximation. We also discuss the associated problems of testing hypotheses about the intersection.

Bhattacharya, G. K. and Johnson, Richard A. (1968-1)
Nonparametric tests for shift at unknown time point
The Annals of Mathematical Statistics, 39, 1731-1743

Abstract: This work is an investigation of a nonparametric approach to the problem of testing for a shift in the level of a process occurring at an unknown time point when a fixed number of observations are drawn consecutively in time. We observe successively the independent random variables $X_1, X_2, \cdots, X_N$ which are distributed according to the continuous cdf $F_i, i = 1, 2, \cdots, N$. An upward shift in the level shall be interpreted to mean that the random variables after the change are stochastically larger than those before. Two versions of the testing problem are studied. The first deals with the case when the initial process level is known and the second when it is unknown. In the first case, we make the simplifying assumption that the distributions F_i are symmetric before the shift and introduce the known initial level by saying that the point of symmetry γ_0 is known. Without loss of generality, we set $\gamma_0 = 0$. Defining a class of cdf's $F_0 = \{F : F \text{ continuous}, F \text{ symmetric about origin}\}$, the problem of detecting an upward shift becomes that of testing the null hypothesis

$$H_0 : F_0 = F_1 = \cdots = F_N, \quad \text{some} \quad F_0 \varepsilon F cross - se_0,$$

against the alternative

$$H_1 : F_0 = F_1 = \cdots = F_m > F_{m+1} = \cdots = F_N, \quad \text{some} \quad F_0 \varepsilon F_0$$

where $m(0 \leq m \leq N-1)$ is unknown and the notation $F_m > F_{m+1}$ indicates that X_{m+1} is stochastically larger than X_m. For the second situation with unknown initial level, the problem becomes that of testing the null hypothesis $H_0^* : F_1 = \cdots = F_N$, against the alternatives $H_1^* : F_1 = \cdots = F_m > F_{m+1} = \cdots = F_N$, where $m(1 \leq m \leq N-1)$ is unknown. Here the distributions are not assumed to be symmetric. The testing problem in the case of known initial level has been considered by Page [1955, Biometrika 42, 523-526], Chernoff and Zacks [1964, Ann. Math. Statist. 35,909-1018] and Kander and Zacks [1966,Ann. Math. Statist. 37,1196-1210]. Assuming that the observations are initially from a symmetric distribution with known mean γ_0, Page proposes a test based on the variables $\text{sgn}(X_i - \gamma_0)$. Chernoff and Zacks assume that the F_i are normal cdf's with constant known variance and they derive a test for shift in the mean through a Bayesian argument. Their approach is extended to the one parameter exponential family of distributions by Kander and Zacks. Except for the test based on signs, all the previous work lies within the framework of a parametric statistics. The second formulation of the testing problem, the case of unknown initial level, has not been treated in detail. The only test proposed thus far is the one derived by Chenoff and Zacks for normal distributions with constant known variance. In both problems, our approach generally is to find optimal invariant tests for certain local shift alternatives and then to examine their properties. Our optimality criterion is the maximization of local average power where the average is over the space of the nuisance parameter m with respect to an arbitrary weighting $\{q_i, i = 1, 2, \cdots, N : q_i \geq 0, \sum_{i=1}^{N} q_i = 1\}$. From the Bayesian viewpoint, q_i may be interpreted as the prior probability that X_i is the first shifted variate. Invariant tests with maximum local average power are derived for the case of known initial level in Section 2 and for the case of unknown initial level in Section 3. In both cases, the tests are distribution-free and they are unbiased for general classes of shift alternatives. They all depend upon the weight function $\{q_i\}$. With uniform weights, certain tests in Section 3 reduce to the standard tests for trend while a degenerate weight function leads to the usual two sample tests. In Section 4, we obtain the asymptotic distributions of the test statistics under local translation alternatives and investigate their Pitman efficiencies. Some small sample powers for normal alternatives are given in Section 5.

Hildreth, Clifford and Houck, James P.(1968-2)
Some Estimators for a Linear Model with Random Coefficients
Journal of the American Statistical Association, 63, 584-595.

Abstract: The linear model given by

$$y_t = \sum_{k=1}^{K} z_{tk}(\beta_k + v_{tk}) \quad t = 1, 2, \cdots, T$$

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is considered. The β_k represent average responses of y_t the dependent variable to unit changes in the independent variables, z_{tk}. The v_{tk} are independently distributed random errors. A number of consistent estimators of the coefficients, β_k, and the variances of the errors are developed and a few properties of the estimators are noted. Further investigations of sampling properties are needed.

Mustafi, Chandan (1968-3)

Inference problems about parameters which are subjected to changes over time

The Annals of Mathematical Statistics, 39, 840-854

Keywords: Empirical Bayes

Bather,J. A.(1967-1)

On a Quickest Detection Problem

The Annals of Mathematical Statistics, 38, 3, 711-724.

Abstract: In a recent paper A. N. Shiryaev [1963, Theory Prob. 8, 22-46] discusses the problem of detecting the arrival of a "disorder" in an observed stochastic process, as quickly as possible subject to a limitation on the number of false alarms. He considers two versions of a simple model. In the first, the disorder arrives at a discrete instant θ according to a geometric distribution. The process disturbed by this event consists of a sequence of independent observations $\{\xi_t\}$, such that $\xi_1, \xi_2, \cdots, \xi_{\theta-1}$ arise from a certain distribution F_0, whereas $\xi_\theta, \xi_{\theta+1}, \cdots$ come from a different distribution F_1. In the continuous time version of the model, the a-priori distribution of θ is exponential:

$$P(\theta > t) = e^{-\lambda t} \quad (t \geq 0)$$

and the disorder is represented by a change in the mean drift of an observed Wiener process $\{\eta(t)\}$. More precisely, for any given value of θ, this process has independent normal increments $\delta\eta = \eta(t + \delta t) - \eta(t)$, with

$$
\begin{aligned}
E(\delta\eta) &= 0 \quad (0 \leq t < \theta), \\
E(\delta\eta) &= \delta t \quad (t \geq \theta), \\
\mathrm{Var}(\delta\eta) &= \delta t \quad (t \geq 0).
\end{aligned}
$$

In both versions it may be decided at any instant t to carry out a detailed inspection in order to ascertain whether or not the disturbance has occurred. Then, if it is found that $\theta < t$ the process terminates but observation must be resumed immediately after a false alarm. Within these rules it is required to find a decision procedure which determines the instants at which a thorough inspection is worthwhile. Assuming that N, the expected number of false alarms, is specified in advance Shiryaev establishes the general form of policy which

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minimizes τ, the expected delay in verifying the arrival of the disorder. The a posteriori distribution of θ at any time, does not depend on anything which took place before the last false alarm. For example, in continuous time

$$p(t) = P(\theta \leq t \mid \eta(t'), 0 \leq t' \leq t) = P(\theta \leq t \mid \eta(t'), s \leq t' \leq t),$$

where s is the instant of the most recent false alarm. The geometric and exponential distributions have the useful property that

$$P(\theta > t + s \mid \theta > s) = P(\theta > t \mid \theta > 0).$$

He deduces that the optimal policy for the period following any false alarm must correspond exactly with the procedure applied initially, before the first inspection. In addition, he proves the existence of a critical level $p^* = p^*(N)$ such that, in general, observations should continue so long as $0 \leq p(t) < p^*$ with an immediate inspection as soon as $p(t) \geq p^*$. These and other similar results are established first for the discrete time model and then extended to the continuous time version. For the latter, the paper also gives more explicit calculations: the evaluation of τ in terms of p^*, for example. But no attempt is made to determine the critical level $p^*(N)$ for the optimal policy. In fact, as we shall see, a very simple relation holds:

$$p^*(N) = (N + 1)^{-1}.$$

However, the aim here is to show how the optimal policy can be found for a more realistic specification of the minimization problem, involving given delay and inspection costs. We shall concentrate entirely on the continuous time model and suppose that each inspection incurs an instantaneous cost $K > 0$, not depending on its outcome, whereas any delay in detecting the arrival of the disorder leads to a cost $c > 0$ per unit time. Hence the total expected cost is $K(N + 1) + c\tau$, which depends both on the decision procedure and on the initial condition $p(0) = 0$. The minimization will be based on the calculation of the minimum expected future cost $f_*(\varphi)$, as a function of the current state $\varphi(t) = p(t)/(1 - p(t))$, by solving a certain differential equation with special boundary conditions. A heuristic argument, in which one simply assumes that $f_*(\varphi)$ is suitably differentiable, can be given without much difficulty. But, strictly speaking, it is not clear that risk functions such as $f_*(\text{var})$, each of whose values is defined as the infimum of a class of expectations, are sufficiently well behaved. This difficulty is often encountered in statistical applications of dynamic programming to processes in continuous time. Typically, it is extremely difficult to establish the required differentiability properties directly and it is necessary to seek an indirect justification by means of existence and uniqueness theorems. In our case the formal solution can be produced explicitly and, because of this, its justification is much easier. Nevertheless, the approach is complicated by the need to establish several preliminary results, and the discussion of these special properties is limited to a brief indication of the main steps, in the hope that the essential structure of the argument will be more generally useful. Section 2 is concerned with the information process $\{\varphi(t)\}$. Its relation to the observed process $\{\eta(t)\}$ is described and certain properties of its increments are collected for later use. The main argument begins

in Section 3 with a discussion of sub-optimal decision procedures defined by specifying an open continuation set **C** within the space $[0, \infty)$ of possible "initial" states φ. It is shown that any risk function $f(\varphi)$ can be determined for each sub-interval of the corresponding set **C** by solving the basic differential equation appropriately. Sections 4 and 5 consider the special solution $f_*(\varphi)$ which represents the optimal decision procedure, and give the required verification that $f_*(\varphi)$ is uniformly minimal. The final section contains a brief analysis of the operating characteristics of the optimal policy and indicates the importance of evaluating the particular minimum risk $f_*(0)$.

Hudson, Derek J. (1966-1)
Fitting segmented curves whose join points have to be estimated
Journal of the American Statistical Association, 61, 1097-1129
Keywords: Spline; Least squares; Regression.

Abstract: The overall least squares solution is found when a complete curve to be fitted consists of two or more submodels, and these have to be joined at points whose abscissae have to be estimated. Under certain standard conditions, each fitted submodel is itself a local least squares solution, and the overall least squares solution can be found quite easily. The exceptions to this rule are studied. The easiest case to handle occurs when a join point coincides with an abscissa of the given data. In that case it is possible to modify local least squares estimates so that they satisfy an obvious linear constraint (the requirement that the curves do actually join there). If the model is not made up entirely of a mixture of straight lines and constants, we will deal separately with the case where the submodels join together with equal slopes. The solution then requires iterative techniques.

Kander; Z.; Zacks, S.(1966-2)
Test Procedures for Possible Changes in Parameters of Statistical Distributions Occurring at Unknown Time Points
The Annals of Mathematical Statistics, 37, 1196-1210.

Abstract: The present study is concerned with the properties of a test statistic proposed by Chernoff, H. and Zacks, S. [1964, Ann. Math. Statist. 35, 999-1018] to detect shifts in a parameter of a distribution function, occurring at unknown time points between consecutively taken observations. The testing problem we study is confined to a fixed sample size situation, and can be described as follows: Given observations on independent random variables $X_1, \cdots, X_n$, (taken at consecutive time points) which are distributed according to $F(X; \theta_i); \theta_i \varepsilon \Omega$ for all $i = 1, \cdots, n$, one has to test the simple hypothesis: $H_0 : \theta_1 = \cdots = \theta_n = \theta_0$ (θ_0 is known) against the composite alternative:

$$H_1 : \theta_1 = \cdots = \theta_m = \theta_0 \theta_{m+1} = \cdots = \theta_n = \theta_0 + \delta; \quad \delta > 0,$$

where both the point of change, m, and the size of the change, δ, are unknown ($m = 1, \cdots, n-1$), $0 < \delta < \infty$. A Bayesian approach led Chernoff and Zacks in [1964] to propose the test statistic $T_n = \sum_{i=1}^{n-1} i X_{i+1}$, for the case of normally distributed random variables. A generalization for random variables, whose distributions belong to the one parameter exponential family, i.e., their density can be represented as $f(x; \theta) = h(x) \exp[\psi_1(\theta) U(x) + \psi_2(\theta)], \theta \varepsilon \Omega$ where $\psi_1(\theta)$ is monotone, yields the test statistic $T_n = \sum_{i=1}^{n-1} i U(x_{i+1})$. In the present paper we study the operating characteristics of the test statistic T_n. General conditions are given for the convergence of the distribution of T_n, as the sample size grows, to a normal distribution. The rate of convergence is also studied. It was found that the closeness of the distribution function of T_n to the corresponding normal distribution is not satisfactory for the purposes of determining test criteria and values of power functions, in cases of small samples from non-normal distributions. The normal approximation can be improved by considering the first four terms in Edgeworth's asymptotic expansion of the distribution function of T_n (see H. Cramer [1946, Mathematical Methods of Statistics. Prinston Univ. Press.] p. 227). Such an approximation involves the normal distribution, its derivatives and the semi-invariants of T_n. The goodness of the approximations based on such an expansion, and that of the simple normal approximation, for small sample situations, were studied for cases where the observed random variables are binomially or exponentially distributed. In order to compare the exact distribution functions of T_n to the approximations, the exact forms of the distributions of T_n in the binomial and exponential cases were derived. As seen in Section 4, these distribution functions are quite involved, especially under the alternative hypothesis. Tables of coefficients are given for assisting the determination of these distributions, under the null hypothesis assumption, in situations of samples whose size is $2 \leq n \leq 10$. For samples of size $n \geq 10$ one can use the normal approximation to the test criterion and obtain good results. The power functions of the test statistic T_n, for the binomial and exponential cases, are given in Section 5. The comparison with the values of the power function obtained by the normal approximation is also given. As was shown by Chernoff and Zacks [1964], when X is binomially distributed the power function values of T_n are higher than those of a test statistic proposed by E. S. Page [1955, Biometrika, 42,523-526], for most of the m values (points of shift) and δ values (size of shift). A comparative study in which the effectiveness of test procedures based on T_n relative to those based on Page's and other procedures will be given elsewhere for the exponential case, and other distributions of practical interest.

Rao, C. R.(1965-1)
The Theory of Least Squares When the Parameters are Stochastic and Its Application to the Analysis of Growth Curves
Biometrika, Vol. 52, 447-458.

Chernoff, H. and Zacks, S. (1964-1)

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Estimating the current mean of a normal distribution which is subjected to changes in time
The Annals of Mathematical Statistics, 35, 999-1018

Robison, D. E.(1964-2)
Estimates for the Points of Intersection of Two Polynomial Regressions
Journal of the American Statistical Association, 59, 305., 214-224.

Abstract: Let t_* denote an unknown abscissa of intersection of two true regression functions $\mu_1(t)$ and $\mu_2(t)$. Under normality assumptions with no restraints on t_* the maximum likelihood estimator of t_* is shown to be the corresponding intersection of the sample regressions. When this estimate exists confidence intervals J can usually be obtained for t_* by an application of the Student t-distribution. When t_* is restrained to some known interval I, the ML estimate may or may not fall in I. A restrained ML estimate proposed is the limiting point of $I \bigcap J$ as the length of $I \bigcap J$ approaches zero. Confidence limits are obtained for the restrained estimate. Many practical difficulties are discussed.

Shiryaev, A.N.(1963-1)
On Optimum Methods in Quickest Detection Problems
Theor. Probability Appl. 8, 22 - 46

Abstract: In this paper optimum methods are developed for observing a process (1), in which the moment when a disorder θ appears is not known. The basic quantity characterizing the quality of this observation method is the mean time delay τ for detection of a disorder. After making assumption (4) it is shown that for a given false alarm probability ω or for a given $\mathbf{N}$ mathematical expectation of false alarm numbers occurring up to the moment the disorder appears the observation method minimizing $\tau = \tau(\omega)$ or $\tau = \tau(\mathbf{N})$ is based on an observation of a posteriors probability (23). In 3 a case is considered, wherein, the disorder appears on the background of steadystate conditions arising when the disorder is absent. A method is found for minimizing $\boldsymbol{\tau} = \boldsymbol{\tau}(\mathbf{T})$ for a set $\mathbf{T}$ mathematical expectation of the time between two false alarms. The dependence $\tau = \tau(\mathbf{T})$ is given by formula (36).

Sprent, P.(1961-1)
Some Hypotheses Concerning Two Phase Regression Lines
Biometrics, Vol. 17, No. 4., pp. 634-645.

Chow, G. C. (1960-1)
Tests of equality between sets of coefficients in two linear linear regressions
Econometrica, 28, 591-605

Abstract: Having estimated a linear regression with p coefficients, one may wish to test whether m additional observations belong to the same regression. This paper presents systematically the tests involved, relates the prediction interval (for m = 1) and the analysis of covariance (for m ¿ p) within the framework of general linear hypothesis (for any m), and extends the results to testing the equality between subsets of coefficients.

Quandt, Richard E. (1960-2)
Tests of the hypothesis that a linear regression system obeys two separate regimes
Journal of the American Statistical Association, 55, 324-330
Keywords: Spline regression; Time-varying parameter

Abstract: Several approaches are explored for testing the hypothesis that no switch has occurred in the true values of the parameters of a linear regression system. The distribution of the relevant likelihood ratio λ is analyzed on the basis of the empirical distribution resulting from some sampling experiments. The hypothesis that -2 log λ has the χ^2 distribution with the appropriate degrees of freedom is rejected and an empirical table of percentage points is obtained. Finally some small sample tests are suggested.

Barnard, G. A.(1959-1)
Control Charts and Stochastic Processes
Journal of the Royal Statistical Society. Series B (Methodological), Vol. 21, No. 2. pp. 239-271.

Abstract: A new approach to control charts for industrial processes is suggested, based on the idea that the underlying stochastic process should be specified, its parameters estimated, and the control procedure related to these. Optimum estimators are derived, and practical approximations to these are indicated.

Quandt, Richard E. (1958-1)
The estimation of the parameter of a linear regression system obeying two separate regimes
Journal of the American Statistical Association, 53, 873-880
Keywords: Spline regression; Time-varying parameter

Abstract: In attempting to estimate the parameters of a linear regression system obeying two separate regimes, it is necessary first to estimate the position of the point in time at which the switch from one regime to the other occurred. The suggested maximum likelihood estimating procedure is based upon a direct examination of the likelihood function. An asymptotic and a small-sample test are suggested for testing the hypothesis that no switch occurred against

the single alternative that one switch took place. The procedure is illustrated with a sampling experiment in which the true switching point is correctly estimated.

Page, E. S.(1957-1)
On Problems in which a Change in a Parameter Occurs at an Unknown Point
Biometrika, 44, No. 1/2., 248-252.

Abstract: Given a sample of n independent observations in the order in which they were obtained, $x_1,...,x_n$; we consider the problem of testing the null hypothesis that the observations come from the same population with distribution function $F(x|\theta)$ against the alternatives that the first m ($0 \leq m < n$), $x_1,...,x_m$ come from $F(x|\theta)$ and $x_{m+1},...,x_n$ come from $F(x|\theta')$ ($\theta' \neq \theta$), where m is unknown. In an earlier paper (Page, 1955) a one-sided test for a change in the mean of a distribution was proposed; the procedure was to record the cumulative sum $S_r = \Sigma_{j=1}^r (x_j - \theta)$, and, if the mean path after the change had a greater slope than that before the change, to use as a test statistics the rise in the cumulative sum above its least value, i.e. $\max_{0 \leq r \leq n}(S_r - \min_{0 \leq i < r} S_i)$, $S_0 = 0$, large values being significant. Some critical values were given for the case where the x_i are 0 or 1 binomial variables. Here, a discrimination approach is adopted for the more general case and the procedure is shown to yield a modification of the previous test.

Page, E. S.(1955-1)
A Test for a Change in a Parameter Occurring at an Unknown Point
Biometrika, 42, No. 3/4., 523-527.

Abstract: Consider a sample of independent observations in the order in which they were obtained, $x_1 ...x_n$; it is sometimes required to test the null hypothesis that all the observation are drawn from the same population with distribution function $f(x|\theta)$ against the alternative that $x_1,...,x_m$ come from $F(x|\theta)$, and $x_{m+1}, ..., x_m$ from $F(x|\theta')$ ($\theta' \neq \theta$). If m is unknown; this raises new problems. A test is proposed for a case where θ is known and some comments are made on the problems presented by other cases.

Page, E. S.(1954-1)
Continuous Inspection Schemes
Biometrika, 41, No. 1/2., 100-115.

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